University Physics Competition 2014

Topic A: Circumbinary Planets

Team 726

November 17, 2014

Abstract

This paper investigates the stability of planetary orbits around a binary star system. We constructed a simulation of the system's behavior over time based on Kepler's and Newton's laws of mechanics and gravitation. The resulting system of equations was solved numerically using Mathematica and the results were analyzed using Mathematica and Matlab. From the results, we can conclude that the shortest distance from the center of mass of the given system where orbits are still consistently stable after 1000 years is 0.6 AU.

1 Background

Stars can exist as single stars or as components of a system of multiple stars gravitationally bound to each other and orbiting around their center of mass. When pairs of stars are bound in such a way, they are called binary stars. Observational studies in the last few decades have shown that there are at least as many binary stars in the observable universe as single ones [2] [6]. This naturally makes it of interest whether or not planetary orbits are stable around such star systems. Through the observation of circumbinary planets, it is known that these are possible [8]. However, the parameters for stability are not very well known.

A planet in a binary star system is a three body problem and analytical solutions for the general case are not possible with our present mathematical tools.

Of particular interest are planets in the habitable zone of the binary star system. For circumbinary planets, the irradiance of the star system fluctuates more strongly as a function of time than for a similarly located planet in a single star system and this must be taken into account.

2 Introduction

This paper looks at the orbit of a planet around a binary star consisting of one star with one solar mass and the other with half a solar mass. The period of the star system is 30 Earth days.

The scope of this paper is further narrowed down to planetary motion in the plane of the star system, effectively making this a 2-dimensional problem. Planets may be found orbiting the entire star system, one of the two stars or at the Lagrange points L_4 and L_5 [4]. This paper focuses on *P*-type orbits, i.e., planets orbiting the entire star system.

We have used a numerical integration method to model the motion of the planet. We defined the time dependent gravitational field of the star system. Then we formulated the differential equations describing the motion of the planet. We solved these differential equations for different initial conditions and ran our simulation up to a time period of 10 years, and for a smaller range of values up to 1000 years. At the end of the simulation, we checked whether the planet was still in orbit around the star system.

3 Theory

3.1 Equations of motion of the binary star system

First, we look at the equations of motion of the stars in the star system. This is a two body problem that can be converted into an equivalent one body problem by placing one of the stars at the origin. Kepler's third law of motion for elliptical orbits is obtained from the equations that follow [10, p 12]:

$$T^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)} \tag{1}$$

where M_1 and M_2 are the masses of the two stars.

Fixing the time period T fixes the separation a between the stars in accordance with (1):

$$a = \sqrt[3]{\frac{G(M_1 + M_2)T^2}{4\pi^2}} \tag{2}$$

Transforming back to the center of mass system, one finds that both stars move in elliptical orbits of period T given by (1), with the center of mass at one focus. The semi-major axes of each orbit is weighted by the mass of each star and adds up to a [10, p 14].

$$a_1 = \frac{M_2}{M_1 + M_2} a$$
(3)

$$a_2 = \frac{M_1}{M_1 + M_2} a \tag{4}$$

With the definition of the center of mass, we must require that the eccentricities of both the orbits are the same and that the two stars are always opposite in phase.

3.1.1 Circular orbits

For a star system with no eccentricity, we find that the two stars orbit the center of mass in circular orbits. The radii of the orbits are simply the semi major axes given by (3).

The equation of motion of the stars in Cartesian coordinates are then written down as:

$$\vec{r_1}(t) = a_1 \cos\left(\frac{2\pi}{T}t\right)\hat{x} + a_1 \sin\left(\frac{2\pi}{T}t\right)\hat{y}$$
(5)

and setting the phase shift on M_2 as π :

$$\vec{r_2}(t) = -a_2 \cos\left(\frac{2\pi}{T}t\right)\hat{x} - a_2 \sin\left(\frac{2\pi}{T}t\right)\hat{y} \tag{6}$$

3.1.2 Eccentric orbits

The semi-major axes of the elliptical orbits are given by (3). A constant term is added to the x coordinate to put the center of mass at the origin.

Then the equations of motion of the stars in Cartesian coordinates, are given by:

$$\vec{r_1}(t) = \left[a_1 \cos\left(\frac{2\pi}{T}t\right) + a_1\varepsilon\right]\hat{x} + \sqrt{1-\varepsilon^2}a_1 \sin\left(\frac{2\pi}{T}t\right)\hat{y}$$
(7)

$$\vec{r_2}(t) = -\left[a_2 \cos\left(\frac{2\pi}{T}t\right) - a_2\varepsilon\right]\hat{x} - \sqrt{1-\varepsilon^2}a_2 \sin\left(\frac{2\pi}{T}t\right)\hat{y}$$
(8)

3.2 Gravitational field of the binary star system

The gravitational field of the two stars can be found at any point around the star system using the superposition principle. Under the assumption that the planet's field is weak compared to the field of the two stars, the field at the position of the planet $\vec{r}(t)$ at time t is given by:

$$\vec{E}(t) = G \frac{M_1}{|\vec{r}_1(t) - \vec{r}(t)|^2} + G \frac{M_2}{|\vec{r}_2(t) - \vec{r}(t)|^2}$$
(9)

This allows us to write the differential equations describing the motion of the planet around the star system:

$$\ddot{\vec{r}}(t) = \vec{E}(t) \tag{10}$$

4 Method

Ordinary differential equation (10) was solved numerically using Mathematica for various initial conditions up to a time period of 1000 years. Orbits were considered stable if:

1. At final time t_f , the planet had not reached escape velocity

$$\frac{1}{2}(\dot{r}(t))^2 - G\frac{M_1}{|\vec{r_1}(t) - \vec{r}(t)|} - G\frac{M_2}{|\vec{r_2}(t) - \vec{r}(t)|} < 0$$
(11)

2. At any time in its trajectory, the planet was never within one solar radius of the position of either of the stars

For initial conditions, the smallest distance from the center of mass considered in the simulation was $R_2 + 10$ solar radii. All smaller distances were deemed too close to the outer star to have stable orbits¹.

The largest distance considered was 2 AU. At this distance, the field calculated by taking the two stars separately is within 3% of the simplified field with the mass of both stars located at the center of mass. It was assumed that since the system can be approximated as a two-body system, a stable configuration exists.

50 values of planet distances were used starting from 0.19 AU in steps of 0.04 AU.

In this simulation, the initial velocities were always chosen to have no radial component with respect to the center of mass. For each radius, a rough estimate for initial velocity that would give a stable configuration was found by approximating all the mass of the star system at the center of mass.

$$v_{\text{estimate}} = \sqrt{\frac{G(M_1 + M_2)}{|\vec{r}(0)|}} \tag{12}$$

The range of initial velocities were chosen such that the v_{estimate} for each radius was included with a grace interval on either side. The range of initial velocities used were 5.0-20.0 AU/yr.

150 values of initial velocity were taken starting from 5.0 AU/yr in steps of 0.1 AU/yr.

To reduce computation time for the simulation up to 1000 years, the range of the radii was reduced to 0.19 - 0.75 AU and the step size in velocities was increased by a factor of 3.

The above was implemented using Mathematica 9. Annotated code has been included in Appendix A.

Plots were made using Mathematica 9 and Matlab R2013b.

5 Results

The following are Mathematica created plots for some specific configurations at a given time that illustrate three general cases of orbits that we obtained:

¹This was verified by the results a posteriori.



Figure 1: Snapshot of a stable orbit at t = 1 yr: initial distance from center of mass = 0.4 AU, initial velocity = 12.0 AU/yr.



Figure 2: Snapshot of an unstable orbit at t = 2.6 yr: initial distance from center of mass = 0.4 AU, initial velocity = 7.5 AU/yr. Eventually planet slingshots around star system and reaches escape velocity.



Figure 3: Snapshot of an unstable orbit at t = 0.351 yr: initial distance from center of mass = 0.4 AU, initial velocity = 8.0 AU/yr. Planet collides with the smaller star shortly after snapshot.

5.1 Binary star with circular orbits



Figure 4: Stable configurations in the r_0 - v_0 phase space after simulation of 10 years. Black squares represent data points still considered stable after a simulation period of 10 years.



Figure 5: Stable configurations in the r_0 - v_0 phase space after simulation over 1 month, 1 year and 10 years. Black squares represent data points still considered stable after a simulation period of 10 years.



Figure 6: Stable configurations in the r_0 - v_0 phase space for $\varepsilon = 0$ after simulation of 1000 years. Black squares represent data points still considered stable after a simulation period of 1000 years.

5.2 Binary star with $\varepsilon = 0.4$



Figure 7: Stable configurations in the r_0 - v_0 phase space for $\varepsilon = 0.4$ after simulation of 10 years. Black squares represent data points still considered stable after a simulation period of 10 year.



Figure 8: Stable configurations in the r_0 - v_0 phase space for $\varepsilon = 0.4$ after simulation over 1 month, 1 year and 10 years. Black squares represent data points still considered stable after a simulation period of 10 years.



Figure 9: Stable configurations in the r_0 - v_0 phase space for $\varepsilon = 0.4$ after simulation of 1000 years. Black squares represent data points still considered stable after a simulation period of 1000 years.

5.3 Binary star with $\varepsilon = 0.6$



Figure 10: Stable configurations in the r_0 - v_0 phase space for $\varepsilon = 0.6$ after simulation of 10 years. Black squares represent data points still considered stable after a simulation period of 10 years.



Figure 11: Stable configurations in the r_0 - v_0 phase space for $\varepsilon = 0.6$ after simulation over 1 month, 1 year and 10 years. Black squares represent data points still considered stable after a simulation period of 10 years.



Figure 12: Stable configurations in the r_0 - v_0 phase space for $\varepsilon = 0.6$ after simulation of 1000 years. Black squares represent data points still considered stable after a simulation period of 1000 years.

6 Data analysis

The same trends are seen in plots of all three values of eccentricity. Lower radii have stable orbits at higher velocities. The band of stable configurations gets thinner for smaller radii. This corresponds to the fact that eccentric orbits are less stable, particularly at smaller radii. A smaller range of initial velocities lead to stable orbits.

The upper bound on each plot is a well defined curve. This is because the upper limit is defined by configurations where the initial conditions give the planet escape velocity. This curve is simply given by (11).

The lower bound displays asymptotic behaviour near initial radius = 0.3 AU. This would be the minimum radius for which the a planetary orbit could be stable. From figure 6, 9 and 12, we see that the minimum radius for stable configuration is further pushed up to 0.45 AU.

Comparing the configurations for stable data around star systems with different eccentricities, we observe that the upper bound is identical. However, the band of stable configurations is thinner for higher eccentricities, indicating that there are fewer stable orbits when the star system is more eccentric.

7 Planets in the habitable zone of the binary star system

Our binary star system has stars with fixed masses and fixed separation. This allows us to define limits for a habitable zone around the star system. From our earlier results on whether a stable planetary orbit is possible in the region of interest, we can deduce whether or not habitable planets are possible in the system.

We assume that both stars are in their main sequence. For simplicity we assume zero eccentricity for the binary star system.

From literature, a good approximate for the habitable zone of a star is given by the following [7]:

Inner radius =
$$\sqrt{\frac{L_{\text{star}}}{1.1}}$$
 (13)

Outer radius =
$$\sqrt{\frac{L_{\text{star}}}{0.53}}$$
 (14)

with the radii in astronomical units, and L denoting the absolute luminance.

From this, we derived the maximum and minimum flux that define the boundaries of the habitable zone:

$$Flux_{\min} = \frac{L}{4\pi r_{\min}^2} = \frac{1.1}{4\pi}$$
(15)

$$Flux_{max} = \frac{L}{4\pi r_{max}^2} = \frac{0.53}{4\pi}$$
 (16)

with flux in units associated with AU.

Good approximates for the luminosity of the two stars can be found in terms of solar luminosity, $L_{\rm sol}$, using the mass-luminosity relation. The luminosity of a star with mass close to half of the solar mass is given by $(\frac{M}{M_{\rm sol}})^4 L_{\rm sol}$ [3, p 458] [7].

$$L_1 = L_{\rm sol} \tag{17}$$

$$L_1 = L_{\text{sol}} \tag{17}$$
$$L_2 = \frac{L_{\text{sol}}}{16} \tag{18}$$

This allows us to calculate the flux at the planet at various positions, or specifically, the maximum and minimum flux at a certain distance from the center of mass depending on the relative position of the stars.



Figure 13: The vector (r, ϕ) represents the position of the planet.

Using polar co-ordinates and setting M_1 at $(a_1, 0)$ and M_2 at (a_2, π) , we found the range of irradiance values at the planet located at (r, ϕ) for a given r and ϕ ranging from 0 to 2π .

Flux
$$(r, \phi) = \frac{L_1}{4\pi (r^2 + a_1^2 - 2ra_1 \cos(\phi))} + \frac{L_2}{4\pi (r^2 + a_2^2 - 2ra_2 \cos(\pi - \phi))}$$
(19)

The following graph records the flux in $r - \phi$ phase space.



Figure 14: Graph of flux in phase space $r - \phi$. The boundaries of the light purple band are the equipotentials corresponding to the minimum and maximum flux. The region in the light purple band is the habitable zone.

Figure 14 shows the habitable zone around the binary system, with the light purple band representing the habitable zone. The plot shows the farthest habitable zone, with higher inside and outside radius, at the position $\phi = 0$, whilst the closest habitable zone, smallest inside and outside radius, is found at $\phi = \pi$. As habitable planets should be inside the habitable zone at all times, the habitable region will be adjusted such that the inner radius will be the maximum value of the inner radius (at $\phi = 0$), and the outer radius will be the minimum value of the outer radius (at $\phi = \pi$). Inserting these values into equation (19), this results in an inner radius of 1.05AU and an outer radius of 1.36AU for the habitable zone. Comparing this to the habitable zone from our sun, with inner radius of 0.95 AU and outer radius 1.15 AU [5]. Our habitable zone is shifted outwards, which is reasonable as the total luminosity from our binary star system is larger.

We conclude that there are stable configurations in the habitable region (1.05-1.36 AU) of the star system in question. The range of valid orbital

speeds in this region for all three plots is roughly between 5 and 12 AU/yr which is comparable with Earth's orbital speed of 2π AU/yr (approximately 6 AU/yr).

8 Scope and limitations

8.1 The stars do not interact with the planet's field

The simulation assumes the star system to be static over time and, in particular, unaffected by the gravitational field of the planet. This doesn't conform to conservation of energy as the planet can gain a lot of energy from the star system without any change in the star system. In most cases, the effect should not be very significant since most planets are much less massive than half a solar mass.

This assumption is shaky in the case of bigger planets (such as gas giants). Our simulation recorded cases of close flybys which would result in the planet getting slingshot around the star system. With a massive enough planet, this could cause significant effects in the dynamics of the star system.

8.2 Limited time frame

Limitations of computing power and time meant that the maximum period for which the simulation could be run was only 1000 years. On the astronomical scale, this is a relatively short period and the stability of a dynamic system cannot be unquestionably determined.



Figure 15: Snapshot of a unstable orbit for $\varepsilon = 0.6$ at t = 325 yr: initial distance from center of mass = 0.75 AU, initial velocity = 11.0 AU/yr.

Figure 15 shows an obit that collides with the star system at roughly t = 326 yrs. Simulation up to 10 years still returned stable. It is likewise also possible that orbits returning stable after 1000 years are unstable in the longer run.

Simulation over a longer period would be preferable for thoroughness.

8.3 Limited starting positions

Our initial conditions are not strictly exhaustive. Under the assumption that our initial position and velocity grid is adequately tight to show the significant results, we are only considering one particular orientation explicitly. Our initial conditions always have both stars and the planet in one line on the plane and initial velocity perpendicular to that line. The validity of this lies in the expectation that the other starting conditions will be represented in the subsequent time steps for at least one of the other orbits tested. Even so, there is no result (stable or not stable conclusion) for these initial conditions from other starting orientations.

This study can be extended to include the starting orientation as another free parameter to check for differences.

8.4 The characteristics of stable orbits are not known

Although we can conclude from these results which starting conditions lead to stable orbits, these are not necessarily the defining characteristics of the stable orbits that they lead to. The stable orbits may be elliptic, having a range of radii and orbital speeds. In the case of small initial radii, this is less problematic since non circular orbits are inherently unstable here. The periapsis brings the planet too close to the dynamic star system causing significant trajectory alterations every period.

This study can be extended to track the trajectories of the orbits found to be stable and identify the shape and possibly parameters (radii/semi-major axis, average orbital speed).

8.5 S-type orbits

S-type orbits are those with planets that orbit around one of the component stars of a binary star system. The same method and program can be extended to include these kind of orbits. The radii of S-type orbits are naturally much smaller relatively (roughly between a few solar radii and the shortest distance between the two stars) and the velocities relatively higher. For our given system, the stable radii are much smaller than the habitable zone.



Figure 16: Snapshot of a stable p-type orbit at t = 0.07 yr: initial distance from center of mass = 0.77 AU, initial velocity = 30.0 AU/yr.

9 Conclusion

Stable orbits are more likely to be found further away from the binary star where the combined effect of the two stars closely resembles that of a single star. Closer in, the relative motion of the stars and the resultant changing field leads to the destabilization of orbits. A stable configuration is reached by smaller intervals of velocity (the remaining free parameter) for smaller radii.

There are many temporarily stable configurations that go around the star system in volatile orbits and collide with the stars or escape the system's gravitational pull over longer periods. In this simulation, a drastic decrease in the number of stable configurations was observed from the 10 year simulation to the 1000 year simulation.

For our given system, after 1000 years, the smallest consistently stable radii were around 0.6 AU for all three values of eccentricities.

In general, it was observed that there were fewer stable configurations available for higher eccentricities of the star system.

Lastly, it was found that the habitable zone for this star system is between 1.05 and 1.36 AU and that there are in fact stable configuration available in this region.

Appendix A *Mathematica* Code with annotations

ClearAll["Global`*"] (* All units in astronomical units *) (* Input instrinsic system parameters *) m := 1 (* mass of planet*) M1 := 1 (* mass of star 1*) M2 := 0.5 (* mass of star 2*) $G := (4 \pi^2)$ (* gravitational constant *) $\omega := \frac{2\pi}{m} \; (* \text{ angular frequency } *)$ $T := \frac{30}{365.25} (* \text{ orbital period } *)$ a := $\left(\left(\frac{30}{265,25}\right)^2 1.5\right)^{1/3}$ (* semi-major axis/separation distance *) R1 := $\frac{1}{2}$ a (* center of mass radius Star 1 *) R2 := $\frac{2}{2}$ a (* center of mass radius Star 2 *) $\epsilon := 0.4 (* \text{ eccentricity } *)$ b := a $\sqrt{1 - \epsilon^2}$ (* semi-minor axis *) rstar := 0.005; (* approximate radius of the stars *) (* Parameterized motion of stars *) x1[t_] := $\frac{1}{2}$ a * Cos[ω t] + $\frac{1}{2}\sqrt{a^2 - b^2}$ (*Position of Star 1 x- component*) $y1[t_] := \frac{1}{2}b * Sin[\omega t] (*Position of Star 1 y- component*)$ x2[t_] := $-\frac{2}{3}a * \cos[\omega t] - \frac{2}{3}\sqrt{a^2 - b^2}$ (*Position of Star 2 x- component*) $y_2[t_] := -\frac{2}{2}b * Sin[\omega t]$ (*Position of Star 2 y- component*) (* Mechanism for planetary collision *) (* These terms create force fields around the position of the stars which will deflect the planet to infinity in the case of collision *) (* The influence on the motion of the planet outside the stars surface is zero *) $cpl[t_] := \left(10^3 \star \frac{G(M1 \star M2)}{ratar^2}\right) /$

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\left(\left(1 + E^{-100\,000\,(-(x[t]-x1[t])+rstar)}\right)\,\left(1 + E^{-100\,000\,(x[t]-x1[t]+rstar)}\right)\right) \, \star \right)
     \left(10^{3} \star \frac{G (M1 \star M2)}{rstar^{2}}\right) / \left(\left(1 + E^{-100\,000 \,(-(y[t]-y1[t])+rstar)}\right) \left(1 + E^{-100\,000 \,(y[t]-y1[t]+rstar)}\right)\right) \star \left(1 + E^{-100\,000 \,(y[t]-y1[t])+rstar}\right) + E^{-100\,000 \,(y[t]-y1[t])+rstar}\right) + E^{-100\,000 \,(y[t]-y1[t])+rstar}
     \left(\frac{1}{1+E^{-100\,000\,(x1[t]-x[t])}}-\frac{1}{1+E^{100\,000\,(x1[t]-x[t])}}\right)\star
    \left(\frac{1}{1 + E^{-100\,000\,(y1[t] - y[t])}} - \frac{1}{1 + E^{100\,000\,(y1[t] - y[t])}}\right)
cp2[t_] := \left(10^{3} * \frac{G(M1 * M2)}{rstar^{2}}\right) / \left(\left(1 + E^{-100\,000\,(-(x[t] - x2[t]) + rstar)}\right) (1 + E^{-100\,000\,(x[t] - x2[t] + rstar)})\right) *
     \left(10^{3} \star \frac{G (M1 \star M2)}{rstar^{2}}\right) / \left(\left(1 + E^{-100\,000 (-(y[t]-y2[t])+rstar)}\right) \left(1 + E^{-100\,000 (y[t]-y2[t]+rstar)}\right)\right) \star \left(1 + E^{-100\,000 (y[t]-y2[t]+rstar)}\right) + E^{-100\,000 (y[t]-y2[t])+rstar}\right) + E^{-100\,000 (y[t]-y2[t])+rstar}
     \left(\frac{1}{1 + E^{-100\,000\,(x2[t] - x[t])}} - \frac{1}{1 + E^{100\,000\,(x2[t] - x[t])}}\right) \star
     \left(\frac{1}{1+E^{-100\,000\,(y^2[t]-y[t])}}-\frac{1}{1+E^{100\,000\,(y^2[t]-y[t])}}\right)
(* Input initial conditions *)
(* The test planet will be positioned on the positive x-
  axis with a positive initial velocity along the y direction *)
initialy0 := y[0] == 0
initialvx0 := x'[0] == 0
(* Simulation parameters *)
ti = 0; (* Initial time of simulation *)
tf = 1000; (* Final time of simulation (in years) *)
maxstep = 30000; (* maximum number of steps used by the NDSolve algorithm *)
cf = 50; (* Number of trials for radius *)
vf = 150; (* Number of trials for velocity *)
Success = Table[2, {i, 1, cf}, {j, 1, vf}]; (* Final table for results *)
(* Zero = planet was not on a stable orbit *)
(* One = planet was on a stable orbit *)
i = 1;
j = 1;
While i ≤ cf,
  initialx0 := x[0] == 0.15 + i * 0.04;
  (* parameters for the mesh grid spacing - initial position *)
  While j \leq vf,
    initialvy0 := y'[0] == 4.9 + j * 0.1;
    (* parameters for the mesh grid spacing - initial velocity *)
```

(* Force terms in x and y components *)

$$fx3[t_] := G \frac{mM1}{\left((x[t] - x1[t])^2 + (y[t] - y1[t])^2\right)} \sqrt{\frac{1}{1 + \left(\frac{y[t] - y1[t]}{x[t] - x1[t]}\right)^2}} \\ \left(\frac{1}{1 + E^{-100\,000\,(x1[t] - x[t])}} - \frac{1}{1 + E^{100\,000\,(x1[t] - x[t])}}\right) + G \frac{mM2}{\left((x[t] - x2[t])^2 + (y[t] - y2[t])^2\right)} \\ \sqrt{\frac{1}{1 + \left(\frac{y[t] - y2[t]}{x[t] - x2[t]}\right)^2}} \left(\frac{1}{1 + E^{-100\,000\,(x2[t] - x[t])}} - \frac{1}{1 + E^{100\,000\,(x2[t] - x[t])}}\right) + cp1[t] + cp2[t];$$

(* The terms containing the exponential effectively model a sign function but without discontinuous behavior at the origin *)

$$fy3[t_] := G \frac{mM1}{\left((x[t] - x1[t])^2 + (y[t] - y1[t])^2\right)} \frac{Abs[(y[t] - y1[t])]}{\sqrt{(x[t] - x1[t])^2 + (y[t] - y1[t])^2}} \\ \left(\frac{1}{1 + E^{-100\,000\,(y1[t] - y[t])}} - \frac{1}{1 + E^{100\,000\,(y1[t] - y[t])}}\right) + G \frac{mM2}{\left((x[t] - x2[t])^2 + (y[t] - y2[t])^2\right)} \frac{Abs[(y[t] - y2[t])]}{\sqrt{(x[t] - x2[t])^2 + (y[t] - y2[t])^2}} \\ \left(\frac{1}{1 + E^{-100\,000\,(y2[t] - y[t])}} - \frac{1}{1 + E^{100\,000\,(y2[t] - y[t])}}\right) + cp1[t] + cp2[t];$$

difequx := m x ''[t] == fx3[t]; (* Newtons second law *)
difequy := m y ''[t] == fy3[t];
(* Numerical solution by NDSolve algorithm *)
NAx = x /. First[NDSolve[{difequx, difequy, initialx0, initialy0,

initialvx0, initialvy0}, $\{x, y\}$, $\{t, ti, tf\}$, MaxSteps \rightarrow maxstep]];

$$If \left[-G \frac{MI}{\sqrt{(NAx[tf] - x1[tf])^2 + (NAy[tf] - y1[tf])^2}} - \frac{MI}{\sqrt{(NAx[tf] - x2[tf])^2 + (NAy[tf] - y2[tf])^2}} + \frac{1}{2} (NAx'[tf]^2 + NAy'[tf]^2) < 0,$$

$$G \frac{M2}{\sqrt{(NAx[tf] - x2[tf])^2 + (NAy[tf] - y2[tf])^2}} + \frac{1}{2} (NAx'[tf]^2 + NAy'[tf]^2) < 0,$$

$$Success[[i, j]] = 1, Success[[i, j]] = 0];$$

$$j + +];$$

$$j = 1;$$

$$i + +]$$

$$Export["Results.dat", Success]$$

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