Launching nuclear residuals and its risks

Team # 412
Problem A

Abstract

In this paper, extraterrestrial disposal is proposed as a solution to the problem of nuclear residuals. We start analyzing six different ways of extraterrestrial disposal comparing them, based on their cost and on their security. After concluding that the optimal disposal method consists on sending the nuclear waste on an heliocentric orbit we start our modeling work. Both deterministic and stochastic modeling techniques have been used to solve different stages of the problem. The dangers involved in leaving the planet with radioactive waste have been analyzed together with the probability of this dangers to actually become true, comparing debri collision and flight failure using a pertinent figure of merit. Our model studies two possible heliocentric orbits where the radioactive waste could be stored and methods for reaching these destinations are proposed. We have studied and compared the possible dangers that radioactive waste could encounter in their new location (such as unwanted asteroid collision with unpredictable results) as well as the energetic costs involved in bringing them there. According to our model, the asteroid belt represents the best option when considering storage or disposal of radioactive waste during the 250 000 years that it takes for them to become innocuous again. State of the art techniques have been used to develop our models, like Lagrangian points, libration oscillations and the Hoffman maneuver. Rkf45 integration has been used to obtain high precision on our computations, which is important as our system is highly chaotic.

Contents

1 Introduction

Since the construction of the first nuclear plant the disposal of high level nuclear waste (HLW) generated by nuclear fission has been an important problem. Nowadays the current production rate of HLW is of 27 tons per nuclear plant and year. According to [1] the international consensus about disposal of HLW is geological disposal. In other words burying the radioactive wastes deeply underground. However, this disposal methods encounters a variety of inconveniences. Two of this are represented by security and social and political acceptance. Yucca Mountain and the Waste Isolation Pilot Plant have demonstrated that researchers cannot guarantee completely environmental isolation for the thousands of years needed until these wastes cease to be a threat and the well known ”Not in my backyard” syndrome constitutes an example of social nonacceptance towards the present dominant method of radioactive waste disposal.

In this paper we will propose and study a different, more definitive, disposal method; sending the HLW into outer space.

2 First insights

After a brief research, we have encountered in [2] six different destinations in outer space to be considered. We will compare this destinations mainly considering the energetic costs necessary in order to reach them (which will be represented by the values of speed variations that would be necessarily made) and, more importantly, the safety problems that reaching each destination could carry. It will also be considered as a slight advantage the possibility of recovering these residuals in case we find a way to use them.

2.1 High Earth orbit

The first option would be to keep the HLW in the terrestrial orbit. The main advantage of this method is its low expense (according to NASA studies it would require a variation in speed of only around 4 km/s when you are above the Shuttle orbit, at 297 km from Earth surface). Another small advantage consists in the relatively easy retrieval of the HLW in a later time if we were to find a way to productively use this wastes.

On the other hand, this location supposes a great danger. In fact, a periodic storage of nuclear waste in Earth’s orbit would greatly increase the transit in the already cluttered Earth orbital space which would difficult later space expeditions. More importantly, plutonium and other radioactive wastes that we would like to get rid of are not deemed to be considered safe again until 250 000 years [2] have passed. Since it is highly unlikely that a waste canister can be constructed that will last for the required 250 000 years this option supposes a great danger for humanity. The canisters will probably erode with time under the influence of internal radiation as well as space encountered radiation, and the solar wind could drive the small particles back into the Earth’s atmosphere contaminating it. Due to this fact, we considered this option a non desirable one.

2.2 Lunar orbit

This option demands a higher energetic cost than the later. It supposes, in fact, a speed variation of 4250 m/s above Shutter orbit in order to reach a circular lunar orbit of radius 21700 km. The transportation time would be higher that that of the previous option (6 days compared to the 18 hours) but it still wouldn't be very high (which implies lesser chances of accidents). A high improvement of this option from the later would be that once reached the lunar orbit, even if the canisters were to malfunction due to prolonged radiation exposure the radioactive, particles able to escape would mostly remain in Moon's orbit. Furthermore, due to higher distances, the few particles that would escape Moon's orbit would disperse leaving only a very small portion reaching the Earth. This could seem a small problem concern at first. However, considering the huge amounts of HLW that we are planning to store, this small portion could represent an excessively large quantity. Another counterpart is that this option (as every option involving leaving Earth orbit) implies, as explained in [2], the need of a non perfect waste-retrieval plan to be executed in case of failure on the transportation process.

2.3 Solar system escape

This option presents the advantage that the disposal of the waste would be definitive with no chance of going back to Earth. Furthermore, the transportation method would be relatively simple implying a single burn of the propulsion system. This fact would allow us to operate with easily reusable ground based propulsion systems such as electromagnetic launchers, mass drivers, gas guns and laser propulsion which, even if they are currently not very efficient due to the high development and construction costs, would in the long term and after more investigation probably turn out to be proven cheaper. Unfortunately we cannot ignore the current inefficiency of this methods so we cannot consider this later argument as an advantage. A counterpart to this method is the huge energy cost that it implies. In fact, we can use a simplified model where the only relevant gravitational fields at the earth surface are the ones from the earth and from the sun. The condition that the system must have is that the total energy must be zero, since we want to put that body at the infinite and still.

$$\frac{1}{2}v_e^2 = \frac{GM_S}{IAU} + \frac{GM_T}{R_T} \Leftrightarrow v_e = \sqrt{2G\left(\frac{M_S}{AU} + \frac{M_T}{R_T}\right)} \approx 43591\frac{\text{m}}{\text{s}} \tag{1}$$

where $G = 6.67 \cdot 10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}$ is the universal gravitational constant $M_S = 1.99 \cdot 10^{30}$ kg is the Sun's mass and $IAU = 1.50 \cdot 10^{11}$ is the international astronomical unit.[1] If given that any object in Earth has a mean orbital velocity of $v_{mean} = 29780$ km/s we will need to give our satellite a speed variation of

$$\Delta v = v_e - v_{mean} = 13811\text{m/s} \tag{2}$$

According to calculations made in [4] this needed velocity variation can be reduced to 7010 m/s using Jupiter swing-by to escape the solar system. Either way the needed velocity variation is much higher that the one needed for the methods exposed up until now so we will consider this option as unpractical until there have been improvements in ground based propulsion systems.

[1] This and other planetary constants have been extracted from [3].

2.4 Solar impact

Despite the fact that this option seems the best one in a first approach, taking the rocket to the sun is not an efficient option. The main reason is that it is need to decelerate the rocket into a lower speed in order to go to the sun. Firstly, it is possible to fully decelerate the rocket and then it would do a Sun dive. However, this option requires a speed variation equal to the orbital velocity of the Earth $v_T = 30.03$km/s. On the other side, there is a more efficient method, describing an elliptical orbit with a perihelion at the Solar surface and an aphelion at the Earth surface. The speed increase of that path is given by the energy equation (3) and the semi-major axis for that specific orbit 4, where the sun radius is $R_s = 6.957 \cdot 10^8$ m:

$$E/m = -\frac{GM}{\mathrm{UA}} + 1/2v^2 = -\frac{GM}{2a} \tag{3}$$

$$a = \frac{\mathrm{UA} + R_s}{2} \tag{4}$$

Rearranging both equations and keeping in mind that the velocity in the equation is the orbital velocity plus the speed increase that we need to calculate, we get to the final equation:

$$\Delta \mathrm{v} = \sqrt{2GM\left(\frac{1}{2\mathrm{UA}} - \frac{1}{R_s + \mathrm{UA}}\right)} - v_T \tag{5}$$

Introducing the values we get that it is need an spread orbit of Δv=-27.164 km/s. The negative tells us that we need to decelerate in order to get into the sun. Both results require a high speed increase, that's why a solar impact it's not the most efficient idea. Furthermore, a solar impact is a non-return action, so the waste would never return to Earth in the hypothetical case that we found a way to use it.

2.5 Lunar soft landing

The process followed in order to safely land the HLW very much resembles the one followed to in orbit around the moon. However, it supposes higher energetic cost since it is necessary to decrease the Orbit Transfer Vehicle (OTV) speed in order to safely land on the Moon. It is also needed to avoid the dispersion on the Moon surface of the nuclear waste which would difficult the explorations of the Moon surface. Moreover, the building of such an OTV would increase the expedition economical expense. Despite its higher energetic cost respect the moon orbiting option, the energetic cost presented by landing the HLW on the Moon surface would, according to [2], be lower that that Solar system escape and solar impact options.

A discrete advantage presented by this option is that the HLW would be reachable and could be extracted in case we should find a way to make it useful in the future. Another advantage is that, even if the canisters should be rendered useless due to prolonged radiation exposure the wastes would not contaminate the Earth since they would remain in the Moon surface due to gravity attraction. In fact the only way that the radioactive waste should return to Earth without active human intervention would be as a Moon meteorite which, even if deemed improbable (there are only 118 registered moon meteorites) it still represents a possibility which should be considered. Another disadvantage of this option is that using the Moon to store all our nuclear waste would difficult the Moon's surface exploration which could be somehow attenuated placing the nuclear residuals in a place of low exploration interest.

2.6 Heliocentric orbit

This last option represents, like the Solar system escape and the Solar impact options, a definitive way to dispose of the nuclear waste. Compared with the lunar soft landing option, the possibility of retrieval of the nuclear waste in this option is harder due to the greater distances involved. Despite this moderate disadvantage this option presents many positive facts. The first is that the even if difficult, the retrieval is not impossible given that the waste would remain in a known stable orbit inside the solar system. An advantage worth of notice of this option consists in the fact that, in this kind of expeditions, escape from Earth would occur within the first 1700 s according to [2]. Besides, if a failure occurred after this brief period and before reaching the new orbit, the nuclear canister would remain in a heliocentric orbit that would have a probability of re-encountering Earth within 250 000 years (the period of time during with the nuclear waste would be dangerous) of only 0.1%. Flying one recovery mission would reduce further this low probability making it irrelevant. The only factor that could really be considered a disadvantage for this option is the high energetic expense required to send the wastes at a far enough distance from Earth to make the HLW take a heliocentric orbit, completely separated from that of Earth. This high cost is however lower than the one necessary to achieve Solar impact or Solar system escape and could be further diminished using planetary assisted flights.

2.7 Comparison of destinations

From the the advantages and disadvantages exposed above we have reached the conclusion that the optimal destination for our nuclear wastes is an heliocentric orbit. That is why it has the important advantage of being a definitive disposal method of nuclear waste with a low possibility of waste going back to Earth. Moreover, is the cheapest and the safest way from the ones studied. The lunar soft landing option almost reaches this parameter, but building an OTV able to soft landing Moon requires a higher economical expense. Furthermore, the risk of meteorites impacting the Moon and sending the nuclear waste back to Earth as nuclear meteorite is also higher than the probability to bring the waste back from a different heliocentric orbit. The only doubt remains in deciding which heliocentric orbit should be used to store the HLW. The criterion to use is the stability. It is obvious observing figure 1 that the most stables orbits of the inner solar system are the ones represented by the asteroid belt and the orbits of the Lagrange points of Jupiter. The proof is simple, there are plenty of asteroids there. In our model we will study the stability of this orbit and methods used to reach this orbits.

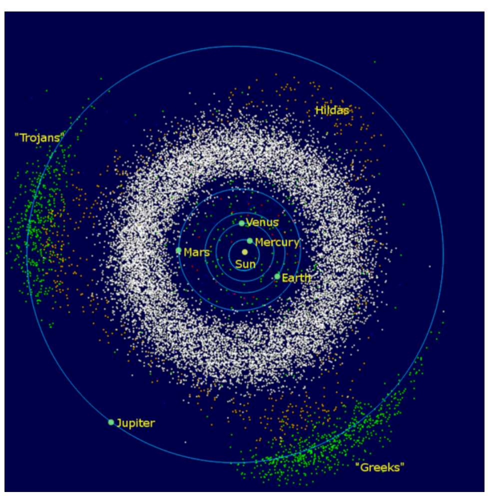

Figure 1: Inner Solar system.

3 Modeling the disposal of the residuals in space

3.1 General assumptions

In this section we will discuss the main assumptions that we will consider when modeling the motion of a rocket:

- We will use Newton's law of universal gravitation to describe the interaction of the planets with the rocket.
- The motion will be planar: we neglect the possible variations in the axis perpendicular to the ecliptic.
- The Sun will be in a fixed position and the planets will go around the Sun in a circular orbit. This assumption is justified due to the fact that the eccentricity of the orbits of the planets considered in the model is really low.
- We will not consider the gravitational field of Mercury, Venus, Saturn, Uranus and Neptune.
- When we kick the rocket, we can give an instantaneous increase of speed (we neglect the acceleration time with respect to the time scale of the problem).

3.2 Abandoning Earth

The biggest dangers of space travel are concentrated in the moments of leaving and coming back to Earth. It is therefore important to consider the problems involved in exiting our planet and which is the probability of an uncontrolled incident happening.

3.2.1 Probability of disaster

In this section we will compute through a simple model the probability of collision with a debri (space junk orbiting the earth at low earth orbit). Since any object abandoning Earth needs to deal with debris surrounding Earth this is a problem worth considering. The number of debris is constantly increasing, and in future time will become a big problem (see [5]). In our model we will consider a uniform distribution of debris in the lower earth orbit. We will also assume that the rocket will be traveling through the lower earth orbit at constant speed and through a radial line, and that the debris are orbiting at a constant an equal speed. Although this assumptions are not completely realistic, as we are doing a stochastic model we can just use the average values.
To compute the probability of collision, we are going to consider the problem from the reference system of a debri. On this system, the debri is still and it will see the rocket moving in a non-radial path. That is, if the debri have velocity $\vec{v}_{debri}$, it will see the rocket moving at velocity $\vec{v}_{rocket} - \vec{v}_{debri}$, being $\vec{v}_{rocket}$ the velocity of the rocket in the Earth reference system. That is what we see in figure 2, where $\theta = \arctan \frac{v_{debri}}{v_{rocket}}$.[2]

Of course, the rocket has a cross-sectional area that will provide the possibility of colliding with a debri. As the rocket is much bigger, we will assume the debris to be point-sized. As the rocket has a cross section and travels through the low earth orbit, its path will have an effective volume, being it the length of the

[2] We write v to express the norm of $\vec{v}$

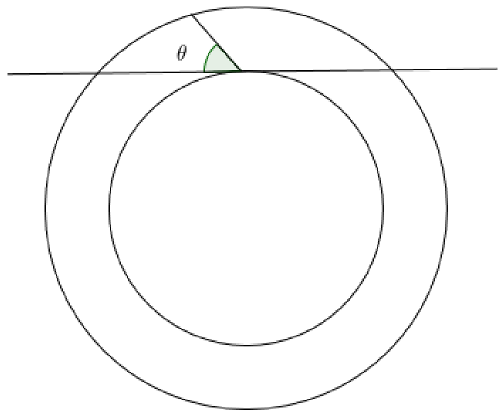

Figure 2: Path of the rocket from a debri reference system.

path times the area of the cross-section. Now, the probability that it collides with a certain debri is the probability of the debri being in the effective volume of the path. Using basic trigonometry, one can easily work out the length of the path, being it $(R_{max} - R_{min})/\sin\theta$. This means that the probability of colliding with a certain debri is:

$$p = \frac{\sigma(R_{max} - R_{min})/\sin(\theta)}{\frac{4\pi}{3}(R_{max}^3 - R_{min}^3)} \tag{6}$$

being σ the cross section of the rocket. Finally, the probability of colliding with n debris can be easily computed as

$$p_{total} = 1 - (1-p)^n \tag{7}$$

where n is the number of debris. Using the values found in [5] and average speed of rockets at the lower Earth orbits, one obtains

$$p_{total} \approx 2.02 \cdot 10^{-7} \tag{8}$$

we have set $R_{max} \sim 2000$ km, $R_{min} \sim 300$ km, $n \sim 1.4 \cdot 10^4$, $\sigma \sim \pi 8^2$ m^2, $v_{debri} \sim 8\frac{\text{km}}{\text{s}}$ and $v_{rocket} \sim 8\frac{\text{km}}{\text{s}}$.

This result means that the collision probability is currently extremely low and there is no actual danger. However, our model is slightly optimistic, as it considers the debris point-sized. Moreover, we should evaluate our model in the launching time, as the number of debris is constantly increasing.

Furthermore, we must consider the fact that there are launching failures can happen. Using the database from 1994 into 2014 showed in [6], we estimate that the probability of a launch failure is 0.79%. This is not a low probability, so before implementing the waste extraterrestrial disposal it is necessary to reduce the launch failure.

3.2.2 Consequences of a disaster

It is very important to take care of the radioactive residual storage. In case of collision. It is extremely important to have a container with a high melting point, so that the nuclear waste does not get scattered over the Earth surface. Here we have an illustrative example of what happens when there is a collision with a debris and all nuclear waste gets scattered:

We can imagine our rocket colliding at a height h against a debri and exploding as a result.

Aiming a simple approximation to the real solution, we can separate the total amount of particles in two groups. The first one we will continue having a high velocity and we will consider that they will leave the Earth and forget about them. For the second group we will optimistically consider that will have its up momentum compensated by the explosion force being therefore left with a 0 radial speed but thrown like a Gaussian distribution (9) in every other direction. We know that the second group must be much lower in number than the first one due to the linear moment conservation. However, that smaller amount will still be significantly dangerous for the earth-livings. In real life, this division is not continuous, but for the approximation that we look for is perfect.

Therefore, we will look for a measure of the dispersion of that second group over the Earth Surface. Let us consider the x Cartesian coordinates of the velocity, being the z coordinate the height. The distribution of velocities will be the following:

$$\rho(v_x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{v_x^2}{2\sigma^2}} \tag{9}$$

The explosion shock wave that will push all particles in every direction is just a pressure wave, just as the sound waves (see [7]). For this reason, the shock wave will push the particles at the speed of sound $v_s = 342.3m/s$. Therefore, the estimated value for average v_x is 0.That means that 68.27% of the particles will have a speed $\in [-\sigma, \sigma]$ If we choose $\sigma = 300m/s$ we get that 68.27% of particles have a speed $\in [-300, 300]m/s$, so with this sigma we cover all the range of velocity and we will have a complete dispersion.

For the remaining particles in the Earth, $\langle v \rangle = 0$, we get that the average time to get from the height of the particles $\langle h \rangle$ to the Earth surface is $\langle t_f \rangle = \sqrt{\frac{2\langle h \rangle}{g}}$, where we have estimated that gravity is constant at this height, a reasonable approximation because the actual value of the gravity in the low earth orbit is $\approx 7.5\text{m/s}^2$. Assuming there is no wind that changes our earth-particles speed, we change the density probability for a lateral dispersion density (x coordinates of the particles) using $x = v_x t$, where

$$\rho(x) = \rho\left(v_x = \frac{x}{t}\right)\frac{dv_x}{dx} = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2 t}} \tag{10}$$

The dispersion approximation gives us a new dispersion for a given time

$$\sigma_x = \sigma t = \sigma\sqrt{\frac{2\langle h \rangle}{g}} \tag{11}$$

According to [5], the average height of the low earth orbit is $\langle h \rangle = 1150\text{km}$, so the estimated dispersion at the earth surface is

$$\sigma_x \approx 145\text{km}$$

As a first approximation, this estimation gives as the idea that a rocket collision with the debris at a 1150 km, will scatter more than 95.57 percent of the remaining radioactive particles over a surface of $S_{\text{dispersion}} = \pi(2\sigma)^2 \approx 264000\text{km}^2$, which approximately the area of Colorado. The affected area by 95.45% will become our Figure of merit. We take all the particles in the 2σ interval, because we consider that particles that have more speed than the 95.45% of them are negligible.

However, we have not considered the effects of wind and radioactive ashes that would spread even more the radioactive particles.

As we have explained before, the probability of a launch failure is 0.79%, we can also make the same wild guess as before and obtain the affected area in case of launch disaster. As the process is the same, we can use the equation 11. A good example of a launch disaster is Space shuttle Challenger disaster from 1986, that exploded at an altitude of 15 km. Using this altitude as the average altitude of disaster, it is obtained that the surface of dispersion the 95.57% of radioactive particles from a launch disaster is

$$S_{\text{dispersion}} \approx 346.16\text{km}^2$$

To sum up, we have two possible accident causes, one less probable but more lethal, and the other more possible but less dangerous and still fatal. Both must be taken in care, but we can make a wild guess to know which is the most dangerous. The figure of merit to determine which one is the most dangerous that we have taken is effective area destructed per launch (S_{eff}). This magnitude is described by

$$S_{eff} = \text{Probability}_{\text{disaster}} \cdot S_{\text{dispersion}} \tag{12}$$

Applying the values of each possible accident cause, we get that the effective areas are $S_{\text{launch}} = 2.7347\text{km}^2$ ans $S_{\text{debris}} = 0.0533\text{km}^2$. This magnitudes gives as the theoretical area that it is sprayed by 95.45% of the radioactive particles in each rocket launch. As a result, we can see that as a first approximation, the launch disaster is 51.28 times worse than the debris collision. However, both disasters would be lethal and devastating.

3.3 Putting the rocket in a stable orbit

3.3.1 Aiming at the Lagrangian points

In this section, we will develop and implement a model to put the rocket carrying the nuclear waste in a very stable orbit. The main focus will be on putting the rocket on a Lagrangian point of the Jupiter-Sun system.

First of all let us talk about what is a Lagrangian point. The Lagrangian points are the critical points of a star-planet dynamical system. Although the Lagrangian points are unstable critical points, as seen in [8], the fact that they are moving in a non-inertial reference frame makes some of them stable. Actually the L_4 and L_5 Lagrangian points are effectively stable when considering that they are a non-inertial system. As explained in [8], when a point in L_4 starts moving away from equilibrium, the Coriolis force takes it back to L_4. This is the reason of the existence of Trojans [9], being objects orbiting the L_4 and L_5 points, which are in the asteroid belt. One can see the geometry of the problem in figure 3.

Our aim will be to put the rocket on a Lagrangian point, as other locations are not safe due to the fact that the N-body problem is chaotic. The point is that this is the most stable that we can aim from a physical viewpoint. L_4 and L_5 guarantee equilibrium if we just consider the sun and Jupiter, and not all the planets. However, every other planet field will just add a correction to the path that will be negligible.

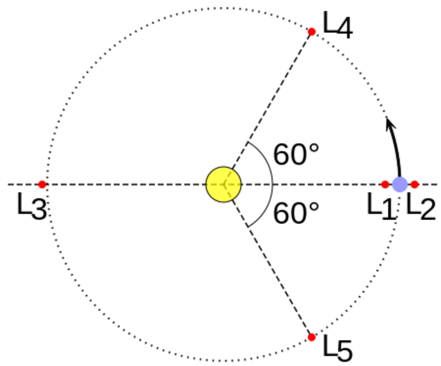

Figure 3: Critical points of the earth-sun dynamical system.

In this model we will assume that Jupiter is orbiting around the sun in a circular path. This is well justified, as the eccentricity of Jupiter is almost 0 (it is 0.048498). We will also consider the Earth going in a circular orbit, is also well justified (Earth's eccentricity is 0.0167). The gravitational bodies that affect the path of the rocket will be Jupiter, the Earth and the Sun. Our model will assume planar motion of all the bodies.

The most efficient way to kick a rocket is tangential to its velocity. All other kicks will be less energetically efficient. In rocket science, this kind of techniques are known as Hoffman's maneuvers.

We will divide our problem in a two-step kick of the rocket. First of all, we will kick it to go out from Earth and approaching Jupiter. Both in this and the following methods, our initial conditions will be at a distance of 10000 from the center of the Earth. Secondly, when we are close to Jupiter's orbit, we will push the satellite to obtain the same speed as Jupiter. In order to do this, we will integrate the motion equations of the rocket and impose that after the second kick it will be moving on L_4 in the sun reference frame. Now, let us consider the geometric situation shown in figure 4.

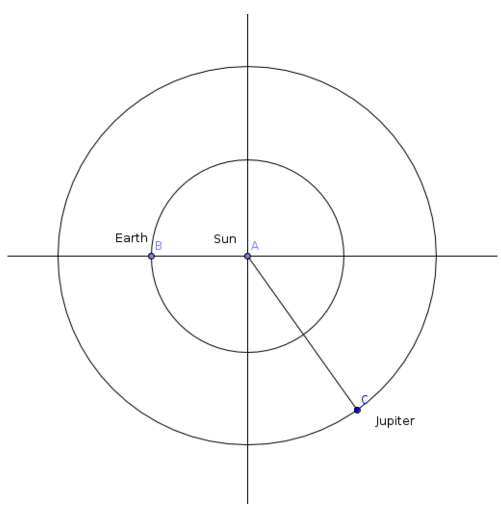

Figure 4: Geometric situation.

Without loss of generality, we can assume the Earth to be in the position $(-AU, 0)$, where AU is the distance from Earth to Sun (astronomic unit). We launch the rocket with a speed $(0, v_1)$, being $v_1 < 0$ such that it intersects tangentially the path of Jupiter at the position $\vec{r}_0 = (5.2AU, 0)$ (the radius of Jupiter path is $5.2AU$). The initial angle of Jupiter and the x axis must be θ such that when the rocket is in position $\vec{r}_0$, Jupiter is in the following position:

$$\vec{r} = \begin{pmatrix} \cos(\pi/3) & \sin(\pi/3) \\ -\sin(\pi/3) & \cos(\pi/3) \end{pmatrix} \begin{pmatrix} 5.2AU \\ 0 \end{pmatrix}$$

This is due to the fact the L_4 point is at an angle of 60 degrees of the Jupiter. Theoretically, L_4 is not in the path of Jupiter, as by definition the triangle between the Sun, L_4 and Jupiter is equilateral. If we assume a circular orbit of Jupiter, L_4 gets to its path.

At that moment, we will do a second push of velocity $(0, v_2)$ such that the final velocity is the one that Jupiter would have at $\vec{r}_0$.

Using this conditions, we have that setting $v_1 = -12.4\frac{\text{km}}{\text{s}}$, $v_2 = 5.6\frac{\text{km}}{\text{s}}$, $\theta = -145.15°$, we can get to the L_4 point.[3] Actually, this strategy is much more energy consuming than throwing it out of the solar system. However, this strategy provides a possibility to recover the residuals in case of loss. The path of the rocket in the following 1000 years is shown in figure 5. The motion equations have been integrated using a rkf45 numerical method. This method provides the necessary precision to obtain a realistic simulation.

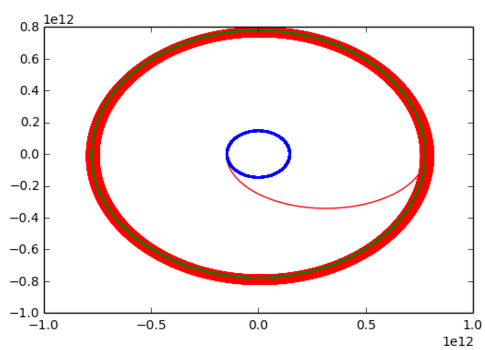

Figure 5: Green:Path of Jupiter. Blue: Path of the Earth. Red: Path of the rocket. The sun is in the origin and the units are meters

3.3.2 Modeling the chances of collisions with Trojans

In this section, we will develop a model to compute the probabilities of collisions with Trojans once we have set our rocket in L_4. This has to be considered, as a collision could set the rocket out of L_4 and a chaotic motion could bring it to the Earth. To develop this model, we will consider libration (see [10] and [11]), which is the process of angular oscillations of the Trojans of Jupiter. That is, the angle between Jupiter and a certain Trojan is not constant as time evolves. As [11] claims, the oscillations have an

[3] This parametrization of θ is equivalent to saying that the angle between the Earth and Jupiter is 35° and that the Earth path is currently following Jupiter's. It is not hard to find a position in the following years where this is satisfied.

average period around 150 years. We will consider the problem from the rocket reference frame. Let us put our three-dimensional axes such that x is parallel to the speed of the rocket. As the period of the oscillations of every Trojan is about $T = 150$ years, every Trojan will cross the $x = 0$ plane about one time every $T/2$ years.[4] After a time t, the number of Trojans that will have crossed the $x = 0$ plane will be, on average, about $\frac{2nt}{T}$, where n is the number of Jupiter Trojans.

The probability of a collision when a Trojan crosses the $x = 0$ plane will be proportional to the cross-section of the Trojan and inversely proportional to the total area of the Trojan orbits. This assumption is justified due to the fact that the area of a Trojan is much bigger than the area of the rocket, so we can consider the rocket punctual. For this reason, the probability of a collision after a time t is given by:

$$P(t) = \frac{2nt}{T} \frac{\sigma}{\text{TotalArea}} \tag{13}$$

where σ is the average cross-section of a Trojan. If we compute this probability with $t \sim 2.5 \cdot 10^5$ years, which is enough time for the Plutonium-239 to decay at its most. Using the data provided by [12], we set $\sigma \sim 1\text{km}^2$, TotalArea$\sim 0.25^2\text{AU}^2$, $n \sim 10^6$, and $T \sim 150$, our probability becomes $P \sim 2.4 \cdot 10^{-6}$. As this probability is really low, one can assume that a collision is not likely to happen and the L_4 and L_5 points are safe points to aim.

3.3.3 Aiming at the asteroid belt

In this section we analyze how a to put a rocket that carries the residuals in the asteroid belt. We will use the same assumptions that in the other model, but we will consider the field of Mars at the rocket position, as Mars' distance to the asteroid belt is low enough. The reason to put it in the asteroid belt is due to the fact that some regions of the asteroid belt are very stable regions. As seen in 6, the regions with higher density of asteroids are the ones which are more stable. On the other hand, the Kirkwood gaps are highly unstable regions.

Again, our aim is to put our rocket in the asteroid belt with a two-step acceleration using the Hoffman maneuver, but now we need to get a semi-major axis in the proper range, considering 6. In this case, we achieve it using an initial speed of 41km/s. Of course, as the Earth speed from the sun frame reference is already 29.8km/s, we only need to give 11.22km/s to the rocket, if we throw tangentially. At the second acceleration, we need to use a propulsion of 2km/s. Using this accelerations one can aim the asteroid belt, and the motion of the bodies are shown in figure 7. Finally, we can compute the semi-major axis, and the result is 2.25 AU. As shown in figure 6, it is a considerably stable zone.

3.3.4 Modeling the chances of collisions with asteroids in the asteroid belt

After putting the rocket in the asteroid belt, we must analyze the safety of that path. As an assumption, we will only take as relevant the collisions with objects of more than 10 km of radius, which are the only ones that are able to give the rocket such a linear moment that it changes its path dramatically, starting a chaotic motion. According to [13], there is a collision of a 10 km radius approximately once every 10^7

[4] In a T period, it turns around doing a "lap" and actually crosses the $x = 0$ plane two times. For this reason, it crosses one time every T years.

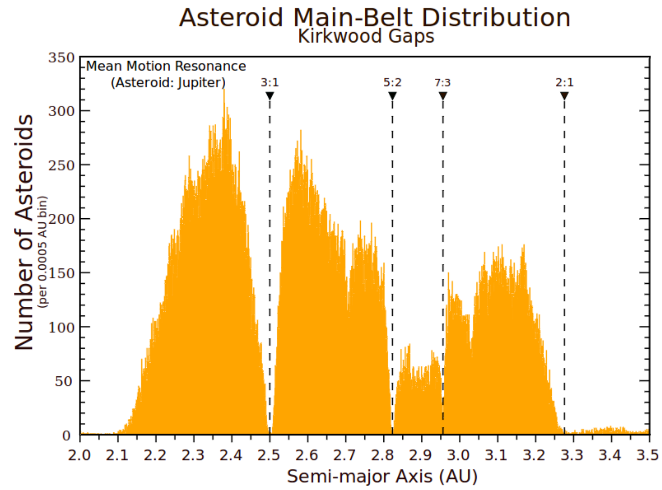

Figure 6: Source: Wikipedia commons

years, and the numbers of objects with that radius is 10^4.

As this is the only data that we have, we will do a modeling trick to obtain the probability of the collision of the rocket with an asteroid. Let $m = 10^4$ be the total amount of asteroids. We know that a collision takes place every $t = 10^7$ years. That is, the average time that we have to wait for a collision taking place is t. Now let us consider the rocket-asteroids system. We will compute which is the average time that we have to wait for a collision taking place with the rocket.

Let us assume that a collision takes place between two bodies of the system. There are an amount of $m(m+1)/2$ possible pairs of bodies.[5] Moreover, there m of them involve the rocket. For this reason, the probability of this collision involving the rocket is $\frac{m}{m(m+1)/2} = \frac{2}{m+1} \approx \frac{2}{m}$. From this we conclude that the average time that we have to wait for a collision taking place with the rocket is $\frac{tm}{2}$.

We can model the probability distribution of the collisions of the rocket through time with an exponential distribution. This assumption is very natural, as the probabilities of collisions during disjoint time intervals are independent. Moreover, one has that the probability distribution will be continuous. With this two hypothesis, one can prove mathematically that the probability distribution will be an exponential distance. If we use an exponential model of parameter λ, we have that its expected value is $\frac{1}{\lambda}$.

Let us join the derivations worked out in the last paragraphs: the expectation value of an exponential distribution is $\frac{1}{\lambda}$, which is the same as $\frac{tm}{2}$ (this is the average time that we have to wait for the rocket to collide!). We can therefore compute $\lambda = \frac{2}{tm} \approx 5 \cdot 10^{11}$, which units are year^{-1}. Now, we will compute the probability that the rocket collides with one of the asteroids in the lifetime of the radioactive particles. As in the other sections, we will set the particles extinguished when $T = 2.5 \cdot 10^5$ years. As the probability

[5] The system has $m + 1$ bodies, as the rocket is one of them. However, this is not important, as $m \approx m + 1 \approx 10^4$.

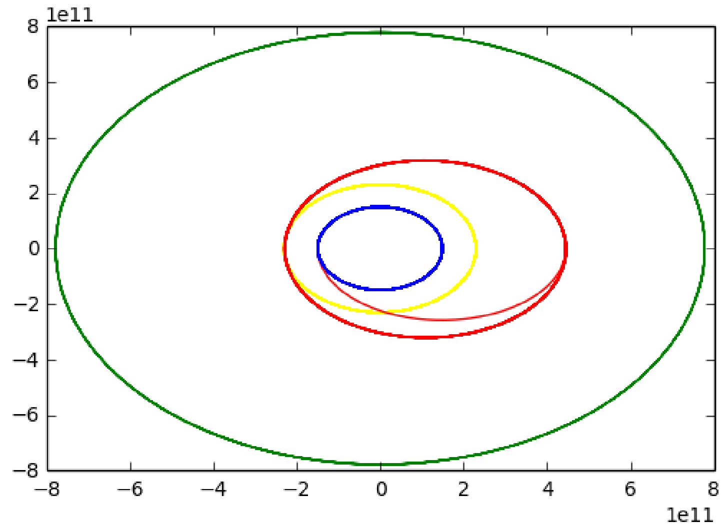

Figure 7: Green: Path of Jupiter. Blue: Path of the Earth. Yellow: Path of Mars. Red: Path of the rocket. The sun is in the origin and the units are meters.

distribution is exponential, one has the probability of having collided at time T is

$$P(T) = 1 - e^{-\lambda T} = 1 - e^{-\frac{2}{tm}T} \tag{14}$$

Giving values to our parameters, we have $P(2.5 \cdot 10^6) = 1 - e^{-5 \cdot 10^{-6}} \approx 5 \cdot 10^{-6}$. Again, the probability is of the order of 10^{-6}. For this reason, we can conclude that the orbit is actually safe.

3.3.5 Comparing models

All the models made before have some pros and cons. We will compare them taking the increase of velocity that we must give to the rocket as the figure of merit. Therefore, we provide a comparison of the methods considering different aims:

Objective	First increase (m/s)	Second increase (m/s)	Total increase (m/s)	Restore
Out Solar System	13811	0	13811	$\times$
L_4 L_5	12400	5600	17000	✓
Asteroid belt	11220	2000	13220	✓

Table 1: Comparison of the different methods. Restore means the possibility of recovering the rocket from its orbit in a future time.

4 Strengths and weaknesses

The strengths of our models are the following:

- Our models are mass independent, which simplify the models extensively, being the assumptions realistic enough.
- We take into account a large variety of methods of extraterrestrial waste disposal and select the most reasonable one (sending the nuclear waste to an heliocentric orbit) from within them.
- Our models have the discrete advantage, with respect to the naive strategy of throwing the residuals away from our galaxy, that there exists the possibility (even if narrow) of waste retrieval. This advantage would become a major one if it was to be found a method to recycle the nuclear waste.
- We use the Hoffman maneuver in order to obtain an optimal usage of our energy.
- We consider and model many problems that could happen in the disposal of the residuals, comparing the danger of them by using an appropriate figures of merit.

The weaknesses of our models are the following:

- The motion of the celestial bodies is assumed to be circular. More realistic simulations could be carried away solving the differential equations of motion for all the bodies and not just for the rocket.
- The energies that we need to supply to our rocket are of the same order of magnitude as the ones that we need to give in order to escape the solar system, so our method does not show a significant improvement in energy expense with respect to the naive strategy. We could probably improve this fact by performing gravity assistance maneuver.

5 Conclusions

The studies made on safety and energy costs for sending nuclear waste have shown us that the best possible option is that of sending the wastes on an heliocentric orbit. Energetically is the optimal one and it is in a safe storage location (unlike a High Earth orbit or a Lunar orbit).

Another remarkable conclusion is that it is very important to make sure that the waste would be able to abandon safely Earth's orbit since an uncontrolled crash in this zone would be fatal. It would be recommendable, even if economically disadvantageous, to plan waste recovery maneuvers for such situations. Moreover, we could simulate a crash with mock cargo to prove their efficiency and reassure the public opinion regarding the dangers of extraterrestrial disposal of nuclear waste.

Our main conclusion is that the most desirable of the two possible heliocentric orbits is the asteroid belt orbit, due to energetic reasons. An important result that we have run into is that the probability of accident for the nuclear waste deposited on the asteroid belt is so low as to be considered negligible.

Moreover, aiming to the Sun with a naive strategy requires really high energies. For this reason, if we want to send it to the Sun we should follow non-trivial strategies.

Finally, it is desirable to comment that one advantage of extraterrestrial nuclear disposal (apart from the obvious one of getting rid of highly contaminating waste) is that such a program would promote founding and investigation in space research techniques reactivating an area of science that has seen a drastic reduction on its advancing rate since the conclusion of the cold war.

6 Future Work

It would be interesting in order to make a more complete model to add and improve the following points in future work

Gravity assisted maneuvers: Planning to take advantage of the gravity field of celestial bodies is a common practice in space travel. This method could help us to place the radioactive waste on the asteroid belt with a much lower fuel expense. On the other hand, this maneuver could suppose a restriction on the day we could extract radioactive waste from Earth since it would be necessary to wait for the celestial bodies used for the gravity assisted maneuver to be on the necessary location at the moment of launching. This would suppose a slower disposal of the radioactive material or a larger fuel expense.

Orbit Transfer Vehicle (OTV) construction: In order to maximize the amount of nuclear waste extracted from Earth with each rocket launch, it would be necessary to build OTV with great capacity and high resistance to radiation. Since it would suppose a periodical necessary expense and it would be very difficult to retrieve the OTV used once it reached the asteroid belt, it would be wise to investigate a method to produce relatively large amounts of identical OTV. It would be interesting in this regard to conduct further investigations on reliability of ground based propulsion methods.

Radioactive waste collection: It would be interesting to conduct plans to recollect the radioactive material storage in stable orbits.

Further improvement of security systems: During our paper we have studied the disasters that would follow a malfunctioning of the OTV extracting the radioactive waste from Earth. It is necessary to be absolutely certain that no such malfunctioning would occur and have a contingency plan in case it happened to occur anyway.

Further alternate extraterrestrial disposal methods: It would be interesting to make further investigations on the variety of extraterrestrial disposal methods such as Lunar hard landing and heliocentric orbiting with a radius of $0.86UMA$ as proposed in [2].

Simulation improvement: In order for the made simulations to be actually useful on the planning of space trips, it would be necessary to make a more complete model. At those scales, it still wouldn't be necessary add more planets nor relativistic effects. However, the model should include progressive accelerations.

Market investigation: It would be an interesting thing to compute how much would suppose to send the radioactive waste to space. It should be compared to the currently dominant politic of underground burial of such wastes. It would also be important to consider how such a project would affect the investigation field of space traveling and what benefits could this bring.

References

[1] "World Nuclear Association radioactive waste management." http://www.world-nuclear.org/information-library/nuclear-fuel-cycle/nuclear-wastes/radioactive-waste-management.aspx. Accessed: 2016-11-12.

[2] W. E. G. R. E. Burns, W. E. Causey and R. W. Nelson, "Nuclear waste disposal in space," Technical Paper 1225, NASA, may 1987.

[3] D. D. R. Williams, "Planetary fact sheet," fact sheet, NASA, november 2015.

[4] J. R. R. R. L. Thompson and S. M. Stevenson, "Study of extraterrestrial disposal of radioactive wastes," Technical Memorandum X-71557, NASA, Cleveland, Ohio 44135, may 1974.

[5] "David Wright presentation on the current debris situation." https://swfound.org/media/99971/wright-space-debris_situation.pdf. Accessed: 2016-11-12.

[6] "Space launch report, all-time launcher results summary." http://www.spacelaunchreport.com/logsum.html. Accessed: 2016-11-13.

[7] "Wikipedia's effects of nuclear explosions." https://en.wikipedia.org/wiki/Effects_of_nuclear_explosions. Accessed: 2016-11-12.

[8] T. Greenspan, "Stability of the lagrange points, L_4 and L_5," Accessed: 2016-11-12.

[9] S. B. Nicholson, "The Trojan Asteroids," *Leaflet of the Astronomical Society of the Pacific*, vol. 8, p. 239, 1961. Provided by the SAO/NASA Astrophysics Data System.

[10] S. C. Levison H., Shoemaker E., "Dynamical Evolution of Jupiter's Trojan Asteroids," *Nat, 385, 42*, 1997.

[11] G. F. Robutel P., "The resonant structure of Jupiter's Trojan asteroids – I. Long-term stability and diffusion," *Mon. Not. R. Astron. Soc. 372, 1463–1482 (2006)*, 2006.

[12] F. Yoshida and T. Nakamura, "Size Distribution of Faint Jovian L4 Trojan Asteroids," *The Astronomical Journal. 130 (6): 2900–11*, vol. 130, pp. 2900–2911, Dec. 2005. Provided by the SAO/NASA Astrophysics Data System.

[13] Anonymous, "Backman, D. Efluctuations in the general zodiacal cloud density"." http://archive.is/C8kw. Accessed: 2016-11-13.

7 Appendix

Here we attach the codes in python that have been used to perform numerical methods:

```
import math
import time
import scipy
import scipy.interpolate
from numpy import *
from scipy.linalg import norm
from scipy import array as vector
from matplotlib import pyplot
%matplotlib inline
year=365*3600*24
UA=1.496*10**11

#Prob of a debri hit
ndebris=1.4*10**4
Vcoet=8.*10**3
Vdebri=8.*10**3
Rmax=2000.*1000
Rmin=300.*1000
CrossSect=pi*8.*8.
Prob1debri=CrossSect*(Rmax-Rmin)/sin(arctan(Vcoet/Vdebri))/(4*pi/3*(Rmax**3-Rmin**3))
print Prob1debri
Probtotal=1-(1-Prob1debri)**ndebris
print Probtotal

DistJUP=5.2*UA
Rsol=6.96*10**8
G=6.67*10**(-11)
Msol=2*10**(30)
Vterrasol=30*10**3
Rasteroids=2.7*UA
MProbe=10**4
MJupiter=2*10**27
T=(4331.572*24*3600) #T jupiter
aJUP=(UA+DistJUP)/2
VJUP=sqrt(2*G*Msol*(2*aJUP-UA)/(2*aJUP*UA))
Tmart=686.971*24*3600
Mmart=6.4*10**23
Distmart=230*10**9

def solve(f, t0, tfinal, y0, tol = 1e-7):

    def F(*args):return vector(f(*args))

    t = t0
    hmax = (tfinal - t0) / 128.0
    h = hmax / 4.0
    y = vector(y0)           # Column vector (nx1).
    out = [(t, list(y))]
    #Cash-Karp parameters
    a = [ 0.0, 0.2, 0.3, 0.6, 1.0, 0.875 ]
```

```
    b = [[],
        [0.2],
        [3.0/40.0, 9.0/40.0],
        [0.3, -0.9, 1.2],
        [-11.0/54.0, 2.5, -70.0/27.0, 35.0/27.0],
        [1631.0/55296.0, 175.0/512.0, 575.0/13824.0, 44275.0/110592.0, 253.0/4096.0]]
    c  = [37.0/378.0, 0.0, 250.0/621.0, 125.0/594.0, 0.0, 512.0/1771.0]
    dc = [c[0]-2825.0/27648.0, c[1]-0.0, c[2]-18575.0/48384.0,
         c[3]-13525.0/55296.0, c[4]-277.00/14336.0, c[5]-0.25]

    while t < tfinal:
        if t + h > tfinal:
            h = tfinal - t
        if t + h <= t:
            raise ValueError('Singularity in ODE')
    # Compute k[i] function values.
        k = [None] * 6
        k[0] = F(t, y)
        k[1] = F(t+a[1]*h, y+h*(k[0]*b[1][0]))
        k[2] = F(t+a[2]*h, y+h*(k[0]*b[2][0]+k[1]*b[2][1]))
        k[3] = F(t+a[3]*h, y+h*(k[0]*b[3][0]+k[1]*b[3][1]+k[2]*b[3][2]))
        k[4] = F(t+a[4]*h, y+h*(k[0]*b[4][0]+k[1]*b[4][1]+k[2]*b[4][2]+k[3]*b[4][3]))
        k[5] = F(t+a[5]*h, y+h*(k[0]*b[5][0]+k[1]*b[5][1]+k[2]*b[5][2]+k[3]*b[5][3]+k[4]*b[5][4]))

    # Estimate current error and current maximum error.
        E = norm(h*(k[0]*dc[0]+k[1]*dc[1]+k[2]*dc[2]+k[3]*dc[3]+k[4]*dc[4]+k[5]*dc[5]))
        Emax = tol*max(norm(y), 1.0)

    # Update solution if error is OK.
        if E < Emax:
            t += h
            y += h*(k[0]*c[0]+k[1]*c[1]+k[2]*c[2]+k[3]*c[3]+k[4]*c[4]+k[5]*c[5])
            out += [(t, list(y))]

    # Update step size
        if E > 0.0:
            h = min(hmax, 0.85*h*(Emax/E)**0.2)

    return out

Vastnova=4.241e+04
epsilon=-10000
x0=vector([-UA,epsilon,0,-Vastnova])
aValue=1/((2/UA)-Vastnova**2/(G*Msol))
semitemps=sqrt(abs(aValue)**3*4*pi**2/(G*Msol))/2
print aValue
theta=-pi/3-2*pi*semitemps/T
year=24*365*3600
Mterra=6*10**24
print DistJUP*2*pi/T
def campVectorial(t,x): #t=tempsReal/(4331.572*25*3600)
    JupiterPosition=DistJUP*vector([cos(theta+2*pi*t/T),sin(theta+2*pi*t/T)])
    sunField=-G*Msol*vector([x[0],x[1]])/sqrt(x[0]**2+x[1]**2)**3
    JupiterField=-G*MJupiter*vector([x[0]-JupiterPosition[0],
    x[1]-JupiterPosition[1]])/sqrt((x[0]-JupiterPosition[0])**2+(x[1]-JupiterPosition[1])**2)**3
```

```
    terraPosition=UA*vector([-cos(2*pi*t/year),-sin(2*pi*t/year)])
    terraField=-G*Mterra*vector([x[0]-terraPosition[0],
    x[1]-terraPosition[1]])/sqrt((x[0]-terraPosition[0])**2+(x[1]-terraPosition[1])**2)**3
    field=sunField+JupiterField+terraField
    if((x[0]-terraPosition[0])**2+(x[1]-terraPosition[1])**2<0): print
        (x[0]-terraPosition[0])**2+(x[1]-terraPosition[1])**2
    return vector([x[2],x[3],field[0],field[1]]);
#solution=solve(campVectorial,0.0,semitemps,x0,1e-10)

#Sortint de la terra

x0=vector([-UA,-10000000,0,-Vastnova])
semitemps=semitemps/80
solution=solve(campVectorial,0.0,semitemps,x0,1e-10)

myspeed=sqrt(solution[len(solution)-1][1][2]**2+solution[len(solution)-1][1][3]**2)
extraSpeed=DistJUP*2*pi/T-myspeed
print myspeed
print DistJUP*2*pi/T
print 'Extraspeed:'
print extraSpeed
print 'theta:'
print theta*180/pi
print 'Initial speed'
print Vastnova
newicons=vector([solution[len(solution)-1][1][0],solution[len(solution)-1][1][1],
              solution[len(solution)-1][1][2],solution[len(solution)-1][1][3]+extraSpeed])
newsolution=solve(campVectorial,semitemps,1000*year,newicons,1e-10)
xcomp=zeros(len(solution))
ycomp=zeros(len(solution))
jupx=zeros(len(solution))
jupy=zeros(len(solution))
terrax=zeros(len(solution))
terray=zeros(len(solution))
for i in range(len(solution)):
    xcomp[i]=solution[i][1][0]
    ycomp[i]=solution[i][1][1]
    jupx[i]=DistJUP*cos(theta+2*pi*solution[i][0]/T)
    jupy[i]=DistJUP*sin(theta+2*pi*solution[i][0]/T)
    terrax[i]=-UA*cos(2*pi*solution[i][0]/(3600*24*365))
    terray[i]=UA*sin(2*pi*solution[i][0]/(3600*24*365))
newxcomp=zeros(len(newsolution))
newycomp=zeros(len(newsolution))
newjupx=zeros(len(newsolution))
newjupy=zeros(len(newsolution))
newterrax=zeros(len(newsolution))
newterray=zeros(len(newsolution))
for i in range(len(newsolution)):
    newxcomp[i]=newsolution[i][1][0]
    newycomp[i]=newsolution[i][1][1]
    newjupx[i]=DistJUP*cos(theta+2*pi*newsolution[i][0]/T)
    newjupy[i]=DistJUP*sin(theta+2*pi*newsolution[i][0]/T)
    newterrax[i]=-UA*cos(2*pi*newsolution[i][0]/(3600*24*365))
    newterray[i]=UA*sin(2*pi*newsolution[i][0]/(3600*24*365))
import matplotlib.pyplot as plt
```

```
plt.plot(xcomp,ycomp, 'r', newxcomp, newycomp, 'r', newjupx, newjupy, 'g', jupx, jupy, 'g',
    terrax, terray, 'b', newterrax, newterray, 'b')

#Let us consdierMars!
Vastnova=4.10e+04
theta=pi
Mterra=6*10**24
print DistJUP*2*pi/T
def campVectorial(t,x): #t=tempsReal/(4331.572*25*3600)
    JupiterPosition=DistJUP*vector([cos(theta+2*pi*t/T),sin(theta+2*pi*t/T)])
    sunField=-G*Msol*vector([x[0],x[1]])/sqrt(x[0]**2+x[1]**2)**3
    JupiterField=-G*MJupiter*vector([x[0]-JupiterPosition[0],
    x[1]-JupiterPosition[1]])/sqrt((x[0]-JupiterPosition[0])**2+(x[1]-JupiterPosition[1])**2)**3
    terraPosition=UA*vector([-cos(2*pi*t/year),-sin(2*pi*t/year)])
    terraField=-G*Mterra*vector([x[0]-terraPosition[0],
    x[1]-terraPosition[1]])/sqrt((x[0]-terraPosition[0])**2+(x[1]-terraPosition[1])**2)**3
    marsPosition=Distmart**vector([cos(theta+2*pi*t/Tmart),sin(theta+2*pi*t/Tmart)])
    marsfield=-G*Mterra*vector([x[0]-marsPosition[0],
    x[1]-marsPosition[1]])/sqrt((x[0]-marsPosition[0])**2+(x[1]-marsPosition[1])**2)**3
    field=sunField+JupiterField+terraField+marsfield
    #if((x[0]-terraPosition[0])**2+(x[1]-terraPosition[1])**2<0): print
        (x[0]-terraPosition[0])**2+(x[1]-terraPosition[1])**2
    return vector([x[2],x[3],field[0],field[1]]);

#solution=solve(campVectorial,0.0,semitemps,x0,1e-10)

#Sortint de la terra
semitemps=1.45*year
x0=vector([-UA,-10000000,0,-Vastnova])
solution=solve(campVectorial,0.0,semitemps,x0,1e-10)

myspeed=sqrt(solution[len(solution)-1][1][2]**2+solution[len(solution)-1][1][3]**2)
extraSpeed=2000

newicons=vector([solution[len(solution)-1][1][0],solution[len(solution)-1][1][1],
                solution[len(solution)-1][1][2],solution[len(solution)-1][1][3]+extraSpeed])
newsolution=solve(campVectorial,semitemps,50*year,newicons,1e-10)
xcomp=zeros(len(solution))
ycomp=zeros(len(solution))
jupx=zeros(len(solution))
jupy=zeros(len(solution))
terrax=zeros(len(solution))
terray=zeros(len(solution))
marsx=zeros(len(solution))
marsy=zeros(len(solution))
for i in range(len(solution)):
    xcomp[i]=solution[i][1][0]
    ycomp[i]=solution[i][1][1]
    jupx[i]=DistJUP*cos(theta+2*pi*solution[i][0]/T)
    jupy[i]=DistJUP*sin(theta+2*pi*solution[i][0]/T)
    marsx[i]=Distmart*cos(2*pi*solution[i][0]/Tmart)
    marsy[i]=Distmart*sin(2*pi*solution[i][0]/Tmart)
    terrax[i]=-UA*cos(2*pi*solution[i][0]/(3600*24*365))
    terray[i]=UA*sin(2*pi*solution[i][0]/(3600*24*365))
newxcomp=zeros(len(newsolution))
```

```
newycomp=zeros(len(newsolution))
newjupx=zeros(len(newsolution))
newjupy=zeros(len(newsolution))
newterrax=zeros(len(newsolution))
newterray=zeros(len(newsolution))
newmarsx=zeros(len(newsolution))
newmarsy=zeros(len(newsolution))
for i in range(len(newsolution)):
    newxcomp[i]=newsolution[i][1][0]
    newycomp[i]=newsolution[i][1][1]
    newjupx[i]=DistJUP*cos(theta+2*pi*newsolution[i][0]/T)
    newjupy[i]=DistJUP*sin(theta+2*pi*newsolution[i][0]/T)
    newterrax[i]=-UA*cos(2*pi*newsolution[i][0]/(3600*24*365))
    newterray[i]=UA*sin(2*pi*newsolution[i][0]/(3600*24*365))
    newmarsx[i]=Distmart*cos(2*pi*newsolution[i][0]/Tmart)
    newmarsy[i]=Distmart*sin(2*pi*newsolution[i][0]/Tmart)
import matplotlib.pyplot as plt
plt.plot(newmarsx, newmarsy, 'yellow', xcomp,ycomp, 'r', newxcomp, newycomp, 'r', newjupx,
     newjupy, 'g', jupx, jupy, 'g', terrax, terray, 'b', newterrax, newterray, 'b')


(newsolution[2][1][0])
maxim=0;
minim=0;
for i in range(len(newsolution)):
    if(newsolution[i][1][0]>maxim): maxim=newsolution[i][1][0]
    if(newsolution[i][1][0]<minim): minim=newsolution[i][1][0]
print (maxim-minim)/(2*UA)
```