

# Analysis of Biological Structures in Hypergravity Environments

Team 611: Problem B

## Abstract

In this paper, we produce an analysis of how life would evolve differently on a planet with a mass eight times that of earth and a radius two times that of earth. We address this analysis by considering the effects of increased gravity on the dynamics and mechanics of the cardiovascular and musculoskeletal systems. We investigate the cardiovascular system by considering laminar flow in large and medium sized vessels, using the Navier-Stokes equations to model the blood pressure gradient. Additionally, we use scaling arguments of allometry to estimate how blood volume and capillary dimensions vary with gravity. We investigate the musculoskeletal system by considering a single bone and muscle extending from the animal body. We decompose bone stress into normal and shear components and treat muscle stress through equilibrium arguments to estimate changes in bone cross sectional area. This model is then used to constrain possible values of bone length and attachment angle. Additionally, we analyze the dynamics of legged locomotion using Froude numbers to demonstrate changes in gait as a function of gravitational acceleration. Finally, the effects additional factors including changes in atmospheric pressure and temperature are considered. Given these considerations, we propose that animals that evolve in this environment would be characterized by bones with twice the cross sectional area and approximately 30% less length than their earth counterparts. Leg bones would attach to the body at shallower angles, blood volume would approximately double and capillary size would increase, organ and heart size would increase, and animals would be shorter and more compact to compensate for higher blood pressure gradients. Additionally, we model gait as a function of gravitational acceleration to demonstrate that it would be increasingly difficult for animals to reach the most efficient modes of transportation, likely decreasing maximum speed.

# 1 Interpretation of the Problem

We will investigate how the structure of animals varies when the mass and surface area of a planet increases of factors of eight and two, respectively. Our main consideration is variation in skeletal structure and blood flow due to the change in surface gravity. We discuss physiological changes relative to an equivalent animal on earth, assuming that these changes will give the new animal the same functionality as the equivalent earth animal.

## 1.1 Introduction

Recent discoveries by instruments such as the Kepler Telescope have revealed an incredible number of exoplanets, many of which are earthlike. Of these, a significant number have been found in the habitable zone, allowing for the possibility of life. While life may exist on these planets, it is possible that the conditions under which it forms and thrives differ significantly from those on earth. In this paper, we consider how creatures on a habitable planet of eight times the mass and twice the radius of earth would differ from those found on earth.

In this paper, we begin by considering gravity and its direct and indirect effects on the biological systems of terrestrial homeotherms. We propose that the factors which most greatly constrain and evolution and structure of life on this exoplanet will be changes in the cardiovascular system, the musculoskeletal system, and atmospheric pressure. We describe each system independently, producing a mathematical model of the effects of gravity on each of these three factors, before combining the results to predict such factors as bone dimensions and blood volume. Throughout the paper, we presume that evolution on this planet will produce animals with approximately the same functions as those on earth.

# 2 Model

## 2.1 Assumptions and Overview

In this paper, we treat the planet as a sphere surrounded with a thin atmosphere. Given an exoplanet with a mass eight times that of earth and a radius that times of earth, it is simple to show that gravitational acceleration at the surface will scale as

$$g \sim \frac{M}{R^2}$$

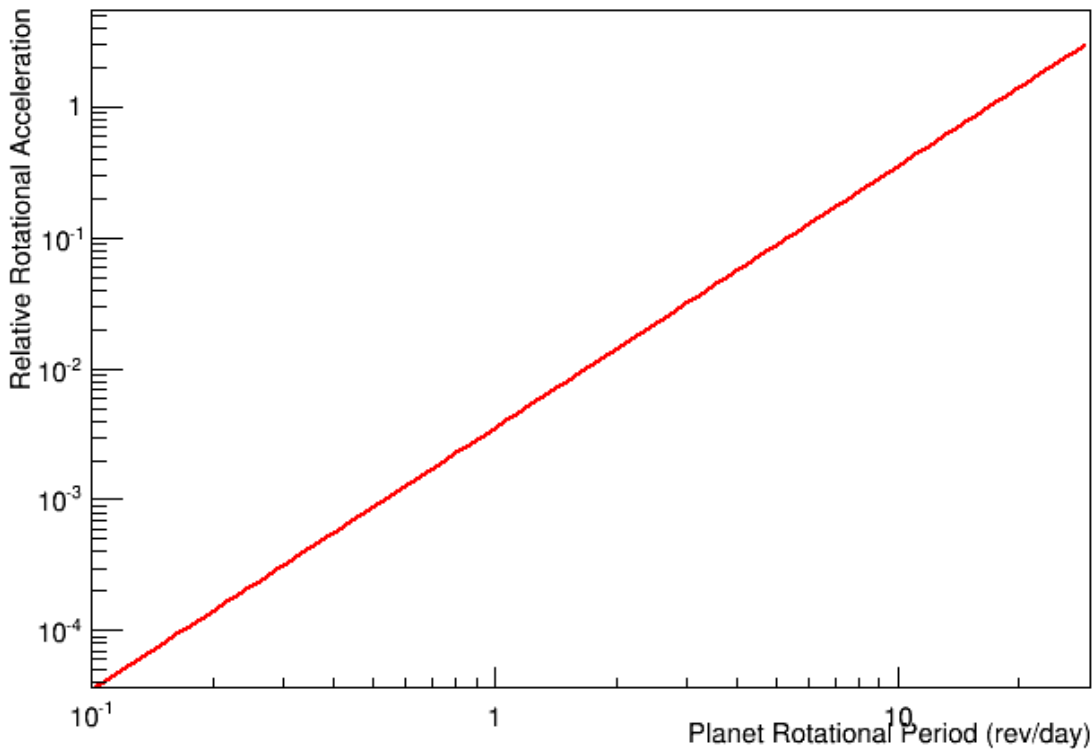
Hence gravitational acceleration at the surface will be two times that of earth. We have also considered the centripetal acceleration caused by the rotational motion of the planet. The centripetal acceleration is given by

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Since  $r$  is larger in the case of the exoplanet, it is important to consider if the centripetal acceleration could become a significant factor. We have plotted  $\frac{a_{c,exo}}{g_{exo}}$  (i.e. the relative rotational

From this plot, we observe that the exoplanet would have to be rotating at least 10 times faster than earth for the centripetal acceleration to factor significantly into the forces experienced at the surface. Hence we assume that the rotation rate does not significantly contribute to the acceleration of objects near the surface of the exoplanet.

### Acceleration vs. Rotation Period



**Fig. 2.1.1** Rotational acceleration as a function of planet rotation.

Given such a drastic change in gravitational acceleration, we assume that difference in gravity accounts for the most significant differences between the organisms on this exoplanet and those on earth. It is thus important to consider the factors which are most influenced by these changes. In this paper, we consider:

- (1) Cardiovascular System: The cardiovascular system must be capable of delivering blood throughout the body and removing waste products. Blood must be pumped against gravity to body parts above the heart and brought back from lower extremities against gravity to prevent pooling. This is accomplished through the force of contraction in the heart and the elasticity and dimensions of arterial blood vessels. We model blood flow mathematically to predict changes in blood pressure and other changes in the cardiovascular system.
- (2) Skeletal System: Perhaps the most obvious structural requirement of an organism is adequate support of body mass. When the force of gravitational loading exceeds

cohesive forces holding together a structure, that structure will collapse. Thus, in a vertebrate it is necessary to increase the strength of supporting skeletal parts to compensate for increased gravitational loading, a phenomenon seen on earth in the scaling of bone cross-sectional area to animal mass [1]. In this paper, we describe a model of the gravitational stresses on a skeletal system and predict how the system will be scaled, and with what efficiency, to compensate for increased gravitational loads.

- (3) Consideration of changes due to atmospheric surface pressure and surface temperature.

Further assumptions are states throughout the paper.

## 2.2 Cardiovascular System

To understand how the cardiovascular system will change, we will model blood flow through the circulatory system using the Navier-Stokes equations and study changes in blood pressure as a function of gravitational acceleration.

We can describe viscous flow with the Navier-Stokes equations, which accurately model the laminar flow experienced in medium or large arteries [4]:

$$\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla P - \nabla \cdot \mu D(\mathbf{u}) = \mathbf{f}, \quad \nabla \cdot \mathbf{u} = 0$$

where  $\mathbf{u}$  is velocity,  $P$  is the fluid pressure,  $\mu$  is the dynamic viscosity of the fluid,  $\mathbf{f}$  is the external force per unit volume, and

$$D(\mathbf{u}) = \frac{\nabla \mathbf{u} + \nabla \mathbf{u}^T}{2}$$

is the strain rate. The second term of the top equation is the convective term and the strain rate

is high the flow is turbulent. During peak systolic velocity, some flow instabilities occur at the aortic valve. In all other cases the blood flow is laminar [4]. We can rewrite the first equation to solve for the pressure gradient,

$$\nabla P = \mathbf{f} + \nabla \cdot \mu D(\mathbf{u}) - \frac{\partial \mathbf{u}}{\partial t} - \rho(\mathbf{u} \cdot \nabla)\mathbf{u}$$

Blood can be assumed to have constant density with constant velocity as a function of  $g$  [4]. In our comparison of blood pressure gradient on earth to our super-earth, we assume the following:

1.  $\mathbf{f}_{\oplus} = m\mathbf{g}$  per unit volume, so  $\mathbf{f}_{sup\oplus} = m2\mathbf{g} = 2\mathbf{f}_{\oplus}$
2. The radial and polar components of the velocity are zero:  $u_r = u_{\theta} = 0$
3.  $\mu_{\oplus} = \mu_{sup\oplus}$ ,  $\mu_{\oplus} = \text{constant}$
4.  $\rho_{\oplus} = \rho_{sup\oplus}$

We can further assume that velocity  $\mathbf{u}$  is only in the direction along the vein, say the  $\hat{\mathbf{z}}$  direction, and that it is a laminar flow. Thus, we assume that velocity only changes in the radial direction. Then  $\mathbf{u}$  is a function of only one variable,  $r$ , so we can write

$$D(\mathbf{u}) = \frac{\frac{\partial \mathbf{u}}{\partial r} + \frac{\partial \mathbf{u}^T}{\partial r}}{2} = \frac{\frac{\partial \mathbf{u}}{\partial r} + \frac{\partial \mathbf{u}}{\partial r}}{2} = \frac{\partial \mathbf{u}}{\partial r}$$

Then

$$\nabla P = \mathbf{f}_{sup\oplus} + \nabla \cdot \mu \frac{\partial \mathbf{u}}{\partial r} - \frac{\partial \mathbf{u}}{\partial t} - \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \mathbf{f}_{sup\oplus} + \mu \frac{\partial^2 \mathbf{u}}{\partial r^2} - \frac{\partial \mathbf{u}}{\partial t} - \rho u \frac{\partial \mathbf{u}}{\partial r}$$

We assume that velocity is time independent, so

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{0}$$

Then

$$\nabla P = \mathbf{f}_{sup\oplus} + \mu \frac{\partial^2 \mathbf{u}}{\partial r^2} - \rho u \frac{\partial \mathbf{u}}{\partial r}$$

By a dimensional argument,

$$\frac{\rho u}{\mu} = \frac{1}{r}$$

So

$$\nabla P = \frac{\partial p}{\partial z} = \mathbf{f}_{sup\oplus} + \mu \frac{\partial^2 \mathbf{u}}{\partial r^2} - \frac{1}{r} \frac{\partial \mathbf{u}}{\partial r}$$

Then

$$\frac{1}{\mu} \frac{\partial p}{\partial z} = \mathbf{f}_{sup\oplus} + \frac{\partial^2 \mathbf{u}}{\partial r^2} - \frac{1}{r} \frac{\partial \mathbf{u}}{\partial r}$$

Solving this differential equation is difficult, particularly when it becomes inhomogeneous from our external force  $\mathbf{f}_{sup\oplus}$ . As seen in derivations from the literature, our force term vanishes after considering the cylindrical momentum equations [9] and we obtain

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = \frac{1}{\mu} \frac{\partial p}{\partial z}$$

Integrating, we find



We deem it probable that a super- " " " " " " " " " alterations in Cases 1 and 2. The creature will most likely have shorter appendages and also have a larger heart than its doppelganger on earth.

### 2.2.1 Blood Volume as a Function of Weight

By basic principles of geometry, it is possible to express the relationship between volume and surface area as a shape grows in size. As an object undergoes a proportional size increase, the final volume is proportional to the cube of the initial volume and the final surface area is proportional to the square of the initial surface area. That is,

$$V_2 = V_1 \left(\frac{l_2}{l_1}\right)^3$$

where  $V_1$  and  $V_2$  are the initial and final volumes, respectively, and  $l_1$  and  $l_2$  are the initial and final lengths of the object. The final area is described as,

$$A_2 = A_1 \left(\frac{l_2}{l_1}\right)^2$$

where  $A_1$  and  $A_2$  are the initial and final areas, respectively, and  $l_1$  and  $l_2$  are the initial and final lengths of the object. This scaling difference between volume and surface area provides the basis of allometry, the study of the relationship of body size to physiology and anatomy.

It is clear that bone cross-sectional area will not scale proportionally, and must instead be described using an allometric relationship [5]. Allometric scaling laws for various physical quantities have been found both theoretically and experimentally. Of particular interest are variations in characteristic radius, length, and number of capillary blood vessels as a function of gravitational acceleration. These quantities were determined by T.H. Dawson [3] to be:

$$r_c \sim g^{\frac{1}{12}}$$

$$l_c \sim g^{\frac{5}{24}}$$

$$n_c \sim g^{\frac{5}{8}}$$

where  $r_c$ ,  $r_c$ , and  $r_c$  are the characteristic radius, length, and number of blood vessels, respectively.  $\sim$  denotes proportionality. Hence as gravitational acceleration increases, blood vessels increase in radius, length, and number. These laws apply both to single pulmonary beds and beds of capillaries in individual organs. Thus, ignoring other factors, organ size would scale with  $g$  in the same proportions. Additionally, we can consider the change in the total volume of blood in the capillaries. If we assume that capillaries can be modeled as cylinders with radius  $r_c$  and length  $l_c$ , then the total volume of blood in the capillaries is,

$$V = \pi n_c l_c r_c^2$$

Of course, capillaries exhibit both curvature and tapering, so we can only conclude that,

$$V \sim n_c l_c r_c^2$$

Hence,

$$V \sim g$$

So blood volume increases proportionally with gravitational acceleration.

### 2.3 Musculoskeletal System

We model the skeletal system by considering the normal and shear stresses on a bone as a function of gravity. Stress describes the internal forces that particles of a continuous material exert on one another. Stress can be broken down into normal and shear components. Here, we consider a bone with cross sectional area  $A$  to be a cylinder of uniform composition. The normal and shear stresses can be expressed as

$$\sigma = \frac{F_N}{A}$$

$$\tau = \frac{F_P}{A}$$

Where  $\sigma$  is the normal stress,  $\tau$  is the shear stress,  $F_N$  is the component of force normal to the cross sectional area of the bone,  $F_P$  is the component of force perpendicular to the bone, and  $A$  is the cross sectional area of the bone. This can be expressed in terms of the gravitational acceleration  $g$  as,

$$\sigma = \frac{mg \cos \theta}{A}$$

$$\tau = \frac{mg \sin \theta}{A}$$

Where  $m$  is the mass of the creature and  $\theta$  is the angle of attachment between the bone and the body.

As an animal walks, a vertical force of  $mg$  acts on the foot, exerting a torque of  $mg l \sin \theta$  about the joint halfway up the leg. To maintain equilibrium, the muscle must exert an equivalent torque. If the muscle has a cross-sectional area  $A$ , a moment arm of  $r$  about the joint, and exerts a stress  $\sigma_m$ , then the torque about the joint exerted by the muscle is,

$$\tau = A \sigma_m r$$

The system is in equilibrium, so,

$$A \sigma_m r = mg l \sin \theta$$

$$\sigma_m = \frac{mg l}{A r} \sin \theta$$



In modeling variations of the musculoskeletal system as a function of gravity, we consider variations in  $A$ ,  $\theta$ , and bone length  $l$ . Considering the decomposed stresses and the above relationship for the stress exerted by muscle, we can constrain these variables. Let us first consider the cross sectional area  $A$ . As gravitational acceleration increases,  $A$  must increase to maintain constant stress to prevent the bone from breaking. To maintain constant total stress,

$$s = \sqrt{\sigma^2 + \tau^2} = \frac{mg}{A}$$

must remain constant as  $g$  changes. Hence, because  $g$  doubles,  $A$  must also double.

Next, we consider the angle  $\theta$  between the bone and body.  $\theta$  can vary between 0 and  $\frac{\pi}{2}$ . On earth, smaller mammals tend to exhibit higher angles, with  $\theta$  decreasing with increasing mass, consistent with the equation for  $\sigma_m$  above assuming constant muscle stress. Elephants, for example, have column-like femurs connecting nearly perpendicularly to their body [2]. We can verify that bones will not break for  $\theta < \frac{\pi}{2}$  by considering shear stress for the case of maximum (breaking point) shear. The maximum allowable  $\theta$  is,

$$\theta_{max} = \sin^{-1} \frac{\tau_{max} A}{mg}$$

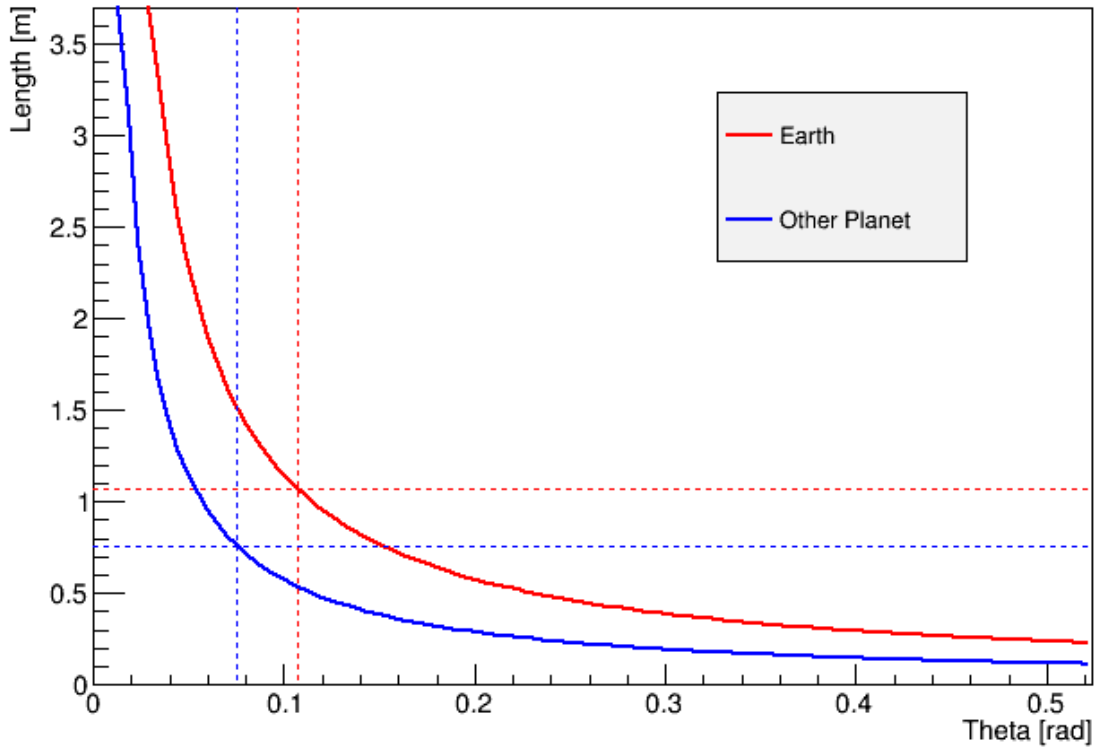
where  $\tau_{max}$  is the maximum shear that bone can withstand before fracturing, experimentally determined to be 51.6 MPa for human femurs [10]. This value is nearly constant between different species. Even for exceptionally large masses, the maximum shear will not be reached at  $\theta_{max}$ , a conclusion which seems both sensible and reassuring. Thus, we can safely conclude that  $0 < \theta < \frac{\pi}{2}$ .

Finally, we consider the length  $l$  of the bone. Longer bones are more susceptible to fracture as gravitational acceleration increases, and thus we constrain bone length to  $0 < l < l_0$  where  $l_0$  is bone length when gravitational acceleration is  $g$ . By assuming that the stress applied by muscle is relatively constant i.e. the physical properties of muscle do not vary as a function of gravitational acceleration and that muscle cross sectional area does not change considerably [1], we can use the equation for  $\sigma_m$  to plot bone length and attachment angle, as shown below in fig. 2.3.1. The constants used to generate these curves were approximated for a human femur and quadriceps, although the same relationship is seen for any number of legs (see Appendix 1).

The corner of this plot represents the point at which the product of  $l$  and  $\theta$  is minimized. Hence, the corner is the point at which muscle stress  $\sigma_m$  is minimized. Presuming that an organism would evolve to minimize muscle stress, this corner represents the ideal values of  $l$  and  $\theta$ . Comparing the curve for  $2g$  with that for  $1g$ , we see that both  $l$  and  $\theta$  decrease as gravitational acceleration increases. Thus, we expect bone length to decrease and the angle between the bones and body to decrease (animal legs will straighten).

From this plot, we can estimate that bone length and attachment angle will both decrease by approximately 30% when surface gravity doubles. As discussed in section 4.1, this estimate is

subject to considerable error, though we can confidently state that both bone length and attachment angle decrease as gravitational acceleration increases.



**Fig. 2.3.1** Plot showing the relationship between bone length and attachment angle on earth (1g) and the other planet (2g). The optimal length and angle for minimizing applied muscle stress is denoted by the corner of each curve, marked by the cross-hairs.

## 2.4 Gait Patterns

Here we analyze the dynamics of legged locomotion in a gravitational field by treating a walking limb as an inverted pendulum. The center of mass of the pendulum swings through a circular arc centered at the base of the limb. The gait can be analyzed by calculating the Froude number, defined as

$$F_r = \frac{v}{c}$$

where  $v$  is a characteristic velocity and  $c$  is a characteristic wave propagation velocity. In our model of locomotion,

$$F_r = \frac{F_c}{F_G}$$

where  $F_c$  is the centripetal force about the center of motion and  $F_g$  is the gravitational force on the walking animal. Hence,

$$F_r = \frac{mv^2}{l} = \frac{v^2}{gl}$$

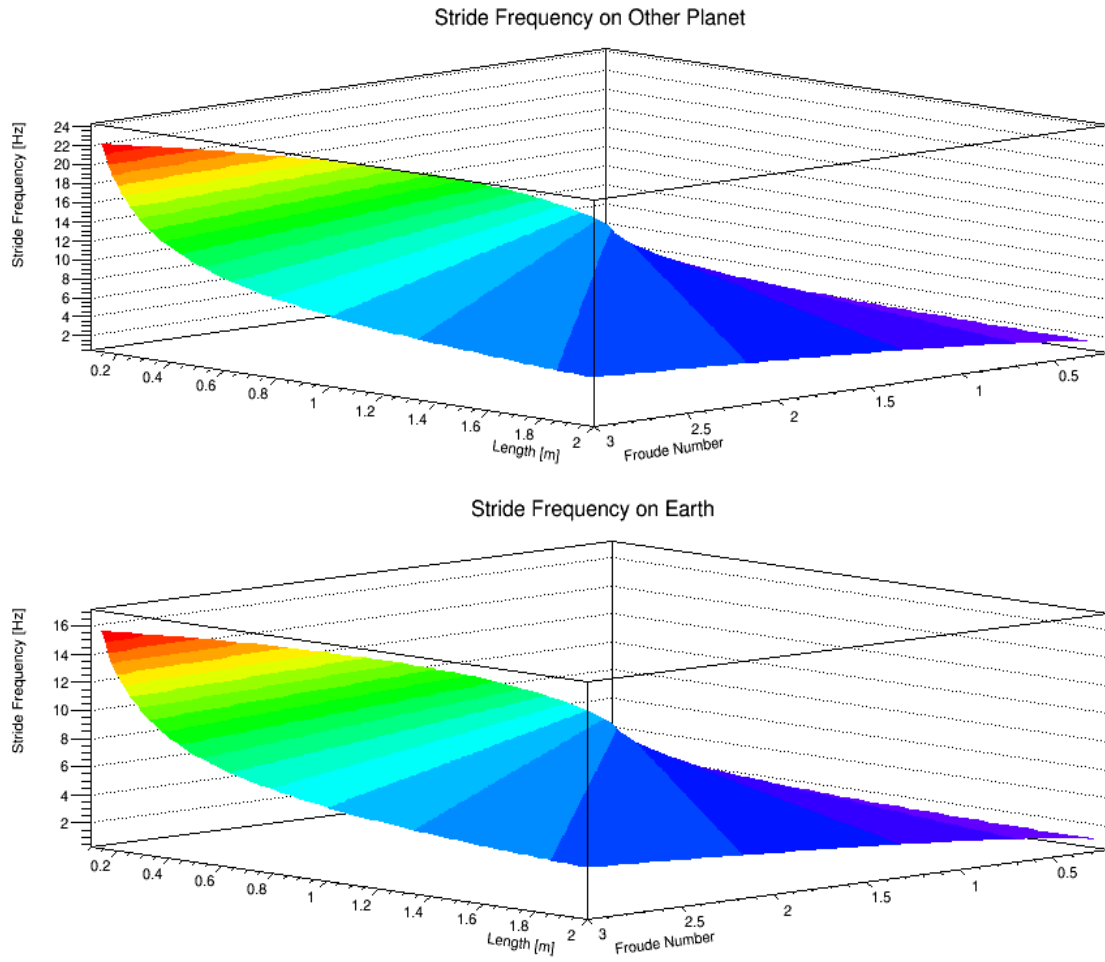
Where  $m$  is animal mass,  $v$  is velocity,  $g$  is acceleration due to gravity, and  $l$  is the characteristic length, which we define as total leg length [1, 2].

From this equation, we can also calculate the stride frequency as,

$$F_r = \frac{v^2}{gl} = \frac{l^2 f^2}{gl} = \frac{l f^2}{g}$$

two objects have the same Froude number, then they are dynamically similar. As Alexander explains, the transition from walking to trotting for quadrupeds on earth appears to happen at a Froude number between 0.3 and 0.8. The transition from trotting to galloping occurs between 2.0 and 3.0 [2].

When looking at the Froude number equation, it is observed that an animal moving at some speed  $v$  with characteristic length  $l$  on earth would have half the Froude number on the exoplanet. At the speed of a gallop on earth, for example, an animal would merely be trotting on the exoplanet. Another implication of Froude numbers is that for similar dynamics the stride frequency of an animal would be larger on the exoplanet than on earth. These results are summarized in the figures below where we have plotted Froude number and characteristic length to derive either the velocity or stride frequency. These results do not give precise information on the maximum speed of the animal. Rather, they suggest that as gravitational acceleration increases, it becomes increasingly difficult for animals to reach more efficient modes of locomotion; running is a more efficient mode of locomotion than walking, and hence animals prefer to run above a certain velocity. Animals on the exoplanet would take more strides and do so in a less efficient mode of locomotion for a given velocity, suggesting that their maximum speed would be lower than that of their earth counterpart.



**Fig. 2.4.1** *Stride length as a function of Froude number and leg length in 1g and 2g*

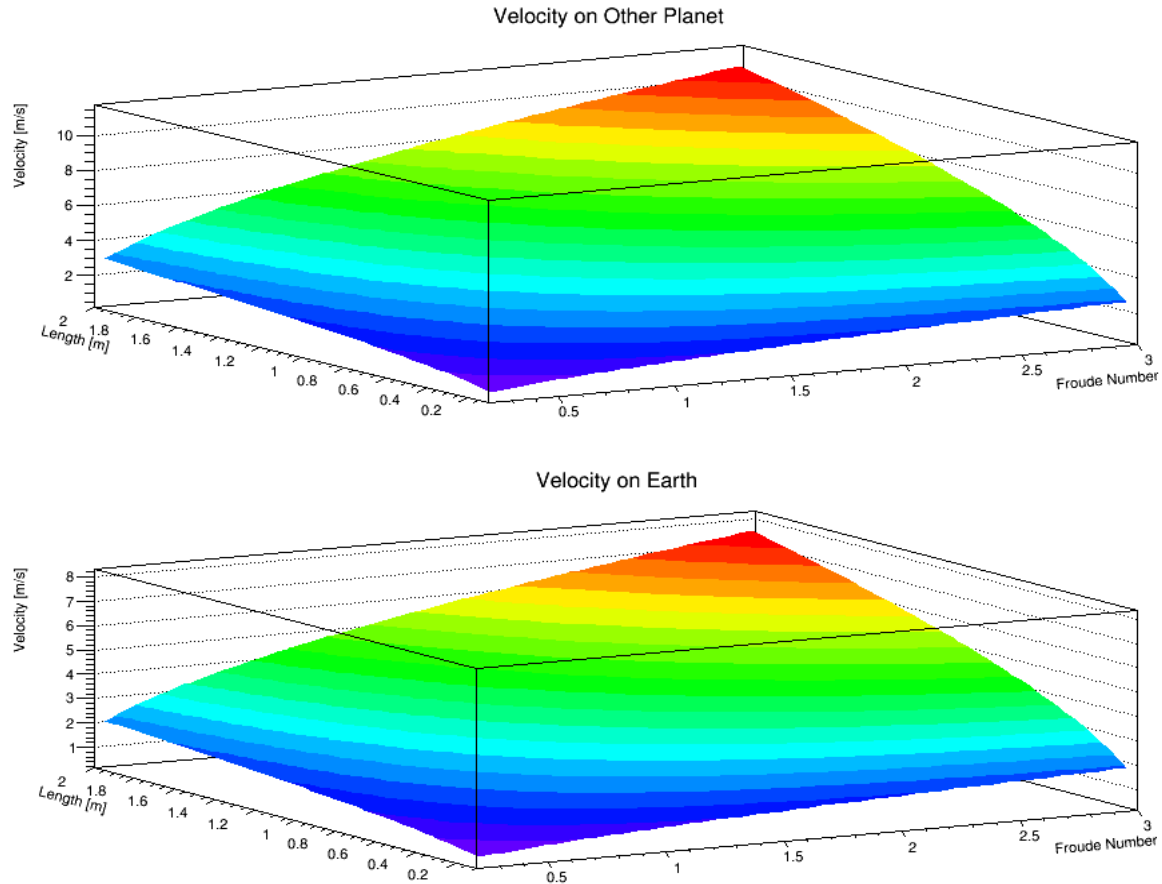


Fig. 2.4.2 Velocity as a function of Froude number and leg length in 1g and 2g

### 3 Additional Considerations

#### 3.1 Effects of surface temperature

Our super-earth has an average temperature of 250K, well below the freezing point of water at 0°C. We know that the amount of insolation the planet receives falls off as  $\cos \theta$ , where theta is the angle from the equator to the pole [11]. Thus, the poles will have a much lower temperature than the equator, just like earth. Earth has an average surface temperature of 288K [12] and ranges from an average temperature of about 215K at the South Pole to an average of about 300K at the equator. Let assume that the average temperature of the super-earth ranges from 190K at the poles to 285K at the equator. We note that the earth still has liquid water in the form of oceans at the poles due to currents transporting thermal energy from the equator. Let us also assume that the super-earth has oceans and currents allowing liquid water to exist at the poles. Life on earth exists at the poles. Polar bears, for example, are able to obtain water by

eating snow and prey. Their bodies break down the blubber from their prey to obtain the water it contains. Creatures living in the poles also have a thick layer of blubber to provide insulation. Thus, if life exists at the poles on the super-earth it must have some sort of insulation in the form of blubber and/or hair coat and be able to obtain water from the food it eats. Further, the poles of the super-earth are much colder than those of earth, so it is likely that creatures will either a) have much more powerful forms of insulation and obtaining water and/or b) live closer to the equator than to the pole of the hemisphere. If the average temperature of the super-earth equator is 285K, then life has the ability to be similar to what we find on earth in terms of temperature adaptations. Liquid water could also be easily found here, so it is probably that the

An interesting note is that the average surface temperature of the super-earth is roughly equivalent to the effective temperature of earth. We assume that the super-earth has an atmosphere and some plate tectonic activity, so the super-earth effective temperature will be much lower than 250K.

The average surface temperature of earth is much larger than its effective temperature of 252K found from equating the absorbed power to the Stefan-Boltzman law:

$$P = A\varepsilon\sigma T^4$$

where  $P$  is the total power radiated from the object,  $A$  is the surface area,  $\varepsilon$  is the emissivity,  $\sigma$  is the Stefan-Boltzman constant, and  $T$  is the black body temperature. The surface temperature of earth is much higher because of the effect of greenhouse gases, in part

### 3.2 Effects of Atmospheric Pressure

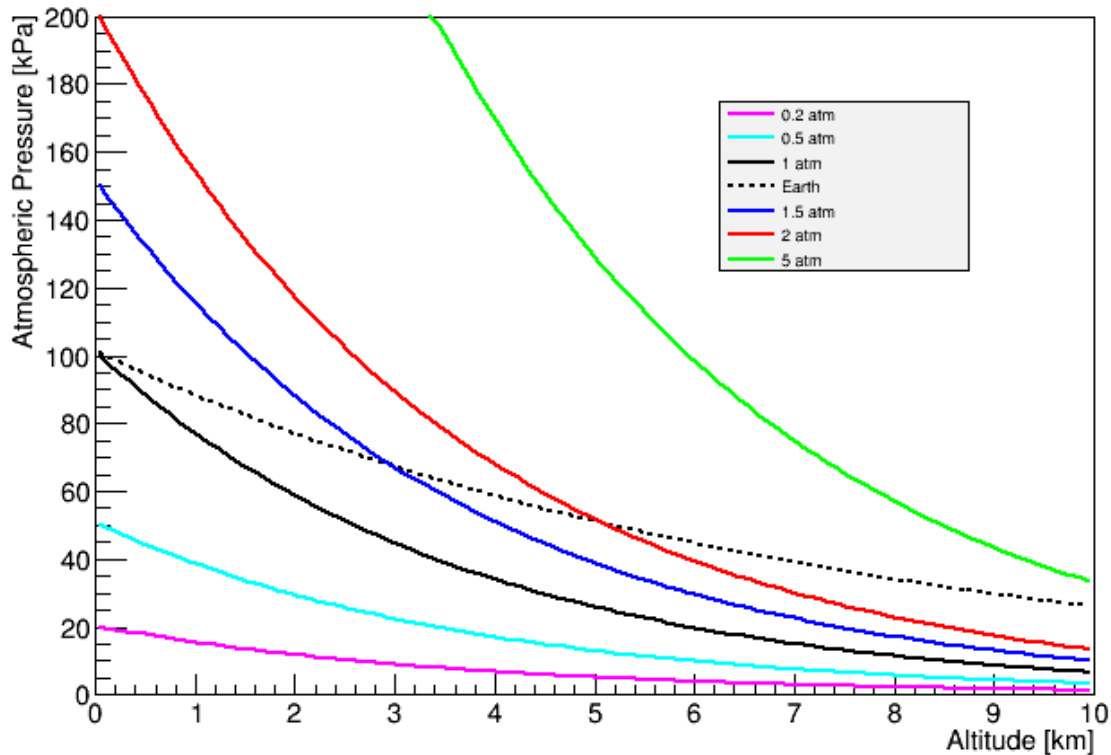
We now consider the changes due to atmospheric pressure on the exoplanet as compared to the earth. It is well known that atmospheric pressure is described as follows:

$$P(h) = P_0 e^{-h/H} \quad H = \frac{k_B T}{mg}$$

where  $P_0$  is the pressure at the surface of the planet,  $H$  is the scale height of the atmosphere,  $k_B$  is Boltzmann's constant,  $T$  is the temperature of the atmosphere,  $m$  is the mean molecular mass of the molecules in the atmosphere,  $g$  is the acceleration due to gravity, and  $h$  is the height above the surface. To model the pressure differences between the earth and the exoplanet, we have made the ansatz that  $m$  will be essentially the same between the two planets. This is supported by the idea that the atmospheric composition of the two planets should be similar if there exist living creatures on both planets. Using  $T=250$  K and  $m = 4.808 \times 10^{-26}$  kg, we obtain a scale height of  $H_{exo} = 3.661$  km while the average  $H_{Earth} = 7.4$  km.

If we assume that the surface pressure of the exoplanet is equal to that of the earth, we observe that the atmospheric pressure of the exoplanet falls off more rapidly than that of the earth (see figure below). This indicates that any airborne creatures living on the exoplanet would have to compensate for a greater pressure difference over the change in height between which they fly

and where they obtain their food and water. Because compensating for an increase pressure gradient requires more metabolic energy expenditure, any flying creature would have to have better methods of expending less energy through this compensation. There is also the possibility that these airborne creatures would just feed on other airborne creatures creating two relatively independent food chains, one ground based and one air based.



**Fig. 3.3.1** Atmospheric pressure as a function of altitude. The solid lines indicate the pressure for the exoplanet given certain surface pressures. The dotted line represents the pressure fall off for earth.

Conversely, if we were to assume that the atmosphere had the same composition as that of earth, then considering a given air column,

$$P = \frac{F}{A} = \frac{mg}{A}$$

Surface pressure would increase, leading to a higher partial pressure of oxygen at the surface and, consequently, less demand for red blood cells to carry oxygen.

## 4 Results Summary

We can combine the results of our above analyses to provide a general picture of life on this planet. For illustration purposes, we consider a four-legged animal living on the equator.







## 6 Closing Remarks

We conclude that on a planet with a mass eight times that of earth and a radius twice that of earth, life would differ significantly. Bone width and length would increase and decrease, respectively, and animals would become more compact with shorter extremities. Organs, including the heart, would increase in size, as would capillaries. Leg bones would meet the body at shallower angles, making the legs straighter and less mobile. Thus, in general, animals would likely be shorter, slower, and stockier.

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## Appendix 1

### Values used for fig. 2.3.1

Variable	Symbol	Quantity	Units
Muscle Stress	$\sigma$	$100 \cdot 10^3$	$\frac{N}{m^2}$
Bone cross-sectional area	$A$	273	$m^2$
Bone radius	$r$	3	$m$
Animal Mass	$M$	74.1	$kg$

Values were approximated based on the human femur and quadriceps. The same arguments from section 2.3 extend to animals with more limbs.

## Appendix 2

### Code used to generate figures (All coding done in C++)

```
void RotationalForce(){
    TCanvas *c1 = new TCanvas("Rotational_Force", "Rotational Force", 0, 0, 700, 500);
    c1->cd();
    TF1 *f1 = new TF1("Length vs. Theta on earth", "1.12/(sin(x)*9.8)", 0,
, TMath::Pi()/6);
    f1->SetTitle("");
    f1->GetYaxis()->SetTitle("Length [m]");
    f1->GetXaxis()->SetTitle("Theta [rad]");
    f1->GetYaxis()->SetRangeUser(0,3.7);
    f1->Draw();

    TLine *l1v = new TLine(.1068,0,.1068,3.7);
    TLine *l1h = new TLine(0,1.12/(sin(.1068)*9.8),TMath::Pi()/6,1.12/(sin(.1068)*9.8));
    l1v->SetLineColor(2);
    l1h->SetLineColor(2);
    l1v->SetLineStyle(2);
    l1h->SetLineStyle(2);
    l1v->Draw("same");
    l1h->Draw("same");

    TF1 *f2 = new TF1("Length vs. Theta on Other Planet", "1.12/(sin(x)*19.6)", 0,
, TMath::Pi()/6);
    f2->SetTitle("Length vs. Theta on Other Planet");
    f2->GetYaxis()->SetTitle("Length");
    f2->GetXaxis()->SetTitle("Theta");
    f2->SetLineColor(4);
    f2->Draw("same");

    TLine *l2v = new TLine(.075557,0,.075557,3.7);
    TLine *l2h = new
TLine(0,1.12/(sin(.075557)*19.6),TMath::Pi()/6,1.12/(sin(.07557)*19.6));
    l2v->SetLineColor(4);
    l2h->SetLineColor(4);
    l2v->SetLineStyle(2);
    l2h->SetLineStyle(2);
    l2v->Draw("same");
    l2h->Draw("same");

    TLegend *leg = new TLegend(.6,.6,.8,.8);
```

```

leg->AddEntry(f1, "earth", "1");
leg->AddEntry(f2, "Other Planet", "1");
leg->Draw();
}

void FroudeNumber(){
    TCanvas *c1 = new TCanvas("Froude_number", "Froude Number", 0, 0, 1300, 650);
    c1->Divide(2,2);

    c1->cd(1);
    TH2F *h2vother = new TH2F("testingplots", "Velocity on Other Planet",50, 1, 3, 50,
0.1, 2);
    for(int i=1; i<=50; i++){
        for(int j=1; j<=50; j++){
            double points = sqrt(h2vother->GetXaxis()->GetBinCenter(i)*h2vother->GetYaxis()-
>GetBinCenter(j)*19.6);
            h2vother->SetBinContent(i,j,points);
        }
    }
    h2vother->GetXaxis()->SetTitle("Froude Number");
    h2vother->GetYaxis()->SetTitle("Length [m]");
    h2vother->GetXaxis()->SetTitleOffset(1.5);
    h2vother->GetYaxis()->SetTitleOffset(1.5);
    h2vother->GetZaxis()->SetTitle("Velocity [m/s]");
    h2vother->SetStats(false);
    h2vother->Draw("SURF2");

    c1->cd(2);
    TH2F *h2vearth = new TH2F("testingplotse", "Velocity on earth",50, 1, 3, 50, 0.1,
2);
    for(int i=1; i<=50; i++){
        for(int j=1; j<=50; j++){
            points = sqrt(h2vearth->GetXaxis()->GetBinCenter(i)*h2vearth->GetYaxis()-
>GetBinCenter(j)*9.8);
            h2vearth->SetBinContent(i,j,points);
        }
    }
    h2vearth->GetXaxis()->SetTitle("Froude Number");
    h2vearth->GetYaxis()->SetTitle("Length [m]");
    h2vearth->GetXaxis()->SetTitleOffset(1.5);
    h2vearth->GetYaxis()->SetTitleOffset(1.5);
    h2vearth->GetZaxis()->SetTitle("Velocity [m/s]");
    h2vearth->SetStats(false);
    h2vearth->Draw("SURF2");

    c1->cd(3);
    TH2F *h2fother = new TH2F("testingplotsf", "Stride Frequency on Other Planet",50, 1,
3, 50, 0.1, 2);
    for(int i=1; i<=50; i++){
        for(int j=1; j<=50; j++){
            points = sqrt(h2fother->GetXaxis()->GetBinCenter(i)/h2fother->GetYaxis()-
>GetBinCenter(j)*19.6);
            h2fother->SetBinContent(i,j,points);
        }
    }
    h2fother->GetXaxis()->SetTitle("Froude Number");
    h2fother->GetYaxis()->SetTitle("Length [m]");
    h2fother->GetXaxis()->SetTitleOffset(1.5);
    h2fother->GetYaxis()->SetTitleOffset(1.5);
    h2fother->GetZaxis()->SetTitle("Stride Frequency [Hz]");
    h2fother->SetStats(false);
    h2fother->Draw("SURF2");

    c1->cd(4);
    TH2F *h2fearth = new TH2F("testingplotfe", "Stride Frequency on earth",50, 1, 3, 50,
0.1, 2);
    for(int i=1; i<=50; i++){
        for(int j=1; j<=50; j++){

```

```

        points = sqrt(h2fearth->GetXaxis()->GetBinCenter(i)/h2fearth->GetYaxis()-
>GetBinCenter(j)*9.8);
        h2fearth->SetBinContent(i,j,points);
    }
}
h2fearth->GetXaxis()->SetTitle("Froude Number");
h2fearth->GetYaxis()->SetTitle("Length [m]");
h2fearth->GetXaxis()->SetTitleOffset(1.5);
h2fearth->GetYaxis()->SetTitleOffset(1.5);
h2fearth->GetZaxis()->SetTitle("Stride Frequency [Hz]");
h2fearth->SetStats(false);
h2fearth->Draw("SURF2");
}

```

```
void CentripitalForce(){
```

```

    TCanvas *c1 = new TCanvas("Centripital_Force", "Centripital Force", 0, 0, 700, 500);
    c1->cd();
    c1->SetLogx();
    c1->SetLogy();

    TF1 *f1 = new TF1("Acceleration vs. Rotation Period", "x^2*.003436", 0.1, 30);
    f1->SetTitle("Acceleration vs. Rotation Period");
    f1->GetYaxis()->SetTitle("Relative Rotational Acceleration");
    f1->GetXaxis()->SetTitle("Planet Rotational Period (rev/day)");

    f1->Draw();
}

```

```
void Pressure(){
```

```

    TCanvas *c1 = new TCanvas("Pressure", "Pressuure", 0, 0, 700, 500);
    c1->cd();

    double scaleheight=1.38*pow(10, -23)*250/(4.808*pow(10, -26)*2*9.8)/1000;
    std::cout<<scaleheight<<std::endl;

    TF1 *f1 = new TF1("1 atm", "101.325*exp(-1*x/3.66099)", 0, 10);
    f1->SetTitle(" ");
    f1->SetLineColor(1);
    f1->GetYaxis()->SetTitle("Atmospheric Pressure [kPa]");
    f1->GetXaxis()->SetTitle("Altitude [km]");
    f1->GetXaxis()->SetRangeUser(0, 10);
    f1->GetYaxis()->SetRangeUser(0, 200);
    f1->Draw();

    TF1 *earth = new TF1("2 atm", "101.325*exp(-1*x/(2*3.66099))", 0, 10);
    earth->SetLineColor(1);
    earth->SetLineStyle(2);
    earth->Draw("same");

    TF1 *f2 = new TF1("2 atm", "2*101.325*exp(-1*x/3.66099)", 0, 10);
    f2->SetLineColor(2);
    f2->Draw("same");

    TF1 *f3 = new TF1("5 atm", "5*101.325*exp(-1*x/3.66099)", 0, 10);
    f3->SetLineColor(3);
    f3->Draw("same");

    TF1 *f4 = new TF1("1.5 atm", "1.5*101.325*exp(-1*x/3.66099)", 0, 10);
    f4->SetLineColor(4);
    f4->Draw("same");

    TF1 *f5 = new TF1(".5 atm", ".5*101.325*exp(-1*x/3.66099)", 0, 10);
    f5->SetLineColor(7);
    f5->Draw("same");
}

```

```
TF1 *f6 = new TF1(".2 atm", ".2*101.325*exp(-1*x/3.66099)" ,0 ,10);  
f6->SetLineColor(6);  
f6->Draw("same");
```

```
TLegend *leg = new TLegend(.6,.6,.8,.8);  
leg->AddEntry(f6, "0.2 atm", "l");  
leg->AddEntry(f5, "0.5 atm", "l");  
leg->AddEntry(f1, "1 atm", "l");  
leg->AddEntry(earth, "earth", "l");  
leg->AddEntry(f4, "1.5 atm", "l");  
leg->AddEntry(f2, "2 atm", "l");  
leg->AddEntry(f3, "5 atm", "l");  
leg->Draw();
```

```
}
```