Analysis of Biological Structures in Hypergravity Environments

Team 611: Problem B

Abstract

In this paper, we produce an analysis of how life would evolve differently on a planet with a mass eight times that of earth and a radius two times that of earth. We address this analysis by considering the effects of increased gravity on the dynamics and mechanics of the cardiovascular and musculoskeletal systems. We investigate the cardiovascular system by considering laminar flow in large and medium sized vessels, using the Navier-Stokes equations to model the blood pressure gradient. Additionally, we use scaling arguments of allometry to estimate how blood volume and capillary dimensions vary with gravity. We investigate the musculoskeletal system by considering a single bone and muscle extending from the animal body. We decompose bone stress into normal and shear components and treat muscle stress through equilibrium arguments to estimate changes in bone cross sectional area. This model is then used to constrain possible values of bone length and attachment angle. Additionally, we analyze the dynamics of legged locomotion using Froude numbers to demonstrate changes in gait as a function of gravitational acceleration. Finally, the effects additional factors including changes in atmospheric pressure and temperature are considered. Given these considerations, we propose that animals that evolve in this environment would be characterized by bones with twice the cross sectional area and approximately 30% less length than their earth counterparts. Leg bones would attach to the body at shallower angles, blood volume would approximately double and capillary size would increase, organ and heart size would increase, and animals would be shorter and more compact to compensate for higher blood pressure gradients. Additionally, we model gait as a function of gravitational acceleration to demonstrate that it would be increasingly difficult for animals to reach the most efficient modes of transportation, likely decreasing maximum speed.

1 Interpretation of the Problem

We will investigate how the structure of animals varies when the mass and surface area of a planet increases of factors of eight and two, respectively. Our main consideration is variation in skeletal structure and blood flow due to the change in surface gravity. We discuss physiological changes relative to an equivalent animal on earth, assuming that these changes will give the new animal the same functionality as the equivalent earth animal.

1.1 Introduction

Recent discoveries by instruments such as the Kepler Telescope have revealed an incredible number of exoplanets, many of which are earthlike. Of these, a significant number have been found in the habitable zone, allowing for the possibility of life. While life may exist on these planets, it is possible that the conditions under which it forms and thrives differ significantly from those on earth. In this paper, we consider how creatures on a habitable planet of eight times the mass and twice the radius of earth would differ from those found on earth.

In this paper, we begin by considering gravity and its direct and indirect effects on the biological systems of terrestrial homeotherms. We propose that the factors which most greatly constrain and evolution and structure of life on this exoplanet will be changes in the cardiovascular system, the musculoskeletal system, and atmospheric pressure. We describe each system independently, producing a mathematical model of the effects of gravity on each of these three factors, before combining the results to predict such factors as bone dimensions and blood volume. Throughout the paper, we presume that evolution on this planet will produce animals with approximately the same functions as those on earth.

2 Model

2.1 Assumptions and Overview

In this paper, we treat the planet as a sphere surrounded with a thin atmosphere. Given an exoplanet with a mass eight times that of earth and a radius that times of earth, it is simple to show that gravitational acceleration at the surface will scale as

$$g \sim \frac{M}{R^2}$$

Hence gravitational acceleration at the surface will be two times that of earth. We have also considered the centripetal acceleration caused by the rotational motion of the planet. The centripetal acceleration is given by

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Since r is larger in the case of the exoplanet, it is important to consider if the centripetal acceleration could become a significant factor. We have plotted $\frac{a_{c,exo}}{g_{exo}}$ (i.e. the relative rotational

acceleration) vs. the period of the planet's rotation. From this plot, we observe that the exoplanet would have to be rotating at least 10 times faster than earth for the centripetal acceleration to factor significantly into the forces experienced at the surface. Hence we assume that the rotation rate does not significantly contribute to the acceleration of objects near the surface of the exoplanet.
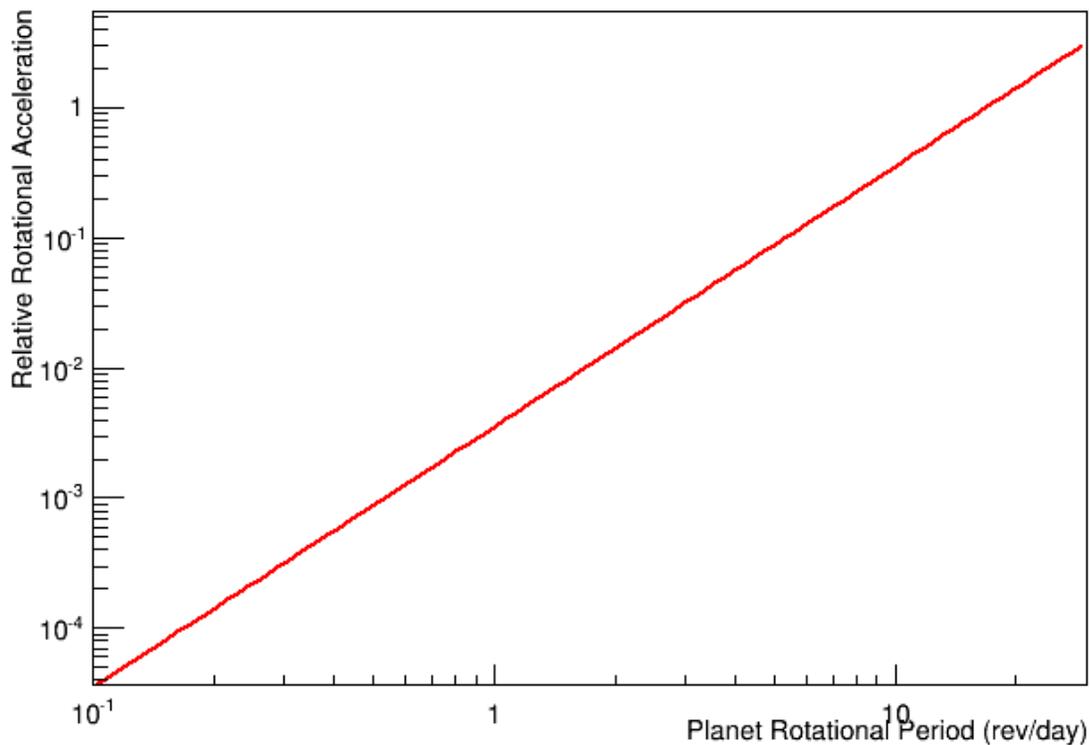

Fig. 2.1.1 *Rotational acceleration as a function of planet rotation.*

Given such a drastic change in gravitational acceleration, we assume that difference in gravity accounts for the most significant differences between the organisms on this exoplanet and those on earth. It is thus important to consider the factors which are most influenced by these changes. In this paper, we consider:

(1) Cardiovascular System: The cardiovascular system must be capable of delivering blood throughout the body and removing waste products. Blood must be pumped against gravity to body parts above the heart and brought back from lower extremities against gravity to prevent pooling. This is accomplished through the force of contraction in the heart and the elasticity and dimensions of arterial blood vessels. We model blood flow mathematically to predict changes in blood pressure and other changes in the cardiovascular system.
(2) Skeletal System: Perhaps the most obvious structural requirement of an organism is adequate support of body mass. When the force of gravitational loading exceeds

cohesive forces holding together a structure, that structure will collapse. Thus, in a vertebrate it is necessary to increase the strength of supporting skeletal parts to compensate for increased gravitational loading, a phenomenon seen on earth in the scaling of bone cross-sectional area to animal mass [1]. In this paper, we describe a model of the gravitational stresses on a skeletal system and predict how the system will be scaled, and with what efficiency, to compensate for increased gravitational loads.

(3) Consideration of changes due to atmospheric surface pressure and surface temperature.

Further assumptions are states throughout the paper.

2.2   Cardiovascular System

To understand how the cardiovascular system will change, we will model blood flow through the circulatory system using the Navier-Stokes equations and study changes in blood pressure as a function of gravitational acceleration.

We can describe viscous flow with the Navier-Stokes equations, which accurately model the laminar flow experienced in medium or large arteries [4]:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla P - \nabla \cdot \mu D(\boldsymbol{u}) = \boldsymbol{f}, \qquad \nabla \cdot \boldsymbol{u} = 0$$

where $\boldsymbol{u}$ is velocity, P is the fluid pressure, µ is the dynamic viscosity of the fluid, $\boldsymbol{f}$ is the external force per unit volume, and

$$D(\boldsymbol{u}) = \frac{\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T}{2}$$

is the strain rate. The second term of the top equation is the convective term and the strain rate is the diffusive term. If the Reynold's number is low (less than 1000) the flow is laminar while if it is high the flow is turbulent. During peak systolic velocity, some flow instabilities occur at the aortic valve. In all other cases the blood flow is laminar [4]. We can rewrite the first equation to solve for the pressure gradient,

$$\nabla P = \boldsymbol{f} + \nabla \cdot \mu D(\boldsymbol{u}) - \frac{\partial \boldsymbol{u}}{\partial t} - \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$$

Blood can be assumed to have constant density with constant velocity as a function of g [4]. In our comparison of blood pressure gradient on earth to our super-earth, we assume the following:

1. $\boldsymbol{f}_{\oplus} = m\boldsymbol{g}$ per unit volume, so $\boldsymbol{f}_{sup\oplus} = m2\boldsymbol{g} = 2\boldsymbol{f}_{\oplus}$
2. The radial and polar components of the velocity are zero: $u_r = u_\theta = 0$
3. $\mu_{\oplus} = \mu_{sup\oplus}$, $\mu_{\oplus}$ = constant
4. $\rho_{\oplus} = \rho_{sup\oplus}$

We can further assume that velocity $\boldsymbol{u}$ is only in the direction along the vein, say the $\hat{\boldsymbol{z}}$ direction, and that it is a laminar flow. Thus, we assume that velocity only changes in the radial direction. Then $\boldsymbol{u}$ is a function of only one variable, r, so we can write

$$D(\boldsymbol{u}) = \frac{\frac{\partial \boldsymbol{u}}{\partial r} + \frac{\partial \boldsymbol{u}^T}{\partial r}}{2} = \frac{\frac{\partial \boldsymbol{u}}{\partial r} + \frac{\partial \boldsymbol{u}}{\partial r}}{2} = \frac{\partial \boldsymbol{u}}{\partial r}$$

Then

$$\nabla P = \boldsymbol{f}_{sup\oplus} + \nabla \cdot \mu \frac{\partial \boldsymbol{u}}{\partial r} - \frac{\partial \boldsymbol{u}}{\partial t} - \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \boldsymbol{f}_{sup\oplus} + \mu \frac{\partial^2 \boldsymbol{u}}{\partial r^2} - \frac{\partial \boldsymbol{u}}{\partial t} - \rho u \frac{\partial \boldsymbol{u}}{\partial r}$$

We assume that velocity is time independent, so

$$\frac{\partial \boldsymbol{u}}{\partial t} = \boldsymbol{0}$$

Then

$$\nabla P = \boldsymbol{f}_{sup\oplus} + \mu \frac{\partial^2 \boldsymbol{u}}{\partial r^2} - \rho u \frac{\partial \boldsymbol{u}}{\partial r}$$

By a dimensional argument,

$$\frac{\rho u}{\mu} = \frac{1}{r}$$

So

$$\nabla P = \frac{\partial p}{\partial z} = \boldsymbol{f}_{sup\oplus} + \mu \frac{\partial^2 \boldsymbol{u}}{\partial r^2} - \frac{1}{r}\frac{\partial \boldsymbol{u}}{\partial r}$$

Then

$$\frac{1}{\mu}\frac{\partial p}{\partial z} = \boldsymbol{f}_{sup\oplus} + \frac{\partial^2 \boldsymbol{u}}{\partial r^2} - \frac{1}{r}\frac{\partial \boldsymbol{u}}{\partial r}$$

Solving this differential equation is difficult, particularly when it becomes inhomogeneous from our external force $\boldsymbol{f}_{sup\oplus}$. As seen in derivations from the literature, our force term vanishes after considering the cylindrical momentum equations [9] and we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{1}{\mu}\frac{\partial p}{\partial z}$$

Integrating, we find

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2\mu}\frac{\partial p}{\partial z} + a$$

$$\frac{\partial u}{\partial r} = \frac{r}{2\mu}\frac{\partial p}{\partial z} + \frac{a}{r}$$

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial z}r^2 + a\ln r + b$$

where r is the radial distance from the center of the vein. We must have a finite u at $r = 0$, so it must be that $a = 0$. We model the blood as a laminar flow, so the no-slip condition requires that u is 0 at $r = R$, where R is the radius of the vein. These boundary conditions yield

$$b = -\frac{1}{4\mu}\frac{\partial p}{\partial z}R^2$$

Then we find a velocity profile of

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial z}(R^2 - r^2)$$

From the above equations we can see that the gradient is roughly proportional to the external force per volume. In an environment with a gravitational acceleration of 2*g*, we expect that the blood pressure gradient will be tighter. That is, there will be a greater difference between the blood pressure at a point *a* and *b* on the creature's appendage in 2*g* than in 1*g*. Let us examine two cases.

Case 1: Assume that the cardiovascular system of the creature on the super-earth is identical to the cardiovascular system of a creature of similar size and build on earth. With an increased pressure gradient, the creature's system has increased difficulty oxygenating and pumping blood. So the body must expend more energy because the heart feels a greater force. Shorter appendages create a shorter distance for the heart to pump blood through. We know $dW = -F \cdot dz$ and that $dE = dQ - dW$, where W is work, F is force, E is energy, and Q is heat. So energy is roughly proportional to $F \cdot dz$. If we decrease the distance *dz* that we pump blood through while holding *F* constant, then we can decrease the energy the body must expend to pump the blood. Consequently, it is likely that a creature on the super-earth with an earth-like cardiovascular system will have a compact body with shorter appendages than its counterpart on earth.

Case 2: Assume that the length of the creature's appendages is comparable to that of a creature on earth with similar size and build. An increased pressure gradient means that the body must expend more energy to oxygenate its blood. If we hold *dz* constant, then in order to minimize the energy the heart expends the organ must become more efficient. This can be achieved by increasing the size, and consequently strength, of the heart. With each pump, a greater amount of blood is sent through the veins, so the heart has to spend less energy per unit of blood pumped. Thus, a creature on super-earth with earth-length appendages will need to develop a larger heart than its counterpart on earth.

We deem it probable that a super-earth creature's body will show a combination of the alterations in Cases 1 and 2. The creature will most likely have shorter appendages and also have a larger heart than its doppelganger on earth.

2.2.1 Blood Volume as a Function of Weight

By basic principles of geometry, it is possible to express the relationship between volume and surface area as a shape grows in size. As an object undergoes a proportional size increase, the final volume is proportional to the cube of the initial volume and the final surface area is proportional to the square of the initial surface area. That is,

$$V_2 = V_1 (\frac{l_2}{l_1})^3$$

where V_1 and V_2 are the initial and final volumes, respectively, and l_1 and l_2 are the initial and final lengths of the object. The final area is described as,

$$A_2 = A_1 (\frac{l_2}{l_1})^2$$

where A_1 and A_2 are the initial and final areas, respectively, and l_1 and l_2 are the initial and final lengths of the object. This scaling difference between volume and surface area provides the basis of allometry, the study of the relationship of body size to physiology and anatomy.

It is clear that bone cross-sectional area will not scale proportionally, and must instead be described using an allometric relationship [5]. Allometric scaling laws for various physical quantities have been found both theoretically and experimentally. Of particular interest are variations in characteristic radius, length, and number of capillary blood vessels as a function of gravitational acceleration. These quantities were determined by T.H. Dawson [3] to be:

$$r_c \sim g^{\frac{1}{12}}$$

$$l_c \sim g^{\frac{5}{24}}$$

$$n_c \sim g^{\frac{5}{8}}$$

where r_c, r_c, and r_c are the characteristic radius, length, and number of blood vessels, respectively. $\sim$ denotes proportionality. Hence as gravitational acceleration increases, blood vessels increase in radius, length, and number. These laws apply both to single pulmonary beds and beds of capillaries in individual organs. Thus, ignoring other factors, organ size would scale with g in the same proportions. Additionally, we can consider the change in the total volume of blood in the capillaries. If we assume that capillaries can be modeled as cylinders with radius r_c and length l_c, then the total volume of blood in the capillaries is,

$$V = \pi n_c l_c r_c^2$$

Of course, capillaries exhibit both curvature and tapering, so we can only conclude that,

$$V \sim n_c l_c r_c^2$$

Hence,

$$V \sim g$$

So blood volume increases proportionally with gravitational acceleration.

2.3   Musculoskeletal System

We model the skeletal system by considering the normal and shear stresses on a bone as a function of gravity. Stress describes the internal forces that particles of a continuous material exert on one another. Stress can be broken down into normal and shear components. Here, we consider a bone with cross sectional area A to be a cylinder of uniform composition. The normal and shear stresses can be expressed as

$$\sigma = \frac{F_N}{A}$$

$$\tau = \frac{F_P}{A}$$

Where σ is the normal stress, τ is the shear stress, F_N is the component of force normal to the cross sectional area of the bone, F_P is the component of force perpendicular to the bone, and A is the cross sectional area of the bone. This can be expressed in terms of the gravitational acceleration g as,

$$\sigma = \frac{mg\cos\theta}{A}$$

$$\tau = \frac{mg\sin\theta}{A}$$

Where m is the mass of the creature and θ is the angle of attachment between the bone and the body.

As an animal walks, a vertical force of mg acts on the foot, exerting a torque of $mgl\sin\theta$ about the joint halfway up the leg. To maintain equilibrium, the muscle must exert an equivalent torque. If the muscle has a cross-sectional area A, a moment arm of r about the joint, and exerts a stress σ_m, then the torque about the joint exerted by the muscle is,

$$\tau = A\sigma_m r$$

The system is in equilibrium, so,

$$A\sigma_m r = mgl\sin\theta$$

$$\sigma_m = \frac{mgl}{Ar}\sin\theta$$

In modeling variations of the musculoskeletal system as a function of gravity, we consider variations in A, θ, and bone length l. Considering the decomposed stresses and the above relationship for the stress exerted by muscle, we can constrain these variables. Let us first consider the cross sectional area A. As gravitational acceleration increases, A must increase to maintain constant stress to prevent the bone from breaking. To maintain constant total stress,

$$s = \sqrt{\sigma^2 + \tau^2} = \frac{mg}{A}$$

must remain constant as g changes. Hence, because g doubles, A must also double.

Next, we consider the angle θ between the bone and body. θ can vary between 0 and $\frac{\pi}{2}$. On earth, smaller mammals tend to exhibit higher angles, with θ decreasing with increasing mass, consistent with the equation for σ_m above assuming constant muscle stress. Elephants, for example, have column-like femurs connecting nearly perpendicularly to their body [2]. We can verify that bones will not break for $\theta < \frac{\pi}{2}$ by considering shear stress for the case of maximum (breaking point) shear. The maximum allowable θ is,

$$\theta_{max} = \sin^{-1} \frac{\tau_{max} A}{mg}$$

where τ_{max} is the maximum shear that bone can withstand before fracturing, experimentally determined to be 51.6 MPa for human femurs [10]. This value is nearly constant between different species. Even for exceptionally large masses, the maximum shear will not be reached at θ_{max}, a conclusion which seems both sensible and reassuring. Thus, we can safely conclude that $0 < \theta < \frac{\pi}{2}$.

Finally, we consider the length l of the bone. Longer bones are more susceptible to fracture as gravitational acceleration increases, and thus we constrain bone length to $0 < l < l_0$ where l_0 is bone length when gravitational acceleration is g. By assuming that the stress applied by muscle is relatively constant – i.e. the physical properties of muscle do not vary as a function of gravitational acceleration – and that muscle cross sectional area does not change considerably [1], we can use the equation for σ_m to plot bone length and attachment angle, as shown below in fig. 2.3.1. The constants used to generate these curves were approximated for a human femur and quadriceps, although the same relationship is seen for any number of legs (see Appendix 1).

The corner of this plot represents the point at which the product of l and θ is minimized. Hence, the corner is the point at which muscle stress σ_m is minimized. Presuming that an organism would evolve to minimize muscle stress, this corner represents the ideal values of l and θ. Comparing the curve for $2g$ with that for $1g$, we see that both l and θ decrease as gravitational acceleration increases. Thus, we expect bone length to decrease and the angle between the bones and body to decrease (animal legs will straighten).

From this plot, we can estimate that bone length and attachment angle will both decrease by approximately 30% when surface gravity doubles. As discussed in section 4.1, this estimate is

subject to considerable error, though we can confidently state that both bone length and attachment angle decrease as gravitational acceleration increases.
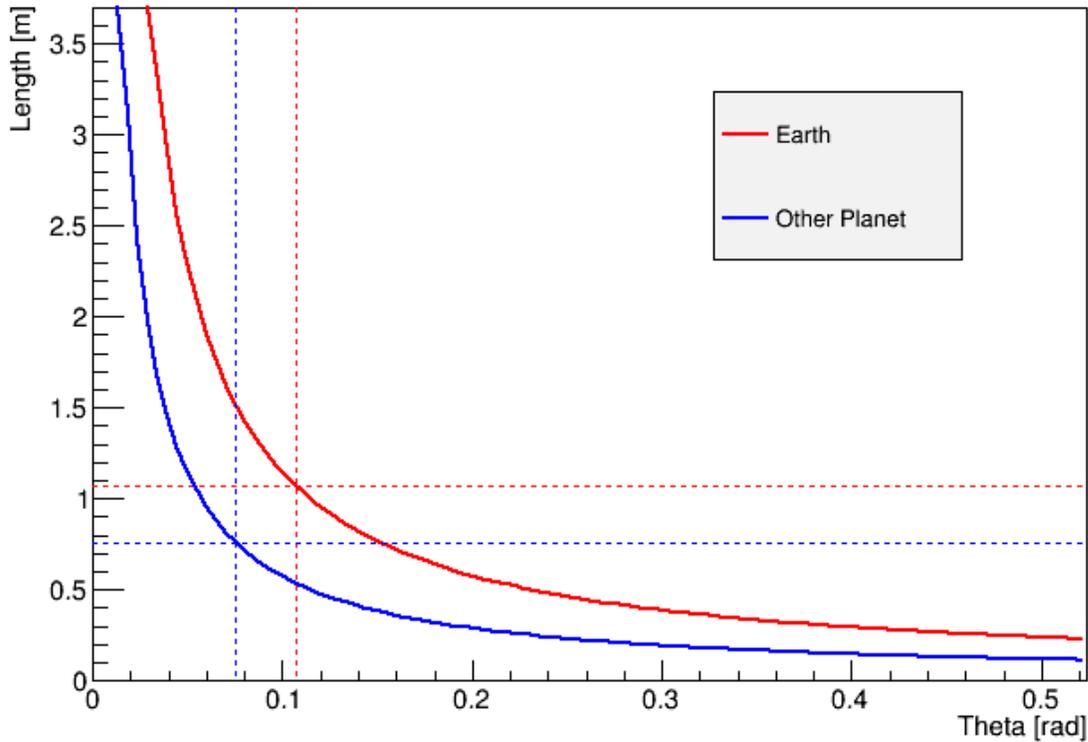

Fig. 2.3.1 *Plot showing the relationship between bone length and attachment angle on earth (1g) and the other planet (2g). The optimal length and angle for minimizing applied muscle stress is denoted by the corner of each curve, marked by the cross-hairs.*

2.4   Gait Patterns

Here we analyze the dynamics of legged locomotion in a gravitational field by treating a walking limb as an inverted pendulum. The center of mass of the pendulum swings through a circular arc centered at the base of the limb. The gait can be analyzed by calculating the Froude number, defined as

$$F_r = \frac{v}{c}$$

where v is a characteristic velocity and c is a characteristic wave propagation velocity. In our model of locomotion,

$$F_r = \frac{F_c}{F_G}$$

where F_c is the centripetal force about the center of motion and F_G is the gravitational force on the walking animal. Hence,

$$F_r = \frac{\frac{mv^2}{l}}{mg} = \frac{v^2}{gl}$$

Where m is animal mass, v is velocity, g is acceleration due to gravity, and l is the characteristic length, which we define as total leg length [1, 2].

From this equation, we can also calculate the stride frequency as,

$$F_r = \frac{v^2}{gl} = \frac{l^2 f^2}{gl} = \frac{lf^2}{g}$$

Froude numbers essentially represent the dynamics of the object's movement. In other words, if two objects have the same Froude number, then they are dynamically similar. As Alexander explains, the transition from walking to trotting for quadrupeds on earth appears to happen at a Froude number between 0.3 and 0.8. The transition from trotting to galloping occurs between 2.0 and 3.0 [2].

When looking at the Froude number equation, it is observed that an animal moving at some speed v with characteristic length l on earth would have half the Froude number on the exoplanet. At the speed of a gallop on earth, for example, an animal would merely be trotting on the exoplanet. Another implication of Froude numbers is that for similar dynamics the stride frequency of an animal would be larger on the exoplanet than on earth. These results are summarized in the figures below where we have plotted Froude number and characteristic length to derive either the velocity or stride frequency. These results do not give precise information on the maximum speed of the animal. Rather, they suggest that as gravitational acceleration increases, it becomes increasingly difficult for animals to reach more efficient modes of locomotion; running is a more efficient mode of locomotion than walking, and hence animals prefer to run above a certain velocity. Animals on the exoplanet would take more strides and do so in a less efficient mode of locomotion for a given velocity, suggesting that their maximum speed would be lower than that of their earth counterpart.
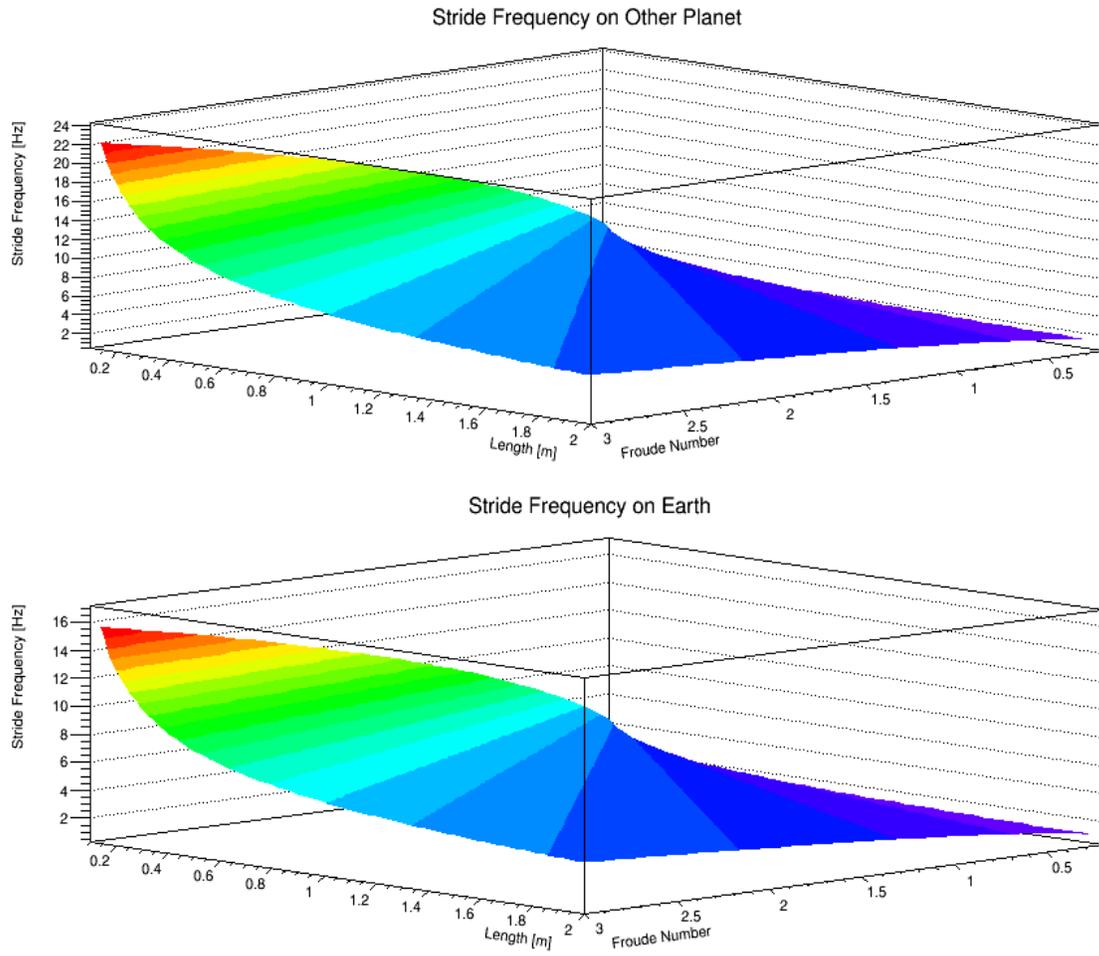
Fig. 2.4.1 *Stride length as a function of Froude number and leg length in 1g and 2g*
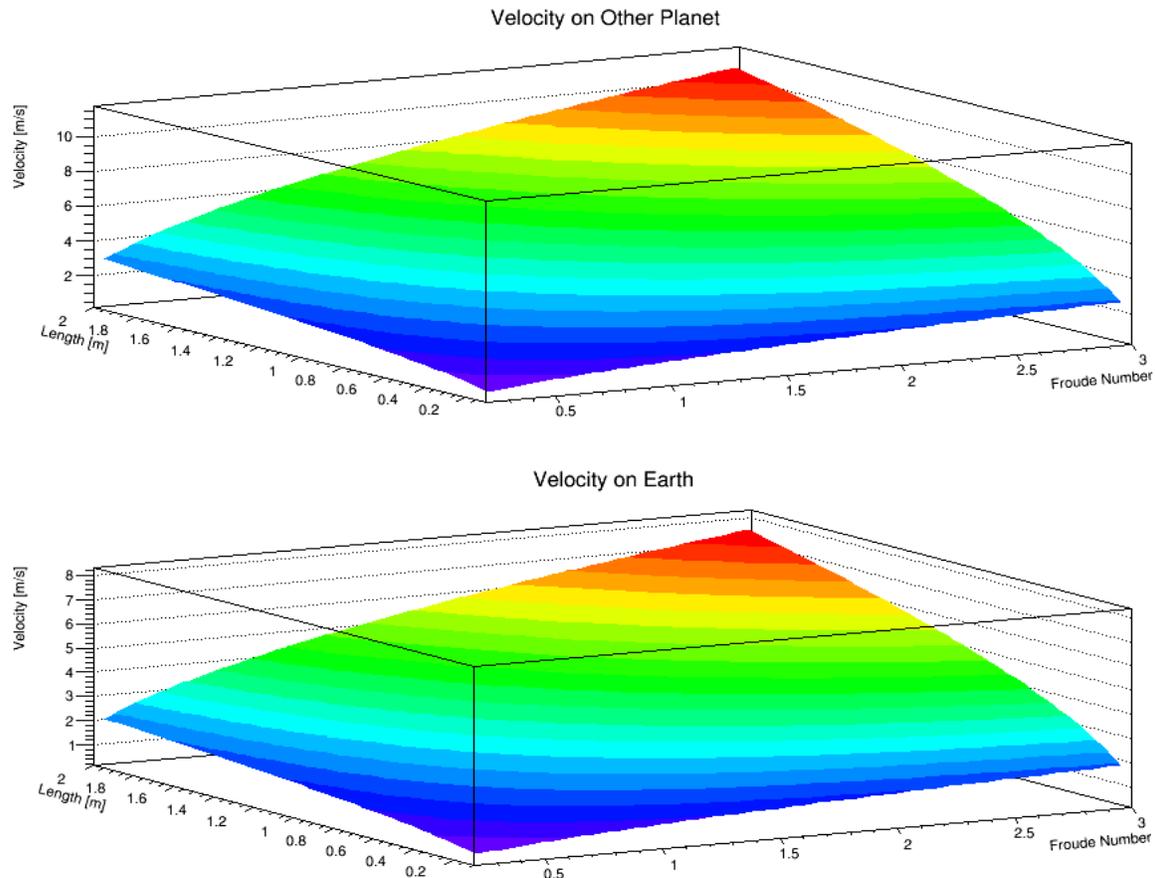
Fig. 2.4.2 *Velocity as a function of Froude number and leg length in 1g and 2g*

3 Additional Considerations

3.1 Effects of surface temperature

Our super-earth has an average temperature of 250K, well below the freezing point of water at 273.15K. This poses a challenge to life's dependence on liquid water. By Lambert's cosine law, we know that the amount of insolation the planet receives falls off as $\cos\theta$, where theta is the sun's rays and the normal to the planet's surface [11]. Thus, the poles will have a much lower temperature than the equator, just like earth. Earth has an average surface temperature of 288K [12] and ranges from an average temperature of about 215K at the South Pole to an average of about 300K at the equator. Let assume that the average temperature of the super-earth ranges from 190K at the poles to 285K at the equator. We note that the earth still has liquid water in the form of oceans at the poles due to currents transporting thermal energy from the equator. Let us also assume that the super-earth has oceans and currents allowing liquid water to exist at the poles. Life on earth exists at the poles. Polar bears, for example, are able to obtain water by

eating snow and prey. Their bodies break down the blubber from their prey to obtain the water it contains. Creatures living in the poles also have a thick layer of blubber to provide insulation. Thus, if life exists at the poles on the super-earth it must have some sort of insulation in the form of blubber and/or hair coat and be able to obtain water from the food it eats. Further, the poles of the super-earth are much colder than those of earth, so it is likely that creatures will either a) have much more powerful forms of insulation and obtaining water and/or b) live closer to the equator than to the pole of the hemisphere. If the average temperature of the super-earth equator is 285K, then life has the ability to be similar to what we find on earth in terms of temperature adaptations. Liquid water could also be easily found here, so it is probably that the bulk of the planet's biomass.

An interesting note is that the average surface temperature of the super-earth is roughly equivalent to the effective temperature of earth. We assume that the super-earth has an atmosphere and some plate tectonic activity, so the super-earth's effective temperature will be much lower than 250K.

The average surface temperature of earth is much larger than its effective temperature of 252K found from equating the absorbed power to the Stefan-Boltzman law:

$$P = A\varepsilon\sigma T^4$$

where P is the total power radiated from the object, A is the object's surface area, ε is the object's emissivity, σ is the Stefan-Boltzman constant, and T is the black body temperature. The surface temperature of earth is much higher because of the effect of greenhouse gases, in part produced from the earth's plate tectonic activity.

3.2   Effects of Atmospheric Pressure

We now consider the changes due to atmospheric pressure on the exoplanet as compared to the earth. It is well known that atmospheric pressure is described as follows:

$$P(h) = P_0 e^{-h/H} \qquad H = \frac{k_B T}{mg}$$

where P_0 is the pressure at the surface of the planet, H is the scale height of the atmosphere, k_B is Boltzmann's constant, T is the temperature of the atmosphere, m is the mean molecular mass of the molecules in the atmosphere, g is the acceleration due to gravity, and h is the height above the surface. To model the pressure differences between the earth and the exoplanet, we have made the ansatz that m will be essentially the same between the two planets. This is supported by the idea that the atmospheric composition of the two planets should be similar if there exist living creatures on both planets. Using T=250 K and $m = 4.808 \times 10^{-26}$ kg, we obtain a scale height of $H_{exo} = 3.661$ km while the average $H_{Earth} = 7.4$ km.

If we assume that the surface pressure of the exoplanet is equal to that of the earth, we observe that the atmospheric pressure of the exoplanet falls off more rapidly than that of the earth (see figure below). This indicates that any airborne creatures living on the exoplanet would have to compensate for a greater pressure difference over the change in height between which they fly

and where they obtain their food and water. Because compensating for an increase pressure gradient requires more metabolic energy expenditure, any flying creature would have to have better methods of expending less energy through this compensation. There is also the possibility that these airborne creatures would just feed on other airborne creatures creating two relatively independent food chains, one ground based and one air based.
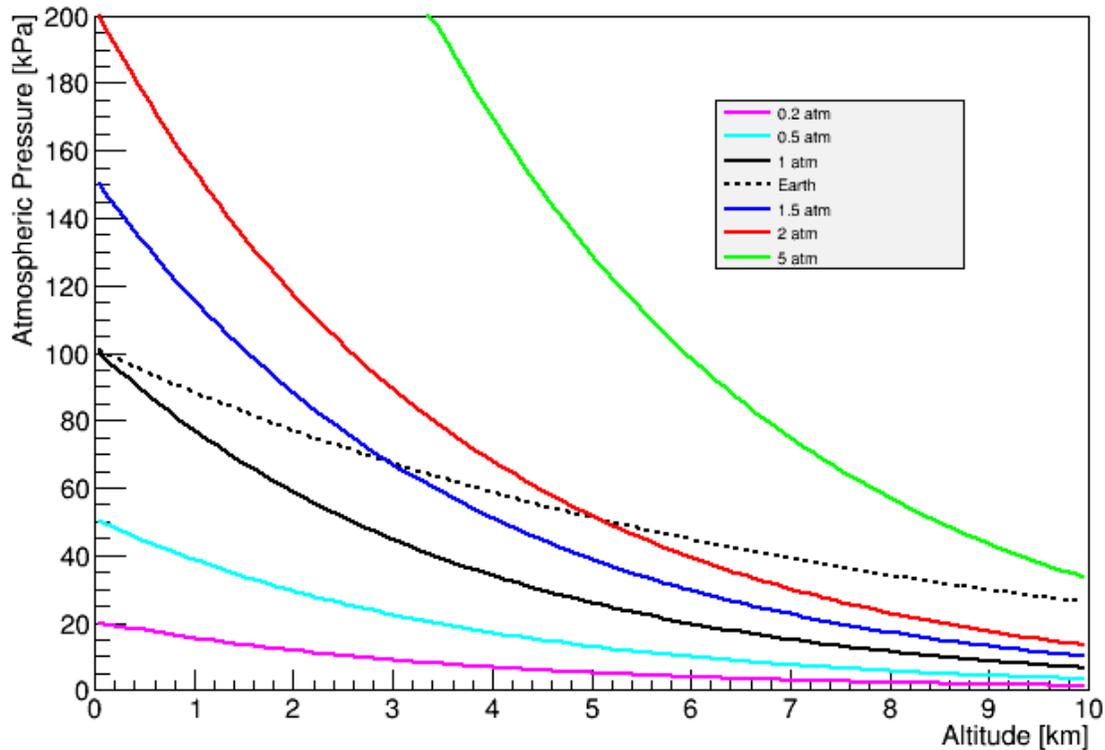

Fig. 3.3.1 *Atmospheric pressure as a function of altitude. The solid lines indicate the pressure for the exoplanet given certain surface pressures. The dotted line represents the pressure fall off for earth.*

Conversely, if we were to assume that the atmosphere had the same composition as that of earth, then considering a given air column,

$$P = \frac{F}{A} = \frac{mg}{A}$$

Surface pressure would increase, leading to a higher partial pressure of oxygen at the surface and, consequently, less demand for red blood cells to carry oxygen.

4   Results Summary

We can combine the results of our above analyses to provide a general picture of life on this planet. For illustration purposes, we consider a four-legged animal living on the equator.

4.1   Changes in Musculoskeletal System and Locomotion

The cross sectional area of supporting bones is expected to double. The length of supporting bones will decrease. Based on figure 2.3.1, we predict a decrease in bone length on the order of 30%. Additionally, the angle between supporting bones (i.e. femurs) and the body will decrease such that legs will become more columnar than those found on earth. While we assumed that the proportion of muscle volume to body volume will remain relatively constant, as is true on earth, there is some allowance for increases in the proportion of muscle volume to counteract increased gravitational acceleration. Finally, it would be more difficult for the animal to run and the top speed would likely be lower.

4.2   Changes in the Cardiovascular System

Increased blood pressure gradients demand that animals in higher-gravity environments have more compact bodies with shorter appendages located closer to the heart. Because the heart must pump harder to send blood the same distance as on earth, we expect heart size to increase due to the demand for stronger heart muscles. Additionally, we expect capillary size to increase, leading to a blood volume approximately twice that of the earth counterpart. This also requires increased organ size. The demand for higher blood volume is consistent with the implication from the Navier-Stokes equations that heart size must increase. Additionally, the likelihood of higher atmospheric surface pressure suggests that in order to function at a similar level as its earth counterpart, the animal would require lower blood-oxygen saturation thanks to the higher partial pressure of oxygen at the surface.

5   Discussion

5.1   Weaknesses

Our approach to solving this problem entails several inherent weaknesses. The cornerstone of our analysis of the cardiovascular system lies with viscous flow described by the Navier-Stokes equations outlined in section 2.2. This partial differential equation proved exceedingly difficult to solve numerically, and we instead had to provide a somewhat qualitative analysis of the equation's consequences, which, while illuminating, is not completely rigorous. We did not consider changes in blood flow as capillary diameter approached the dimensions of red blood cells due to the presence of turbulent flow, but this could impact the functionality of an animal's extremities [4]. Additionally, if blood volume indeed increases in proportion with gravitational acceleration, then volume approaches infinity in the limiting case, indicating that this relationship fails at higher gravitational accelerations.

In analyzing the musculoskeletal system, we assumed that the proportion of muscle volume remains relatively constant as a function of gravitational acceleration, a supposition supported by the literature [1]. However, it is possible for muscle mass to increase exponentially without violating the equilibrium condition used to solve for σ_m. It stands to reason that this and other quantities in our expression for σ_m may be somewhat variable, which could impact fig. 2.3.1.

Perhaps more significantly, we assumed that evolution will limit the muscle stress by selecting for l and θ values found at the corners of the curves in this plot. It is entirely possible, however, for l to remain constant while θ changes more considerably, and vice versa, while the bone remains within the stress threshold.

Although we modeled stride frequency and velocity as a function of leg length and Froude number, this did not provide us with a quantitative assessment of top running speed. It may be possible to assess this quantitatively by considering maximum stride frequency and the normal stresses at which bone fractures.

5.2   Strengths

We began by ruling out factors that could affect the creature's structure. We found that the rotation rate of the planet does not considerably affect the acceleration of objects near the surface. This allowed us to restrict our investigation to the effects of the increase in gravitational acceleration on creatures.

When discussing the effects of the increased blood pressure gradient in our cardiovascular section, we considered multiple cases. Our inclusion of these cases allowed us to more intensively compare super-earth creatures to creatures on earth. By doing so, we were also able to allow for the diversity of life that could be found on the planet from variations in appendage length and heart size.

The independent modelling of the cardiovascular and musculoskeletal systems led us to the same conclusions about changes in shape and stature, which indicates their efficacy in solving this problem. In particular, the cardiovascular model limits height and discourages extremities due to increased blood pressure gradients while the musculoskeletal model limits height due to increased normal and shear stresses. In addition, we maintained a consistent conclusion that blood volume and organ size increase while appendage length decreases. The agreement between these models is encouraging.

In our discussion of the musculoskeletal system, we considered the maximum shear that bone can withstand before fracturing to consider our bounds on the angle between bone and body. One of the most important components of our analysis was the ability to relate bone length and bone-body interface angle using the "l-curve" plot shown in fig. 2.3.1 following the assumption that the skeletal system can be modeled by decomposing shear and normal stresses. This allowed us to express how each quantity varies with respect to the other and to predict, ideally, the value for both quantities given information about muscle stress σ_m.

6   Closing Remarks

We conclude that on a planet with a mass eight times that of earth and a radius twice that of earth, life would differ significantly. Bone width and length would increase and decrease, respectively, and animals would become more compact with shorter extremities. Organs, including the heart, would increase in size, as would capillaries. Leg bones would meet the body at shallower angles, making the legs straighter and less mobile. Thus, in general, animals would likely be shorter, slower, and stockier.

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Appendix 1

Values used for fig. 2.3.1

Variable	Symbol	Quantity	Units
Muscle Stress	σ	$100*10^3$	$\dfrac{N}{m^2}$
Bone cross-sectional area	A	273	m^2
Bone radius	r	3	m
Animal Mass	M	74.1	kg

Values were approximated based on the human femur and quadriceps. The same arguments from section 2.3 extend to animals with more limbs.

Appendix 2

Code used to generate figures (All coding done in C++)

```cpp
void RotationalForce(){

  TCanvas *c1 = new TCanvas("Rotational_Force", "Rotational Force", 0, 0, 700, 500);

  c1->cd();
  TF1 *f1 = new TF1("Length vs. Theta on earth", "1.12/(sin(x)*9.8)" ,0
,TMath::Pi()/6);
  f1->SetTitle(" ");
  f1->GetYaxis()->SetTitle("Length [m]");
  f1->GetXaxis()->SetTitle("Theta [rad]");
  f1->GetYaxis()->SetRangeUser(0,3.7);
  f1->Draw();

  TLine *l1v = new TLine(.1068,0,.1068,3.7);
  TLine *l1h = new TLine(0,1.12/(sin(.1068)*9.8),TMath::Pi()/6,1.12/(sin(.1068)*9.8));
  l1v->SetLineColor(2);
  l1h->SetLineColor(2);
  l1v->SetLineStyle(2);
  l1h->SetLineStyle(2);
  l1v->Draw("same");
  l1h->Draw("same");

  TF1 *f2 = new TF1("Length vs. Theta on Other Planet", "1.12/(sin(x)*19.6)" ,0
,TMath::Pi()/6);
  f2->SetTitle("Length vs. Theta on Other Planet");
  f2->GetYaxis()->SetTitle("Length");
  f2->GetXaxis()->SetTitle("Theta");
  f2->SetLineColor(4);
  f2->Draw("same");

  TLine *l2v = new TLine(.075557,0,.075557,3.7);
  TLine *l2h = new
TLine(0,1.12/(sin(.075557)*19.6),TMath::Pi()/6,1.12/(sin(.07557)*19.6));
  l2v->SetLineColor(4);
  l2h->SetLineColor(4);
  l2v->SetLineStyle(2);
  l2h->SetLineStyle(2);
  l2v->Draw("same");
  l2h->Draw("same");

  TLegend *leg = new TLegend(.6,.6,.8,.8);
```

```cpp
    leg->AddEntry(f1, "earth", "l");
    leg->AddEntry(f2, "Other Planet", "l");
    leg->Draw();

}



void FroudeNumber(){

  TCanvas *c1 = new TCanvas("Froude_number", "Froude Number", 0, 0, 1300, 650);
  c1->Divide(2,2);

  c1->cd(1);
  TH2F *h2vother = new TH2F("testingplots", "Velocity on Other Planet",50, 1, 3, 50,
0.1, 2);
  for(int i=1; i<=50; i++){
    for(int j=1; j<=50; j++){
      double points = sqrt(h2vother->GetXaxis()->GetBinCenter(i)*h2vother->GetYaxis()-
>GetBinCenter(j)*19.6);
      h2vother->SetBinContent(i,j,points);
    }
  }
  h2vother->GetXaxis()->SetTitle("Froude Number");
  h2vother->GetYaxis()->SetTitle("Length [m]");
  h2vother->GetXaxis()->SetTitleOffset(1.5);
  h2vother->GetYaxis()->SetTitleOffset(1.5);
  h2vother->GetZaxis()->SetTitle("Velocity [m/s]");
  h2vother->SetStats(false);
  h2vother->Draw("SURF2");

  c1->cd(2);
  TH2F *h2vearth = new TH2F("testingplotse", "Velocity on earth",50, 1, 3, 50, 0.1,
2);
  for(int i=1; i<=50; i++){
    for(int j=1; j<=50; j++){
      points = sqrt(h2vearth->GetXaxis()->GetBinCenter(i)*h2vearth->GetYaxis()-
>GetBinCenter(j)*9.8);
      h2vearth->SetBinContent(i,j,points);
    }
  }
  h2vearth->GetXaxis()->SetTitle("Froude Number");
  h2vearth->GetYaxis()->SetTitle("Length [m]");
  h2vearth->GetXaxis()->SetTitleOffset(1.5);
  h2vearth->GetYaxis()->SetTitleOffset(1.5);
  h2vearth->GetZaxis()->SetTitle("Velocity [m/s]");
  h2vearth->SetStats(false);
  h2vearth->Draw("SURF2");

  c1->cd(3);
  TH2F *h2fother = new TH2F("testingplotsf", "Stride Frequency on Other Planet",50, 1,
3, 50, 0.1, 2);
  for(int i=1; i<=50; i++){
    for(int j=1; j<=50; j++){
      points = sqrt(h2fother->GetXaxis()->GetBinCenter(i)/h2fother->GetYaxis()-
>GetBinCenter(j)*19.6);
      h2fother->SetBinContent(i,j,points);
    }
  }
  h2fother->GetXaxis()->SetTitle("Froude Number");
  h2fother->GetYaxis()->SetTitle("Length [m]");
  h2fother->GetXaxis()->SetTitleOffset(1.5);
  h2fother->GetYaxis()->SetTitleOffset(1.5);
  h2fother->GetZaxis()->SetTitle("Stride Frequency [Hz]");
  h2fother->SetStats(false);
  h2fother->Draw("SURF2");

  c1->cd(4);
  TH2F *h2fearth = new TH2F("testingplotfe", "Stride Frequency on earth",50, 1, 3, 50,
0.1, 2);
  for(int i=1; i<=50; i++){
    for(int j=1; j<=50; j++){
```

```cpp
      points = sqrt(h2fearth->GetXaxis()->GetBinCenter(i)/h2fearth->GetYaxis()-
>GetBinCenter(j)*9.8);
      h2fearth->SetBinContent(i,j,points);
    }
  }
  h2fearth->GetXaxis()->SetTitle("Froude Number");
  h2fearth->GetYaxis()->SetTitle("Length [m]");
  h2fearth->GetXaxis()->SetTitleOffset(1.5);
  h2fearth->GetYaxis()->SetTitleOffset(1.5);
  h2fearth->GetZaxis()->SetTitle("Stride Frequency [Hz]");
  h2fearth->SetStats(false);
  h2fearth->Draw("SURF2");
}


void CentripitalForce(){

  TCanvas *c1 = new TCanvas("Centripital_Force", "Centripital Force", 0, 0, 700, 500);
  c1->cd();
  c1->SetLogx();
  c1->SetLogy();

  TF1 *f1 = new TF1("Acceleration vs. Rotation Period","x^2*.003436",0.1,30);
  f1->SetTitle("Acceleration vs. Rotation Period");
  f1->GetYaxis()->SetTitle("Relative Rotational Acceleration");
  f1->GetXaxis()->SetTitle("Planet Rotational Period (rev/day)");

  f1->Draw();

}
void Pressure(){


  TCanvas *c1 = new TCanvas("Pressure", "Pressuure", 0, 0, 700, 500);
  c1->cd();

  double scaleheight=1.38*pow(10,-23)*250/(4.808*pow(10,-26)*2*9.8)/1000;
  std::cout<<scaleheight<<std::endl;

  TF1 *f1 = new TF1("1 atm","101.325*exp(-1*x/3.66099)",0,10);
  f1->SetTitle(" ");
  f1->SetLineColor(1);
  f1->GetYaxis()->SetTitle("Atmospheric Pressure [kPa]");
  f1->GetXaxis()->SetTitle("Altitude [km]");
  f1->GetXaxis()->SetRangeUser(0,10);
  f1->GetYaxis()->SetRangeUser(0,200);
  f1->Draw();

  TF1 *earth = new TF1("2 atm", "101.325*exp(-1*x/(2*3.66099))" ,0 ,10);
  earth->SetLineColor(1);
  earth->SetLineStyle(2);
  earth->Draw("same");

  TF1 *f2 = new TF1("2 atm", "2*101.325*exp(-1*x/3.66099)" ,0 ,10);
  f2->SetLineColor(2);
  f2->Draw("same");

  TF1 *f3 = new TF1("5 atm", "5*101.325*exp(-1*x/3.66099)" ,0 ,10);
  f3->SetLineColor(3);
  f3->Draw("same");

  TF1 *f4 = new TF1("1.5 atm", "1.5*101.325*exp(-1*x/3.66099)" ,0 ,10);
  f4->SetLineColor(4);
  f4->Draw("same");

  TF1 *f5 = new TF1(".5 atm", ".5*101.325*exp(-1*x/3.66099)" ,0 ,10);
  f5->SetLineColor(7);
  f5->Draw("same");
```

```cpp
TF1 *f6 = new TF1(".2 atm", ".2*101.325*exp(-1*x/3.66099)" ,0 ,10);
f6->SetLineColor(6);
f6->Draw("same");



TLegend *leg = new TLegend(.6,.6,.8,.8);
leg->AddEntry(f6, "0.2 atm", "l");
leg->AddEntry(f5, "0.5 atm", "l");
leg->AddEntry(f1, "1 atm", "l");
leg->AddEntry(earth, "earth", "l");
leg->AddEntry(f4, "1.5 atm", "l");
leg->AddEntry(f2, "2 atm", "l");
leg->AddEntry(f3, "5 atm", "l");
leg->Draw();

}
```