# Space Tower Launch Costs

Team Number 352

Problem A

#### Abstract

We determine the effects of changing launch altitude on the total cost of launching a 10,000 kg payload on a traditional rocket into low-earth orbit from an from an arbitrarily high tower. We address two aspects of this problem: vertical atmospheric launch and space-based launch tangent to Earth's surface. In the former case, we simulate the atmospheric flight of a representative two-stage medium-launch rocket based on SpaceX's Falcon 9 on the way to a circular orbit with an altitude of 300 km, and we compute the savings in  $\Delta v$  as a function of launch height. In the latter case, we compute the requisite fuel mass to take the 10,000 kg payload from the tower to a Hohmann transfer orbit terminating in a circular orbit at an altitude of 300 km. In both cases, we also compute the cost of lifting the space vehicle to various launch heights. We conclude that because the space-based launcher, with a tower height between 100 and 300 km, has a mass and  $\Delta v$  much lower than the atmospheric rocket, its elevation costs and  $\Delta v$  savings add up to a monetary cost considerably less than the atmospheric launcher. Assuming total mission cost is proportional to the mass of the launcher and elevation costs are negligible, at a payload launch cost of \$15,000/kg we estimate that the cost of launching a rocket from a tower above 100 km is only about \$19 million, or about \$1,900 per kilogram of payload.

# 1 Interpretation of Problem A

Our goal is to determine the total cost of sending a 10,000 kg payload on a traditional rocket into a low-Earth orbit (LEO) by launching from towers of differing heights. To compare these costs, we must design the launch the rocket will take to achieve its final orbit and calculate the amount of fuel required to carry out such a procedure as a function of launch height. We must then estimate total cost of launching the 10,000 kg payload into a given low Earth orbit as a function of launch height. We will use the reference cost of (15,000/kg) × (10,000 kg) =  $1.5 \times 10^8$  to send a 10,000 kg payload into LEO.

# 2 Assumptions

First, we take our target LEO to be a circular orbit 300 km above Earth's surface [5].

The construction of actual rockets is complex and beyond the scope of this analysis, so we chose to base our model on SpaceX's Falcon 9 (described in [2]), a representative two-stage medium-lift rocket capable of transporting the requisite 10,000 kg payload to LEO. We name our fictitious spacecraft Falcon 9 Surrogate.

See Table 1 for the characteristics we chose for Falcon 9 Surrogate. The table includes the frontal cross-sectional area  $A_f$ , the drag coefficient  $C_D$ , the mass of the payload  $m_p$  from the problem statement, the mass of stage 1 of the rocket when empty  $m_e^{(1)}$  and when full of fuel  $m_f^{(1)}$ , and the mass of stage 2 of the rocket when empty  $m_e^{(2)}$  and when full of fuel  $m_f^{(2)}$ .

We also assume the World Geodetic System WGS84 values in [7] for such constants as Earth's radius,  $R_E = 6378$  km, the Earth's angular velocity,  $\Omega_E = 7.29 \times 10^{-5}$  rad/s, and the product of the gravitational constant and Earth's mass, which we denote as  $\mu = GM =$  $3.986 \times 10^5$  km<sup>3</sup>/s<sup>2</sup>. In addition we assume that atmospheric density around Earth at a



Figure 1: Photograph of the Falcon 9 during liftoff.

$A_f (\mathrm{m}^2)$	$C_D$	$m_p$	$m_e^{(1)}$	$m_f^{(1)}$	$m_{e}^{(2)}$	$m_{f}^{(2)}$
$\pi(2.3)^2$	0.3	$10,000 \ {\rm kg}$	$20,000 \ {\rm kg}$	$342,\!859 { m ~kg}$	3000 kg	$54{,}000~{\rm kg}$

Table 1: Specifications of our fictional spacecraft, the Falcon 9 Surrogate.

radius r from Earth's center is approximately

$$\rho = \rho_0 \, \exp\left(\frac{r - R_E}{H}\right),\tag{1}$$

where we use H = 8 km as the approximate scale height of Earth's atmosphere (referring to [7]) and  $\rho_0 = 1.225$  kg/m<sup>3</sup> as the atmospheric density at sea level [1]. Furthermore, when we consider the equations of motion for the Falcon 9 Surrogate during launch, we will only consider the atmospheric drag effects when the rocket is below the Kàrmàn line at 100 km above Earth's surface [4]. This marks the end of the "sensible atmosphere" for our purposes, because drag forces are negligible beyond this line. To work this assumption into our code later on, we will alter our formula for  $\rho$  from the one in equation (1) to

$$\rho = \rho_0 \exp\left(\frac{r - R_E}{H}\right) \times \Theta(h),$$

where  $\Theta(h)$  is the step function

$$\Theta(h) = \begin{cases} 1 & \text{if } h \le 100 \text{ km}, \\ 0 & \text{if } h > 100 \text{ km}. \end{cases}$$

# **3** Launch Mechanics

Since we launch the Falcon 9 Surrogate at the equator, we can restrict our attention to the plane that contains the trajectory of the rocket. Working in two spatial dimensions, we describe the location of the rocket using polar coordinates  $r, \theta$  where r measures distance from the center of Earth and  $\theta$  is the azimuthal angle, increasing eastwardly. We will also use the unit vectors  $\hat{r}, \hat{\theta}$  which point in the directions of increasing  $r, \theta$  respectively. Throughout our analysis of the rocket's launch trajectory, we let v denote its speed.

## **3.1** External Forces

During its launch, several forces act on the Falcon 9 Surrogate. First, gravity pulls the rocket towards the center of the Earth with ([3])

$$\vec{F}_g = -\frac{\mu m}{r^2} \hat{r}.$$
(2)

Here, m varies with time since the Falcon 9 Surrogate expels fuel during the launch, and is given by

$$m(t) = m_0 + \int_0^t \dot{m} \, dt, \tag{3}$$

where  $m_0$  is the total initial mass of the rocket including all its fuel and  $\dot{m}$  is the rate at which this total mass changes. Note that  $\dot{m}$  is negative, since total mass decreases as fuel is expelled.

In addition, at relatively low altitudes the Falcon 9 Surrogate experiences considerable drag. The drag forces are proportional to square of the velocity of the rocket with respect to the atmosphere. We assume the atmosphere rotates with the angular velocity of Earth,  $\Omega_E$ , so we calculate the relative velocity of the rocket with respect to the atmosphere as

$$\vec{v}_{\rm rel} = v_r \,\hat{r} + (v_\theta - r\,\Omega_E)\,\hat{\theta},\tag{4}$$

where  $v_r, v_\theta$  are the polar components of the velocity of the rocket. Then the drag force is given by

$$\vec{D} = -\frac{1}{2}\rho(r)A_f C_D v_{\rm rel}^2 \,\hat{v}_{\rm rel},\tag{5}$$

where  $\hat{v}_{rel}$  is the unit vector in the direction of  $\vec{v}_{rel}$  [3].

Furthermore, while the Falcon 9 Surrogate expels fuel, it creates a thrust force parallel to the body of the rocket. Now, while the rocket travels through the atmosphere, its orientation changes. We let  $\alpha$  denote the angle between  $\hat{r}$  and a unit vector pointing along the body of the rocket, letting  $\alpha$  take a positive value if the rocket has rotated counterclockwise with respect to  $\hat{r}$ . Thus, the thrust force  $\vec{T}$  is given by the so-called "rocket equation,"

$$\vec{T} = -v_e \dot{m} (\cos \alpha \, \hat{r} + \sin \alpha \, \hat{\theta}), \tag{6}$$

where  $v_e$  is the velocity of the exhaust relative to the rocket [3]. From equation (6) we can calculate the change in speed of a rocket when the only force it experiences is thrust. In that case the total external force is  $\vec{T}$ , so change in speed can be found by integrating  $m\dot{v} = T$ . From the time fuel begins burning until the engine cuts off

$$\Delta v = v_e \ln\left(\frac{m_i}{m_f}\right),\tag{7}$$

where  $m_i$  is the total mass of the rocket and fuel and  $m_f$  is  $m_i$  minus the mass of the fuel burned.

Moreover, during the flight, if  $\alpha$  is nonzero and if the Falcon 9 Surrogate is not programmed to keep  $\alpha$  constant, then gravity will cause the spacecraft to rotate with ([3])

$$\dot{\alpha} = \frac{\mu \sin \alpha}{v r^2}.$$
(8)

This is know as a "gravity turn."

## **3.2** Equations of Motion

We will consider several regimes of the launch trajectory. While the Falcon 9 Surrogate is below the Kàrmàn line with fuel burning, we include the drag and thrust forces to obtain the equation of motion,

$$m\ddot{\vec{r}} = \vec{F}_g + \vec{D} + \vec{T},\tag{9}$$

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which gives

$$\ddot{r} = -\frac{\mu m}{r^2} - \frac{1}{2} \rho(r) A_f C_D v_{\rm rel} v_r - v_e \dot{m} \cos \alpha,$$
  
$$r\ddot{\theta} = -\frac{1}{2} \rho(r) A_f C_D v_{\rm rel} (v_\theta - \Omega_E r) - v_e \dot{m} \sin \alpha.$$

If the spacecraft has exited the sensible atmosphere and continues to burn fuel, we disregard  $\vec{D}$  to obtain the equation

$$m\ddot{\vec{r}} = \vec{F}_g + \vec{T},\tag{10}$$

or

$$\ddot{r} = -\frac{\mu m}{r^2} - v_e \dot{m} \cos \alpha,$$
  
$$r\ddot{\theta} = -v_e \dot{m} \sin \alpha.$$

Above the Kàrmàn line, once the rocket engines cut off,  $\vec{T}$  vanishes and our equation of motion reduces to

$$m\ddot{\vec{r}} = \vec{F}_g.$$
 (11)

## 3.3 Free Fall

When its behavior is dictated by equation (11), our spacecraft is in free fall and will take part in a closed elliptical orbit (since it will never reach escape velocity). In such an orbit, we can write the energy of Falcon 9 Surrogate it two ways,

$$E = \frac{mv^2}{2} - \frac{\mu m}{r} = -\frac{\mu m}{2a},$$

where a is the semi-major axis of the ellipse [3]. This gives an equation for the spacecraft's speed,

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a}\right)}.$$
(12)

As the rocket orbits Earth, the center of Earth will coincide with one of the foci of the orbit ellipse. When the spacecraft has a given energy E and angular momentum L, the eccentricity of the orbit ellipse is given by

$$e = \sqrt{1 + \frac{2EL^2}{\mu^2 m^3}}.$$

The apogee  $r_a$  of the orbit is the distance from Earth's center to the farthest point on the orbit ellipse and is given by

$$r_a = (1+e)a,\tag{13}$$

a function of E and L. When the Falcon 9 Surrogate reaches the apogee of its orbit, it's altitude is given by  $h = r_a - R_E$ .

## 3.4 Hohmann Transfer

Later in the analysis, we will need a method to transfer Falcon 9 Surrogate from one circular orbit to another with larger radius. We will use something similar to a "Hohmann transfer." In a Hohmann transfer an instantaneous boost is fired at one point of the initial circular orbit with  $\Delta v_1$  so that the rocket enters an elliptical orbit that will tangentially intersect the target circular orbit after one half-period. At the point of intersection, a second instantaneous boost is fired with  $\Delta v_2$  such that the spacecraft enters desired orbit. According to [3], the  $\Delta v$ 's required for this are given by

$$\Delta v_1 = \sqrt{\frac{\mu}{r_i}} \left( \sqrt{\frac{2r_f}{r_i + r_f}} - 1 \right), \tag{14}$$

$$\Delta v_2 = \sqrt{\frac{\mu}{2_f}} \left( 1 - \sqrt{\frac{2r_i}{r_i + r_f}} \right),\tag{15}$$

where  $r_i$  and  $r_f$  are the radii of the initial and final circular orbits of the rocket.

# 4 Launch Design

We consider two regimes of tower heights and design a distinct launch strategy for each of them. First we consider towers that do not extend above the Kàrmàn line, and next we consider the case of a tower that climbs outside the sensible atmosphere.

## 4.1 Launching from Below the Kàrmàn Line

We design the trajectory Falcon 9 Surrogate will take during each of its two fuel-burning stages.

#### 4.1.1 Stage 1

In Stage 1, we burn with  $\dot{m} \approx 720$  kg/s and  $v_e \approx 4020$  m/s to achieve a thrust of magnitude  $T \approx 2890$  kN.

We begin with a vertical liftoff for the first ten seconds of flight. During this period, the motion of the rocket is dictated by equation (9) with  $\alpha$  held at 0.

At t = 10 s, the spacecraft begins a programmed turn. Since the rocket is still below the Kàrmàn line during this phase, the trajectory is determined again by equation (9), but here  $\alpha$  changes at the prescribed rate of  $\dot{\alpha} = 0.5^{\circ}/\text{s}$ . This continues until  $\alpha = 8.5^{\circ}$ .

At that point, we allow the Falcon 9 Surrogate to undergo a gravity turn during which  $\dot{\alpha}$  is given by equation (8). Before the rocket exits the sensible atmosphere, its motion is dictated by equation (9); however, when  $r - R_E$  exceeds 100 km, we must use equation (10) to determine the spacecraft's motion.

#### 4.1.2 Between Stages

The rocket continues to burn fuel above the Kàrmàn line until it attains an energy and an angular momentum sufficient for an elliptical orbit with apogee equal to the target orbit radius. That is, the engines cut off when E and L are such that  $r_a$  in equation (13) exceeds 300 km. At this point the spacecraft free falls in an elliptical orbit, obeying equation (11). When Falcon 9 Surrogate reaches the apogee of this orbit, we burn fuel once more, and Stage 2 begins.

#### 4.1.3 Stage 2

In Stage 2, we burn with  $\dot{m} \approx 24.3$  kg/s and  $v_e \approx 4530$  m/s to achieve a thrust of magnitude  $T \approx 110$  kN. This thrust is directed along  $\hat{\theta}$ . To simplify calculations, we assume that Stage 2 occurs as an impulsive burn, neglecting the duration of time during which it takes place. At this step, enough fuel must be exhausted to impart to the rocket the  $\Delta v$  required to transfer from its elliptical orbit to the target circular orbit. We calculate the necessary  $\Delta v$  from equation (12). When Falcon 9 Surrogate reaches the apogee of its elliptical orbit,  $r = r_a$ , so its speed is given by

$$v_i = \sqrt{\mu \left(\frac{2}{r_a} - \frac{1}{a}\right)}.$$
(16)

In order to transfer to a circular orbit of radius  $r_a$ , the spacecraft should be moving with speed

$$v_f = \sqrt{\mu \left(\frac{2}{r_a} - \frac{1}{r_a}\right)} = \sqrt{\frac{\mu}{r_a}}.$$
(17)

Subtracting equation (16) from (17) gives the requirement

$$\Delta v = v_f - v_i = \sqrt{\frac{\mu}{r_a}} - \sqrt{\mu \left(\frac{2}{r_a} - \frac{1}{a}\right)}.$$

## 4.2 Launching from Above the Kàrmàn Line

When launching Falcon 9 Surrogate from an altitude h outside the sensible atmosphere, we use a different technique to get the rocket into its target orbit. Since we no longer need to consider drag at this stage, we choose a horizontal launch, with liftoff tangent to Earth's surface. Before liftoff, our spacecraft is traveling in a circular path around Earth's center at a speed  $(R_E + h)\Omega_E$ . We will mimic a Hohmann transfer, first boosting with  $\Delta v_1$  to enter an elliptical orbit that will tangentially intersect the desired circular orbit at altitude 300 km after one half-period, and second boosting with  $\Delta v_2$  when the intersections occurs to enter the target orbit. To write down the equation for  $\Delta v_1$  we must modify equation (14) because the rocket must first accelerate to the correct speed for a circular orbit at the launch altitude. Since  $\Delta v_1$  must include this change in velocity, we modify equation (14) to get

$$\Delta v_1 = \left(\sqrt{\frac{\mu}{R_E + h}} - (R_E + h)\,\Omega_E\right) + \sqrt{\frac{\mu}{R_E + h}}\left(\sqrt{\frac{2r_f}{R_E + h + r_f}} - 1\right),$$

where  $r_f = R_E + 300$  km is the radius of the target circular orbit. This simplifies to

$$\Delta v_1 = \sqrt{\frac{\mu}{R_E + h}} \sqrt{\frac{2r_f}{R_E + h + r_f}} - (R_E + h) \Omega_E.$$

We take  $\Delta v_2$  straight from equation (15) to get

$$\Delta v_2 = \sqrt{\frac{\mu}{r_f}} \left( 1 - \sqrt{\frac{2(R_E + h)}{R_E + h + r_f}} \right).$$

# 5 Numerical Analysis

# 5.1 Simulation and Results



Figure 2: Launch trajectories of our Falcon 9 surrogate from various altitudes between 0 and 100 km viewed from the south in a co-rotating frame. Each trajectory is plotted from launch until the first engine cutoff. All trajectories lead to an apogee of 300 km.

We divide our analysis into two parts: We first model the flight of Falcon 9 Surrogate at various starting altitudes below the Kàrmàn line, propagating its equations of motion forward until the state vector is such that the rocket will have a ballistic trajectory with an apogee at the target orbit's altitude. Above the Kàrmàn line, the effects of atmospheric drag are negligible; thus we modeled launches from above 100 km as impulsive maneuvers in the tangential direction using an ideal rocket with a mass commensurate to the  $\Delta v$  required to enter a Hohmann transfer orbit with apogee at the target orbit.



**Figure 3:** Launch trajectories of Falcon 9 Surrogate from various altitudes between 0 and 100 km viewed from the north in an inertial frame. Each trajectory is plotted from launch until the first engine cutoff. All trajectories lead to an apogee of 300 km. Note the effect of the rotation of Earth on the initial rocket velocity.

Because the equations of motion for the rocket through the Earth's atmosphere cannot be solved analytically, we chose to write a Python integrator to model the flight of the rocket.

At the start of each atmospheric launch, we give the rocket a thrust in the vertical direction. After 10 s, the rocket pitches to the east at  $0.5^{\circ}$ /s until its zenith angle is  $8.5^{\circ}$ . At this point, the rocket begins a gravity turn defined by Equation (8). The values of the pitch rate and threshold pitch were determined by manual fine tuning. The model propagated the equations of motion until the position and velocity of the rocket were such that it had a trajectory with an apogee at the target orbit.

Above the Kàrmàn line, the rocket was modeled as an ideal rocket with only a payload and fuel mass. We make no assumptions of empty stage or payload fairing masses. The rocket's trajectory was as a Hohmann transfer orbit between the tower rotating with the earth and the target orbit. The rocket's mass was computed such that it had exactly enough fuel to achieve this. This meant that the initial total mass was, on average 51,000 kg.

### 5.2 Cost Analysis

The cost of building and launching a rocket has fixed costs and variable costs. The fixed costs have to do with labor, insurance, etc., while the variable costs depend on the type of mission, the rocket, and the payload. This analysis ignores all fixed costs and assumes a cost model dependent only on the mass of the spacecraft and the energy used to transport it. We also assume that the cost of building and maintaining the tower is a sunken cost and



**Figure 4:** Acceleration (a), velocity (v), altitude (h), zenith angle ( $\alpha$ ), and mass (m) for the Falcon 9 Surrogate launched from h = 0 km, all versus flight time. In the first two figures, the blue line represents radial component while green line shows the azimuthal component.

does not factor into this analysis. We use

$$C_T = C_0 + C_{\Delta v} + C_m + C_{\Delta h}$$

The total cost,  $C_T$ , is the sum of the fixed cost,  $C_0$ ; the mission-specific cost,  $C_{\Delta v}$ ; the cost of manufacturing, assumed to grow linearly with the mass of the launch vehicle,  $C_m$ ; and the cost of elevating the spacecraft,  $C_{\Delta h}$ . With the atmospheric Falcon 9 Surrogate,  $C_m$  is absorbed into the fixed costs because the spacecraft's initial mass does not change. Given



Figure 5: Various  $\Delta v$  vs. launch altitude plots for the tower-launched Falcon 9 Surrogate ( $h_0 < 100$  km) and the ideal tower-launched rocket  $h_0 > 100$  km. The burnout  $\Delta v$  is an ideal value computed from the fuel burned getting the rocket to the requisite altitude and velocity for an apogee at a 300 km altitude. The "burnout v" is the actual velocity of the rocket at burnout; the difference between the burnout v and the burnout  $\Delta v$  illustrates the effects of atmospheric drag and gravity. The apogee  $\Delta v$  is the  $\Delta v$  at apogee required to circularize the orbit. The sum of the apogee and burnout  $\Delta v$  is the total  $\Delta v$ , which decreases as a function of launch altitude. Above 100 km, the elevator  $\Delta v$  is the sum of both  $\Delta v$ s required to get the ideal tower-launched rocket in and out of a Hohmann transfer orbit terminating in a circular orbit with a 300 km altitude.

a \$ 6.1 million difference in launch costs for a Falcon 9 between LEO and GEO and that the difference in  $\Delta v$  is calculated to be about 4 km/s, we adopt a marginal cost of  $\Delta v$  for the Falcon 9 Surrogate of  $\frac{dC}{d\Delta v} =$ \$1500 per m/s. The cost of elevation has been estimated in work on space elevators at (from [6]) \$ 220/kg at a change of about  $\approx 50$  MJ/kg of potential between the surface and geosynchronous orbit. This means the cost of elevation is \$4.4 × 10<sup>-6</sup> per joule of potential energy gained. From our previous results, we know the Falcon 9 Surrogate saves about 3 m/s of  $\Delta v$  for every additional kilometer of launch height. This means that every additional kilometer of height saves the Falcon 9 Surrogate about \$5,000 worth of  $\Delta v$ . The cost of elevating the fully-loaded Falcon 9 Surrogate, however, increases much more rapidly at about \$17,000/km. The total cost of elevating and launching the Falcon 9 Surrogate is therefore \$12,000/km. The tower thus does not save money below the Kàrmàn line.

The ideal rocket launched horizontally above the Kàrmàn line, however, sees a dramatic reduction in calculated mass as its launch height is increased, reflecting the increasing velocity of the tower due to the rotation of Earth and lower orbital velocities as altitude increases. Given that the cost of elevation is directly proportional to the mass of the rocket, which ranges between 50,000 and 52,000 kg (roughly 12% of the mass of the Falcon 9 Surrogate)



the elevation costs grow much more slowly than those of the Falcon 9 Surrogate at about 2,000/km. The  $\Delta v$  savings grow much more slowly and are swamped by the elevation costs.

Given that changing the altitude of the launch height does not change the total cost by more than \$ 2 million, a small fraction of the actual launch costs, we drop the  $C_m$  and  $C_{\Delta h}$ terms from our cost function and make the cost dependent only on mass and fixed cost. Given a specific payload cost of \$15,000/kg, a 10,000 kg payload, and a total mass of our Falcon 9 Surrogate of 407,000 kg, we compute a cost per unit of bulk launcher mass of \$368 /kg. Multiplied by the average mass of the space-based rocket, 51,000 kg, we get a total cost of about \$19 million, about 13% of the cost of the ground-based launch.

# 6 Discussion

## 6.1 Weaknesses

Our approach to this problem comes with a few weaknesses.

First, our calculations assume a launch from the equator directly towards the east. In the real world, our launch tower would be at least slightly displaced from the equator. In that case, the motion of the spacecraft could not be restricted to a plane and a three-dimensional analysis would be necessary.

Next, we assume the Stage 2 fuel-burn creates a significant  $\Delta v$  in an infinitesimal time  $\delta t$ . In reality, the engines would be on for a significant period of time and a careful treatment of the transfer from an elliptical orbit to a circular one would be required.

In addition, our method of numerical solution of the equations of motion could be better.

We use a basic for-loop with time-step  $\Delta t = 0.1$  s

Most importantly, our approach for towers below the Kàrmànline gives insight into the first-order energy saving effects of launching Falcon 9 Surrogate from a tower rather than the ground. This is because we fix the mass of the spacecraft and compare the  $\Delta v$ 's we require from the fuel during the entire launch trajectory when lifting off from towers of differing heights. To study higher-order effects, we should also vary the mass of the spacecraft for towers of differing heights.

## 6.2 Strengths

When Falcon 9 Surrogate lifts off from a tower instead of the ground, its trajectory changes most notably in the low altitude regime of the launch. Our model limits unnecessary approximations in this phase of the launch so that we can make the most accurate estimate of the cost differences in lifting off from towers of differing heights. To ensure accuracy of our model in the atmosphere, we used a two-dimensional analysis instead of using a simple 1D liftoff. Also, we take into account the rotation of Earth and the rotation of the atmosphere along with it to estimate drag effects most accurately.

In addition, our model uses a procedure similar to that taken for sending real satellites into orbit [3]. Thus, we can claim that we obtain meaningful results that approximate the real economic effects of increasing the liftoff height of a real launch.

Furthermore, when we assume that towers extending above the Kàrmànline could possibly be constructed, we adjust our launch trajectory to account for the change in conditions. A liftoff tangent to Earth's surface is much more energy efficient than a vertical launch in this regime since atmospheric drag is no longer an issue. Thus, our model produces results that have meaning in each regime we studied.

Moreover, we modeled the Falcon 9 Surrogate in the style of a traditional rocket. Therefore the costs we estimated can be directly compared to current costs of sending payload into LEO.

# 7 Closing Remarks

We find that elevation costs and  $\Delta v$  savings increase with altitude in both cases, though the elevation costs increase much more quickly. These costs, however, are negligible compared to the cost savings from the dramatically reduced mass of the space-based launcher. Because the space-based launcher does not have to carry its fuel with it be lifted out of Earth's gravity well and through its atmosphere, it can afford to be much less massive than the atmospheric launcher. Although the incredible costs and engineering challenges of building a tower extending into space cannot be understated, the savings of nearly 90% on launch costs make this option undeniably attractive.

# References

[1] M. CAVCAR. The International Standard Atmosphere (ISA), Anadolu University, Turkey. (2000) URL: http://home.anadolu.edu.tr/ mcavcar/common/ISAweb.pdf

- [2] "Falcon 9 Launch Vehicle Payload User's Guide", Rev. 1, Space Exploration Technologies Corporation. (2009). URL: http://www.spacex.com/Falcon9UsersGuide\_2009.pdf
- [3] F. J. HALE. Introduction To Space Flight, (Pearson Education, New Jersey, 1994).
- [4] "Dryden Flight Research Center", National Aeronautics and Space Administration website. 21 October, 2005. URL: http://www.nasa.gov/centers/dryden/news/X-Press/stories/2005/102105\_Schneider.html
- [5] "NASA Safety Standard 1740.14, Guidelines and Assessment Procedures for Limiting Orbital Debris", Office of Safety and Mission Assurance. 1 August 1995. URL: http://www.orbitaldebris.jsc.nasa.gov/library/NSS1740\_14/nss1740\_14-1995.pdf
- [6] D. RAITT AND B EDWARDS. "The Space Elevator: Economics and Applications", 55th International Astronautical Congress, Vancouver, Canada. (2004).
- [7] World Geodetic System, 1984, The National Geospatial-Intelligence Agency (2011). URL: http://earth-info.nga.mil/GandG/wgs84/index.html.