Establishing a Martian Greenhouse Effect Through Directed Impacts

TEAM 190: Problem A - Terraforming Mars

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Abstract

In this report we investigate the effectiveness of asteroid bombardment on Mars in creating a stable atmosphere which allows for the existence of liquid water. The impacts release thermal energy which temporarily raises the average temperature of the Martian atmosphere, thereby allowing carbon dioxide frozen in the poles to be released. We found total mass of the impactors to be 1.225×10^{13}kg, based on a required energy for atmospheric heating and CO_2 sublimation of 2.14×10^{21}J. The impulse required to move these asteroids such that they collide with Mars is on the order of 10^{16} N*s.

1 Introduction

Humanity has been fascinated with Mars since its discovery by ancient civilizations thousands of years ago. The evidence of the past existence of liquid water on Mars' surface has reinvigorated this fascination, and given it a new focus. Modern scientific understanding puts the colonization of Mars in view by giving humanity the ability to approach the problems created by the inhospitable conditions on Mars' surface. In this report we approach the problem of Mars' low surface temperature. A significant amount of carbon dioxide is frozen at the poles, and if released could thicken Mars' atmosphere enough to maintain a temperature which permits liquid water to form. We hypothesize that driving asteroids into the surface of Mars would increase the average temperature to a point which drives the dry ice in the poles to sublimation. The positive-feedback loop caused by this sublimation would lead to a thicker atmosphere and more stable climate conditions.

2 Required Equilibrium Conditions

2.1 Conditions for Liquid Water

We want Mars to be able to sustain liquid water on its surface. To do this, we require Mars to have a surface air temperature of at least 273K and a surface air pressure of at least 612Pa. This was determined with the location of the triple point on the phase diagram of water[5]. Mars already has a sufficient surface air pressure of about 636Pa[12]; however, Mars' surface temperature is another story.

2.2 Blackbody Temperatures

To achieve sustainable liquid water on Mars, we need Mars to have an average surface air temperature slightly above 273K. Using the Stefan-Boltzmann equation, the blackbody temperature of Mars at equilibrium, T_{Mars}, can be determined. $T_{Sun} = 5778K$[13] is the blackbody temperature of the Sun (or surface temperature). $a = 0.17$[12] is the albedo of Mars. $R_{Sun} = 6.96 \times 10^8 m$[13] is the radius of the Sun. $D = 2.2792 \times 10^{11} m$[12] is the average distance between Mars and the Sun.

$$T_{\text{Mars}} = T_{\text{Sun}}(1-a)^{\frac{1}{4}}\sqrt{\frac{R_{Sun}}{2D}} \tag{1}$$

We determine that the blackbody temperature of Mars is about 215K, which isn't enough to satisfy the conditions for liquid water. As a comparison, Earth's blackbody temperature is about 254K, which is also below 273K. Earth's average surface air temperature is actually much higher than 254K. This is due to the greenhouse effect of various gases in Earth's atmosphere, such as CO_2, H_2O, CH_4, CFCs and so on. Mars' atmosphere is comprised of mostly CO_2, and for the purposes of this report, we will consider that it is all CO_2. Assuming this, we can model the effects of different CO_2 levels on heating up Earth's atmosphere and apply it to Mars.

2.3 Optical Depth of CO_2

We can determine the temperature change due to the greenhouse effect using the optical depth, $\tau(n)$, of certain concentrations, n, of gases in the atmosphere. We used a version of Ramanathan's semi-gray model shown in the equation below[8], where T_S is the surface air temperature after the greenhouse effect, T_B is the blackbody temperature of the planet, and $\beta \sim 0.7$[8] is a constant representing the ratio of electromagnetic radiation that is opaque to the gas.

$$\tau(n) = \frac{2}{3}\frac{2T_S^4 - T_B^4}{T_B^4 - (1-\beta)T_S^4} \tag{2}$$

The optical depth of CO_2 on Earth before the industrial era was determined to be our reference to be about 1.73, when the concentration of CO_2 in the atmosphere was about 280ppm[8]. The effects of other greenhouse gases have already been considered by the source. The number density of CO_2 on Earth at this time can be determined by using the concentration as a ratio of the number density of air (about $2.7 \times 10^{25} m^{-3}$[10]). The number density of CO_2 on Earth before the industrial era was about $7.56 \times 10^{21} m^{-3}$.

The ratio of optical depths of different concentrations of CO_2 is equal to the ratio of those concentrations. This is shown in the equation below[8], where n_0 is the pre-industrial concentration of CO_2.

$$\tau_{CO_2}(n) = \frac{n}{n_0}\tau_{CO_2}(n_0) \tag{3}$$

This can be rewritten in terms of a ratio of number densities. N_0 was determined earlier to be about $7.56 \times 10^{21} m^{-3}$.

$$\tau_{CO_2}(N) = \frac{N}{N_0} \tau_{CO_2}(N_0) \tag{4}$$

Using Equation 2, we can determine the optical depths required for liquid water (we'll consider temperatures slightly higher than 273K).

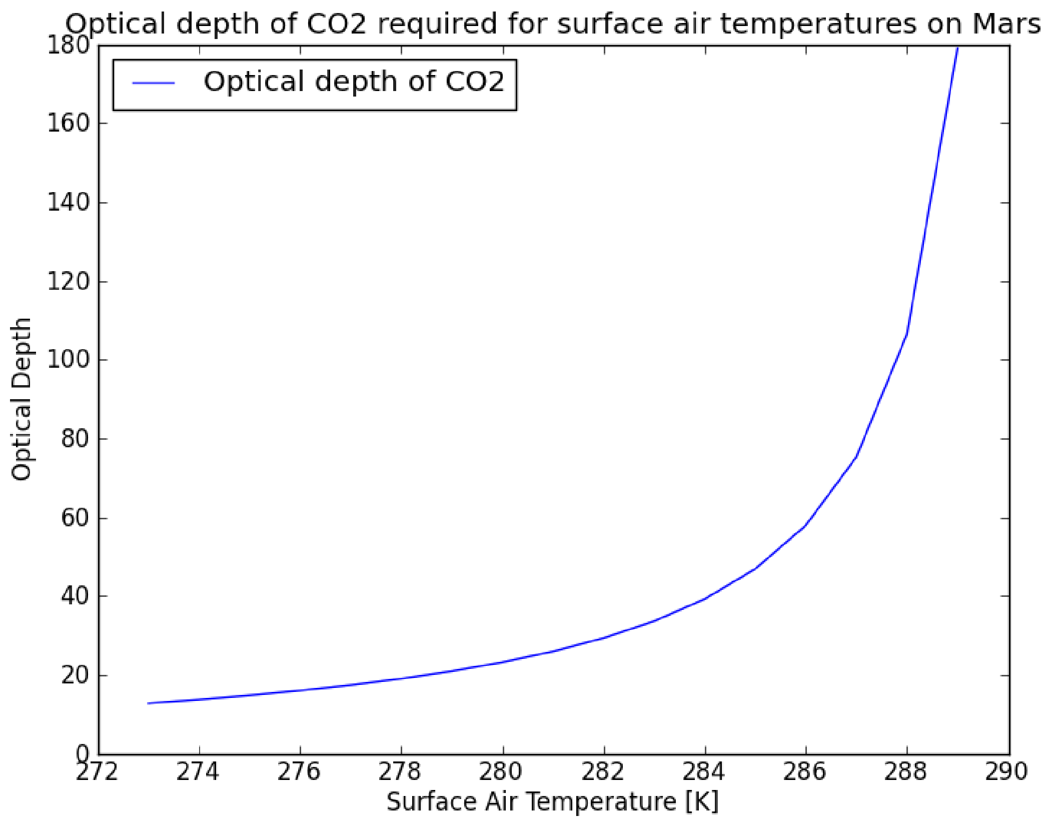

Figure 1: The relation between the optical depth of CO_2 and the surface air temperature

We will need an optical depth of at least 12.72 to provide an equilibrium surface air temperature of 273K on Mars. Using equation 4, we can see that this corresponds to a number density of about $5.56 \times 10^{22} m^{-3}$, or a mass density of about $0.0041 kg/m^3$ (assuming m_{CO_2}~44amu).

In conclusion, Mars requires an atmospheric CO_2 density of over $0.0041 kg/m^3$, as a bare minimum, in order to maintain an average surface air temperature capable of sustaining liquid water.

3 The Current State of Mars

3.1 The Martian Atmosphere

Mars has a current average atmospheric temperature of roughly 210 Kelvin[12]. This is due to the absence of a thick atmosphere which would otherwise trap the sun's energy. Mars' atmosphere has a density of roughly $0.02 kg/m^3$ and a surface pressure of about 6.36mb[12], which are roughly 1.7% and 0.6% of Earth's, respectively. The Martian atmosphere has a composition (by volume) of 95.32% Carbon Dioxide (CO_2), 2.7% Nitrogen (N_2), 1.6% Argon (Ar), 0.13% Oxygen (O_2) and traces of other compounds[12]. The mass of Mars' atmosphere is ~2.5×10^{16}kg, based on a scale height of 11.1km. Both the low temperature and atmospheric pressure prevent the existence of liquid water on

Mars' surface, excepting a limited amount during warmer seasons[1]. We calculate an approximation for the volume of Mars' atmosphere by treating it as a spherical section with a width equal to the scale height. The volume is then given by equation 5 and works out to be $1.59 \times 10^{18}\mathrm{m}^3$.

$$V = \frac{4}{3}\pi(r_{Mars}^3 - r_{Mars+ScaleHeight}^3) \tag{5}$$

3.2 Polar Ice Caps

Mars has two distinct polar caps which are comprised of a surface layer coating of solid carbon dioxide covering a significantly thick layer of water ice[9]. The exact size and shape of these caps has a dramatic seasonal dependence. It is known that during Martian Summers, at times the North pole completely vanishes due to sublimation, while the South pole never completely sublimates. In this paper we are interested only in this surface layer of solid carbon dioxide. The North pole of Mars features a layer approximately 1100km in diameter and 1m thick, while the South pole features a layer approximately 350km in diameter and 8m thick[9]. We compute a volume for the carbon dioxide stored in the North pole by modelling the North pole as a cylinder with a diameter of 1100km and height of 1m. Doing so, we arrive at a volume of 9.50x10^11 m^3. Similarly, the computed volume of carbon dioxide stored in the South pole is found to be $7.69 \times 10^{11}\mathrm{m}^3$. Thus we find that at its coldest, Mars currently stores $1.72 \times 10^{12}\mathrm{m}^3$ of carbon dioxide in its poles. By completely sublimating this stored CO_2, we can thicken the atmosphere and initiate a greenhouse effect which can contribute to further warming the planet to desired levels.

3.3 The Heat Capacity of the Martian Atmosphere

In order to simplify future calculations, in particular that of optical depth and its effect on temperature, we approximate this atmosphere to be comprised of solely carbon dioxide. We take the specific heat capacity of the atmosphere to be equal to that of gaseous carbon dioxide, 735 J/kgK at a temperature of 200K[3]. Multiplying this capacity by the atmospheric mass given in section 3.1 gives an energy value of 1.8375×10^{19}J needed to raise the entire atmosphere's temperature by one Kelvin.

4 Generation of the Greenhouse Effect

A large body colliding with Mars will increase the overall temperature of the planet as energy is added to the system. Not all of the energy of the colliding body will go towards heating the surroundings and eventually, by dissipation, the planet. Following from the results of a simulation done by Maindl et. al. (2015), we expect a weak dependence for energy transfer on size of colliding body and a strong dependence on angle of impact[7]. Adapting the methodology of Maindl et. al. (2015), we also relate the energy available as heat from the collision to be 60% of the kinetic energy of the colliding object[7].

We decided to choose a series of collisions such that the energy available is sufficient to heat the planet enough to amplify the current greenhouse effect by adding all of the stored carbon dioxide from the poles to the atmosphere. This thickening enhances the ability of the atmosphere to absorb and retain heat, leading to higher surface temperatures and the ability to support liquid water.

4.1 Initial Atmospheric Heating

A series of ten impacts heat the atmosphere to a temperature sufficient to start sublimation. The coldest temperature recorded at the polar regions of Mars was 123K, we require that this be raised to 150K to allow for sublimation to occur[9]. The mechanism for generating this heat is the impacts themselves, so we require that the collisions provide enough energy to raise the temperature by 27K. After this initial atmospheric heating, temperatures at the polar regions are high enough that even during the coldest seasonal period of Mars sublimation can occur.

4.2 Sublimation of the Polar Caps

From 3.2, we know that the total volume available for sublimation is $1.72 \times 10^{12}\mathrm{m}^3$. Were the poles to completely sublimate, this would add a large amount of carbon dioxide to the atmosphere. We ignore the timescale over which this sublimation occurs as it is not of interest to us in this first approximation of creation of a suitable environment for liquid water to exist. Using a density of $1600\mathrm{kg/m}^3$ [2], this corresponds to a mass of 2.75×10^{15}kg of carbon dioxide being added to the atmosphere. Thus, the amount of carbon dioxide in the atmosphere after the asteroid impacts is 2.78×10^{16}kg. This will correspond to a new atmospheric density of $0.0175\mathrm{kg/m}^3$.

4.3 The New Optical Depth

The new atmospheric number density of CO_2 will be about 2.38×10^{23}m^{-3}. Using equation 4, we find that the new optical depth of CO_2 is about 54.45. The corresponding surface air temperature appears to be between 285K and 286K, according to Figure 1. This is sufficient for liquid water to exist on the surface of Mars.

5 Required Energy

The energy required to change Mars to a state capable of supporting liquid water on its surface comes from the ten collisions. From 4.1 we require that the collisions provide enough energy to raise the temperature by 27K, and from 4.2 we require that they also provide enough energy to completely sublimate the poles. The energy required to raise the temperature by 27K follows from equation 6, which gives a value of 4.961×10^{20}J.

$$\Delta E = HeatCapacityOfAtmosphere \times 27K \tag{6}$$

The energy required to sublimate the poles is given by equation 7, where m is 1.72×10^{12}m^3 from 3.2, M is the molar mass, ~44 amu and ΔH is the enthalpy of sublimation, 26.3 kJ/mol[2]. This energy works out to be equal to 1.64×10^{21}J.

$$\Delta E = \frac{m_{IceCap}}{m_{CO_2}} \Delta H \tag{7}$$

From the two energies above, we conclude that the impacts must provide a total energy of 2.14×10^{21}J to change the state of Mars. Providing that the energy transfer from the collisions has an efficiency of 60%, as discussed in section 4, an estimate for total mass of colliding objects can be obtained from equation 8.

$$\sum m_i = \frac{2\Delta E}{0.6v^2} \tag{8}$$

Following the equation above and assuming a collision velocity of ~24km/s, we find that the total mass required is 1.225×10^{13}kg. This required collision velocity was chosen arbitrarily, but led to a very physical size estimate for the colliding bodies, and so was kept.

6 Asteroid Hunting

From section 5, it is apparent that we must find 10 bodies whose total mass adds to 1.225×10^{13}kg to collide into Mars. We decide that such candidates are best found in the population of Mars-crossing asteroids and the asteroid belt itself. Prior to searching for specific candidates, we decide to specify an ideal asteroid to which we may compare candidates.

This ideal asteroid is spherical in shape, is solid and of uniform density, a circular orbit on the plane of Mars, and contains one tenth the required total mass (1.225×10^{12}kg). The asteroid must be able to withstand the strains of approaching Mars and travelling through its atmosphere. Upon impacting Mars, this ideal asteroid will transfer most of its kinetic energy into heat, and will have lost no energy in the process of travelling through Mars' atmosphere.

Certainly no real asteroids are expected match the all the criteria. In fact, none are expected to match any of the criteria; however, they serve to set a good baseline. From the NASA data sheet on the asteroid belt, we see that the best candidates (mass-wise) will be asteroids similar to Icarus[11]. Since it is difficult to calculate the mass of an asteroid, it is rarely done and thus any database of asteroids, such as the one used to find candidates, lacks this information. As a result we have to make the assumption that if we choose bodies similar in diameter to Icarus, their masses will be of similar scale. We set a range of diameters of 1.0km - 2.0km as a search criterion and end up choosing the ten bodies listed in table 1, obtained from the Jet Propulsion Laboratory database[6].

7 Asteroid Energies and Impulses

7.1 Virial Theorem

Using the Virial Theorem, we can determine the change in orbital energy of an asteroid when we move it from its initial orbit to a similar, stable, Martian orbit. The orbital energy of an object in the solar system is given by the equation below, where M_{Sun} is the solar mass, a is the semi-major axis of orbit, and μ is the mass of the orbiting body.

$$E_{Total} = \frac{-GM_{Sun}\mu}{2a} = -E_{Kinetic} = \frac{E_{Potential}}{2} \tag{9}$$

Thus, the change in orbital energy can be described below. An extra term has been added to account for the extra relative velocity between the asteroid and Mars that we wish for the bodies to collide with (24km/s as was chosen earlier).

$$\Delta E = \frac{-GM_{Sun}\mu}{2}\left(\frac{1}{a_{Mars}} - \frac{1}{a_{Asteroid}}\right) + \frac{1}{2}\mu v_{rel}^2 \tag{10}$$

7.2 Impulse

The impulse, J, required for this action is described below using changes in total momentum, rather than momentum as a vector. The downside to using this method is that we don't know which direction we are pushing the asteroid, only how hard. Another method that involves slowing down the tangential velocity of the asteroid will be discussed in the next section, but it is incomplete. In our equation, a_{Mars} is the semi-major axis of Martian orbit, and a_{Asteroid} is the semi-major axis of the asteroid's initial orbit. Again, v_{rel} was chosen to be 24km/s.

$$J = \Delta p = \mu((v_f + v_{rel}) - v_i) = \mu\left(\sqrt{\frac{GM_{Sun}}{a_{Mars}}} - \sqrt{\frac{GM_{Sun}}{a_{Asteroid}}} + v_{rel}\right) \tag{11}$$

7.3 Sending an Imaginary Asteroid to Mars

As an example, we will determine the energy and total impulse required to send an ideal asteroid (described in section 6) on a collision course towards Mars. The mass of our asteroid is 1.225×10^{12}kg. Assuming its initial orbit lies within the asteroid belt (a ~ 3AU = 4.5×10^{11}m). Using equation 10, we determine that the energy required to send our rock to Mars is 1.77×10^{20}J. This is equivalent to about 42000 megatons of TNT. As a comparison, the Tsar Bomba was the largest nuclear detonation on Earth. It was recorded to release an energy of about 50 megatons. We would need a significantly larger amount of energy to move even one asteroid in such a way that we could hit Mars hard enough to terraform it.

Using equation 11, we can also determine that the total impulse needed to move our imaginary asteroid would be about 3.79×10^{16}kg*m/s.

7.4 Sending Real Asteroids to Mars

By applying the same steps in 7.3, we can determine the energies and impulses required to smash the asteroid candidates from section 6 into Mars. The mass of each asteroid is estimated to be of the same order as Icarus. This data is shown in the table below.

Asteroid	Semi-major Axis[10] [m]	Eccentricity	Diameter [km]	Mass [kg]	ΔE [J]	J [kg*m/s]
Icarus[11]	1.61267e+11	0.82690	1.4	1e12	4.08e20	1.94e16
152558 (1990 SA)[6.1]	3.01299e+11	0.44154	1.4	~1e12	2.17e20	2.71e16
217628 Lugh (1990 HA)[6.2]	3.81493e+11	0.70353	1.4	~1e12	1.71e20	2.94e16
2201 Oljato[6.3]	3.25304e+11	0.71313	1.8	~1e12	2.01e20	2.79e16
4769 Castalia[6.4]	1.59062e+11	0.48317	1.4	~1e12	4.14e20	1.92e16
85182 (1991 AQ)[6.5]	3.32418e+11	0.77645	1.1	~1e12	1.96e20	2.82e16
(2001 OB74)[6.6]	4.60753e+11	0.49528	2.0	~1e12	1.41e20	3.12e16
153591 (2001 SN263)[6.7]	2.97254e+11	0.47842	2.0	~1e12	2.20e20	2.70e16
3908 Nyx (1980 PA)[6.8]	2.88373e+11	0.45862	1.0	~1e12	2.27e20	2.67e16
7335 (1989 JA)[6.9]	2.64889e+11	0.48425	1.8	~1e12	2.47e20	2.57e16

Table 1: Asteroid Candidates to Terraform Mars

8 Discussion

8.1 Timing of Impacts

Our model assumes that the impact energy in the form of heat (and therefore the change in temperature) is communicated instantly. In reality there is some timescale over which this operates, one that depends on the density of the atmosphere, the difference between the temperature of the impact region and the rest of the atmosphere, and the contents of the atmosphere. If this timescale is comparable to or greater than the timescale of total heat dissipation through the atmosphere, then an impact at an arbitrary location will have a significant-or even total- reduction in efficiency. Depending upon the severity of this reduction it may be enough to simply reduce the delay between impacts. Otherwise, directing the initial impacts toward the poles in order to reduce dissipation might cause the release of enough CO_2 to reduce the dissipation associated with future collisions.

8.2 Molecules Escaping the Atmosphere

In heating up the Martian atmosphere, we need to be careful that we aren't increasing the surface air temperature over the point at which CO_2 molecules begin to gain enough speed to escape Mars' gravitational potential and escape the atmosphere. The rate of particles leaving the atmosphere is given by the following equation[4]. R is the radius of the outer edge of the atmosphere. n is the number density of the atmosphere (determined to be 2.83×10^{23}m^{-3} earlier).

$$\dot{N} = 4\pi R^2 \upsilon n \tag{12}$$

ν is characterized by the following equation[4], where m is the mass of each particle (44 amu), k is the Boltzmann constant, and T is the temperature (we determined earlier that the new equilibrium surface air temperature would be about 286K).

$$\upsilon \equiv \frac{1}{8}\sqrt{\frac{m}{2\pi kT}}\left(v_{esc}^2 + \frac{2kT}{m}\right)e^{-mv_{esc}^2/2kT} \tag{13}$$

v_{esc} is the escape velocity described by the equation below. M is the mass of Mars (about 6.42×10^{23}kg[12]).

$$v_{esc}^2 = \frac{2GM}{R} \tag{14}$$

These equations result in a calculated $\dot{N}$ of about 4.57×10^{-61}s^{-1}. N ~ nV $= 4.5 \times 10^{41}$, where the volume of the atmosphere was determined earlier to be 1.59×10^{18}m^3. We determine that the time for all the CO_2 to leave the atmosphere is $t_{esc} = N/\dot{N} = 9.85 \times 10^{101} s = 3.12 \times 10^{94} years$. We won't need to worry about CO_2 escaping from the Martian atmosphere any time soon.

8.3 Using Angular Momentum to find the Impulse of Asteroids

We can send asteroids towards Mars by slowing down their motions in the tangential direction of their orbits. This would result in the asteroid spiralling inwards. Assuming the impulses to the asteroids are instantaneous, we would have a tangential velocity v_ϑ of the asteroid that remains the same after we push it. This can be determined with our impulse. The final angular momentum can be represented in the following way, assuming that the final stabilized orbit of the asteroid is circular, with a radius r (v_r=0).

$$L^2 = GMr\mu^2 = \frac{\mu^2 v_\theta^2}{r^2} \tag{15}$$

Since the tangential velocity should remain constant after we push it, we can represent it as follows.

$$v_\theta^2 = GMr^3 \tag{16}$$

In order for the asteroid to spiral into a smaller orbit, its radial velocity would need to change. We want it to change in such a way such that the asteroids relative velocity with Mars is about 24km/s as it passes through Martian orbit.

$$v_{rel} = v_{Mars} - \sqrt{v_r^2 + v_\theta^2} = -24km/s \tag{17}$$

Since v_{Mars} is also about 24km/s, we want $v_r^2 + v_\theta^2 = v_r^2 + GMr^3 = (48km/s)^2$. This gives us a relation between v_r and r, the final stable radius of the asteroid's orbit assuming it doesn't collide with Mars. Finding a final orbital radius that allows the asteroid to travel through the Martian orbit such that equation 17 is satisfied would allow us to find the impulse that we need to push the asteroid with. The direction of impulse would simply be in the opposite of its initial tangential motion. We do not know how to model an orbit spiralling inwards, so this method was too difficult for us to use, but it could be used to find a more efficient method of sending the asteroids to Mars.

9 Conclusion

Our analysis suggests that the total energy necessary to sublimate the amount of carbon dioxide needed to create atmospheric conditions conducive to the existence of liquid water is roughly 2.14×10^{21}J, accounting for both the energy necessary to heat the atmosphere and the energy needed to fuel the state transition. Based on our assumption about the impact velocity, this corresponds to a total required mass of 1.225×10^{13}kg. Assuming that this mass is distributed evenly between all of the asteroids, and that these asteroids are located in the asteroid belt, the impulse required to place them on a collision course with Mars is on the order of 10^{16} N*s.

References

[1] Brown, D., Cantillo, L., & Webster, G. (2015, September 28). NASA Confirms Evidence That Liquid Water Flows on Today's Mars. In NASA. Retrieved November 15, 2015. <https://www.nasa.gov/press-release/nasa-confirms-evidence-that-liquid-water-flows-on-today-s-mars>

[2] Carbon Dioxide. NIST, 2011. Accessed November 13, 2015.
<http://webbook.nist.gov/cgi/cbook.cgi?ID=C124389&Mask=4>

[3] Carbon Dioxide - Specific Heat. The Engineering Toolbox. Accessed November 13, 2015.
<http://www.engineeringtoolbox.com/carbon-dioxide-d_974.html >

[4] Carroll and Ostlie, Introduction to Modern Astrophysics (2013). Carroll, Bradley, and Dale Ostlie. An Introduction to Modern Astrophysics. 2nd ed. London: Pearson Education Limited, 2014. Print.

[5] Chaplin, M. (n.d.). Water Phase Diagram. In Water Structure and Science.
<http://www1.lsbu.ac.uk/water/water_phase_diagram.html>

[6] JPL Small-Body Database. Jet Propulsion Laboratory, California Institute of Technology. Accessed November 13, 2015.
<http://ssd.jpl.nasa.gov/sbdb_query.cgi>

[6.1] <http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=152558>

[6.2]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=217628>

[6.3]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2201>

[6.4]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=4769>

[6.5]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=85182>

[6.6]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2001%20OB74>

[6.7]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=153591>

[6.8]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=3908 >

[6.9]<http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=7335 >

[7] Maindl, T. I.; Dvorak, R.; Lammer, H.; Güdel, M.; Schäfer, C.; Speith, R.; Odert, P.; Erkaev, N. V.; Kislyakova, K. G.; Pilat-Lohinger, E. Impact induced surface heating by planetesimals on early Mars. Astronomy & Astrophysics, vol.574. 2015.
<http://arxiv.org/abs/1405.5913>

[8] Miyazaki, K. An Explanation of Global Warming without Supercomputing. University of Texas.
<https://www.ma.utexas.edu/mp_arc/c/10/10-195.pdf>

[9] Russell, Randy. Mars Polar Regions. National Earth Science Teachers Association. July 7, 2008. Accessed November 13, 2015.
<http://www.windows2universe.org/mars/places/mars_polar_regions.html&edu=high>

[10] The Atmosphere and Weather. University of Arizona, 2012.
<http://www.atmo.arizona.edu/students/courselinks/fall12/atmo336/lectures/sec1/structure.html>

[11] Williams, David R. Asteroid Fact Sheet. NASA, 2014. Accessed November 13, 2015.
<http://nssdc.gsfc.nasa.gov/planetary/factsheet/asteroidfact.html>

[12] Williams, David R. Mars Fact Sheet. NASA, 2014. Accessed November 13, 2015.
<http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html>

[13] Williams, David R. Sun Fact Sheet. NASA, 2014. Accessed November 13, 2015.
<http://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html>

Appendix - Code

The following code was used to produce Figure 1.

```
## Team 190 - opticaldepth.py
## This code was used to calculate the optical depth of CO2 and plot a graph
##      of the optical depth required for liquid water to exist on Mars.

import numpy as np
from matplotlib import pyplot as plt

# Optical depth as a function of surface air temp and blackbody temp
def optical(T_surf, T_bb):
        return (2./3.)*((2.*float(T_surf)**4. - float(T_bb)**4.)/\
                        (float(T_bb)**4. - (0.3)*float(T_surf)**4.))

# Range of temperatures allowing for liquid water
ts = np.arange(273, 290)
# Blackbody temp of Mars is about 215K
tbb = 215.
opt = []

for i in ts:
        opt.append(optical(i, tbb))
opt = np.array(opt)
print opt[0], opt[-1]

plt.plot(ts, opt, label='Optical depth of CO2')
plt.xlabel('Surface Air Temperature [K]')
plt.ylabel('Optical Depth')
plt.legend(loc=2)
plt.title('Optical depth of CO2 required for surface air temperatures on Mars')
plt.show()
```