Problem A: Solar Sailing to Mars

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Abstract

Solar sails became reality lately, being far more affordable then most of modern methods of propulsion in space. They obviously have some limitations, as they can work effectively only in close proximity of the Sun and can not propel spacecrafts towards the Sun. Using such solar sails for missions to the Mars has been discussed previously by many groups and seem to have a lot of potential, especially if time is not the most limiting factor. Our limiting factor was a mass of an entire ship fixed at 2000kg. In our work we suggest a heuristic approach to optimize an angle of incidence during flight, while keeping this angle fixed for fixed periods of time. With numerical simulations we found optimal routes for different ratio (q) of sail to ship mass. With such an optimization we concluded, that most promising is to have q ratio equal to 0.46, what gives 1080 kg of payload. Such a spaceship would reach Mars in about 532 days.

1 Introduction

A solar sail is a modern concept of cosmic travel, where a spacecraft is propelled by light pressure from photons emitted from the Sun. It offers obvious advantages of fuel conservation and possible weight reduction. Using solar sail can become the future of cosmic exploration, but it also demands a new approach to planning a mission.

Indeed, the concept significantly differs from conventional methods, where for most part the vessel travels along an elliptical path with gravity being the only acting force, and the orbit is changed by short boosts at perihelion / aphelion. Light pressure constantly affects the sail, which makes equations of motion more complicated and trajectories impossible to derive analytically.

Therefore we evaluated trajectories numerically with variable parameters of sail size and orientation angle with respect to position vector (which can be modified throughout the flight by rotating the sail at low energetic cost). Our goal is to optimize $\eta = \frac{m}{T}$, where m is the mass of a spacecraft, and T the time of a mission.

In our case we consider a mission to Mars, where we start at the moment of planet's closest approach to Earth. Similar problem has been presented previously by [5], but with less constraints than in our case. We decided to use a different with semi-heuristic algorithm attached in Appendix A.

2 Mathematical model of solar sail

We model a solar sail as a flat surface tilted at an angle Θ between line connecting the center of the Sun and normal vector to sail surface. The net force acting on a sail is proportional to an effective cross section of a solar sail and scales with a $\cos(\Theta)$. We assume that sun is a uniform black sphere at a temperature equal to 5 778K [4]. Therefore, solar radiation impulse on a sail can be estimated based on total radiation emitted by black body according to equation:

$$F = \frac{2A_\odot \sigma T_\odot A_S}{4\pi r^2 c}\cos(\Theta)$$

Where $A_\odot$ is total Area of the Sun, $T_\odot$ is temperature of a surface of the Sun, A_s is a total area of the sail, r is distance between aircraft and the Sun, σ is the Stefan–Boltzmann constant and c is speed of light in vacuum. After projecting this force on orthogonal basis in spherical coordinates we receive such components:

$$\begin{cases} F_r = \frac{2A_\odot \sigma T_\odot A_S}{4\pi m r^2}\cos^2(\Theta) \\ F_\phi = \frac{2A_\odot \sigma T_\odot A_S}{4\pi r^2 c}\cos(\Theta)\sin(\Theta) \end{cases} \tag{1}$$

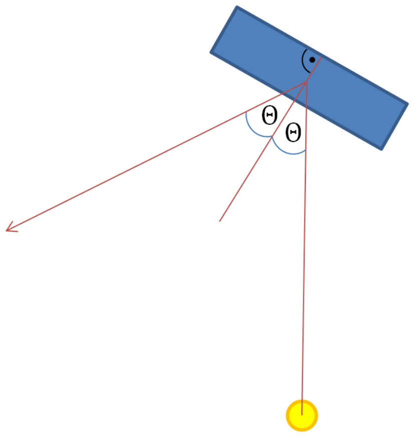

Figure 1: Mathematical model of a solar sail with defined angle

2.1 Equations of motion

We append derived equations to a standard equations of motion for a standard spherical coordinates, assuming that orbit is a circle with the Sun in the center. We therefore consider only positions on a plane, assuming that orbits of the Earth and Mars lie on the same plane.

$$\begin{cases} \ddot{r} = -\frac{GM}{r^2} + \frac{\alpha}{r^2}\cos^2(\Theta) + \dot{\phi}^2 r \\ r\ddot{\phi} = \frac{\alpha}{r^2}\sin(\Theta)\cos(\Theta) - 2\dot{r}\dot{\phi} \end{cases} \tag{2}$$

Where, $\alpha = \frac{2A_\odot \sigma T_\odot A_S}{4\pi m c}$ We can modify radial component of above equations to be a motion in modified gravity field, as all forces are proportional to $\frac{1}{r^2}$

With this assumptions we are able to simulate flight of the satellite with different $\Theta(t)$. Lets also assume, that our satellite at a beginning has a well defined surface of the sail and that changing angle Θ does not affect energy of a satellite. Under such conditions, change of an angle Θ in time is the only parameter for optimization. For every size of sail (which defines also payload size) we are able to propose a best route and measure transit time. Since we want to optimize ratio of payload weigh and mission time we are able to maximize it for every parameter.

Considering now a fixed payload size and fixed sail size we optimize only angle $\Theta(t)$ in time.

Since the launch takes place on a day where Earth and Mars are in the closest proximity, satellite reaches Mars orbit Before Mars and has to decelerate in order to reach it‘s proximity.

Therefore we decided to simulate movement of the satellite by considering long distances with fixed angle Θ, so function $\Theta(t)$ becomes a piecewise function of time. We can denote number of individual parts (with fixed angles) as N. If we will increase N to infinity we can easily convert this approach for a smooth function $\Theta(t)$, but due to nature of a problem piecewise function seem to fit well and highly simplifies optimization process. We keep number of pieces low as complexity of a problem scales with exponent. Moreover, with a few degrees of freedom we obtain highly enhanced time of mission and route. Further increase in number of different settings $\Theta(t)$ does not decrease flight time significantly.

2.2 Flight optimization

Search for optimal ratio has been performed assuming three different angles Θ, so with two changes of an angle during the flight. We define a sail area factor q (mass of sail/entire mass of aircraft)

and we probe best solutions with a resolution of 0.01.
First we try to estimate a range of parameters, which allow to reach the Mars orbit, and then we optimize them in order to decrease distance from Mars and adjust relative speed for it to be below the required 9km/s. By using such a heuristic approach we define some parameter ranges, such that eg. sail angle at the beginning should not be set to decelerate aircraft. This approach narrows the space to be searched and, because of that, lowers the computation time.

2.3 Optimization Algorithm

We had 3 parameters in our optimization process θ_1,θ_2,θ_3 which were respectively angles of incidence for different times of flight (from take off to 6 month mark, then 12 month mark and then to contact with the Mars). We used grid optimization and after finding local minimum for given q we optimized further near previously found minimum.

Initial conditions:
$r(0)$=Earth radius, $\dot{r}(0) = 0$, $\phi(0) = 0$, $\dot{phi}(0) =$ Earth's angular velocity

Constrains for initial parameters

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \tag{3}$$

3 Results

All data and equations has been obtained from following resources: [4, 2, 3, 1] After running initial simulations, a trend emerges. Most of locally optimal solutions follow a specific behavior: first, Mars' orbit is reached; then, the ship decelerates, by changing the direction of the sail, and makes its final approach. Sometimes, a final correction takes place to hit Mars more precisely.

This course of action, which was found by optimizing sail's angle in respect to time, is to be expected. It proves that the simulation gives rational results.

When the aircraft first approaches the Mars orbit it is significantly ahead of the Mars. As the launch happened from Earth orbit and since Earth‘s orbital velocity is higher than Mars', to reach close proximity of Mars and not waist time probe should reach higher (further) orbit from the Sun and decrease its angular speed, "wait for Mars", before next crossing.

Then, when shuttle is in proximity of Mars, we can adjust course to reach Mars within good radius (estimated to 20,400km - near stationary orbit for Mars, between its moons Phobos and Deimos) and with low enough velocity given in a task to be less than 9km/s.

Example trajectory found by our simulation look like the one below:

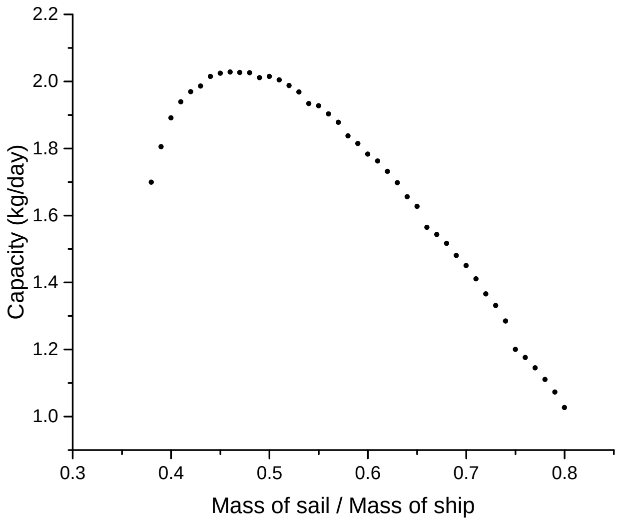

Figure 2: Capacity of sail as function of q

The graph below pictures the relation between loading factor q, and the time for the shuttle to reach Mars by following the optimized trajectory: As it would be expected, the larger the sail,

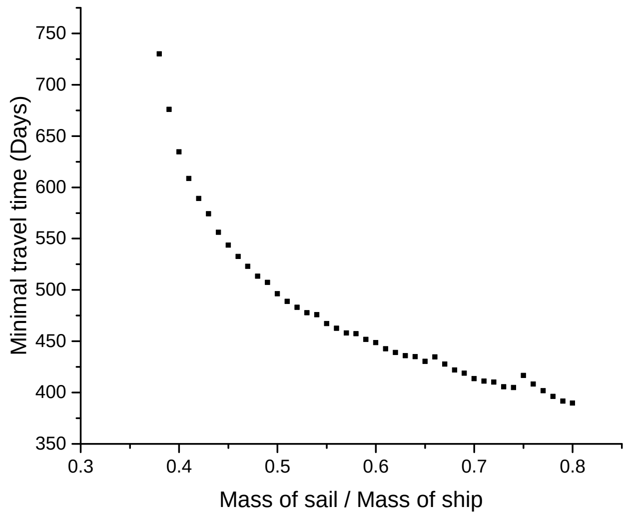

Figure 3: Minimal flight time as function of q

the faster our target can be reached. However, when we consider load mass to travel time ratio (channel capacity), the result is very interesting:

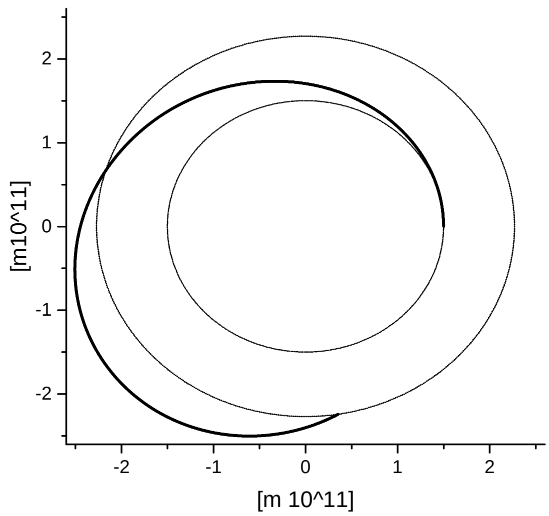

Figure 4: Trajectory of optimal flight path. Two circles represent orbits of Earth and Mars

A clear maximum can be seen around $q = 0.46$. As one would expect the transport efficiency decreases when q reaches 0 or 1. Having a very small sail results in a far too long trip, while a short trip with a huge sail is not fast enough to account for the load capacity lost to the sail.

Optimal transport efficiency was reached for $q = 0.46$ and it was $\eta = 2.028\frac{kg}{day}$. It was reached after 532.517 days, distance from mars at that time was $1.07 \cdot 10^7 m$ and their relative velocity $8.6\frac{km}{s}$.

4 Conclusions

Our results for 3 angle optimization gives a reasonable capacity yield of $2.028\frac{kg}{day}$. With more computational power we could add more optimization points, which would possibly increase the capacity-to-time ratio.

During our simulations we deduced that this particular time is not optimal and we could increase this ratio by changing the launch date and time, for example having Mars ahead of Earth would probably simplify the task. However, we did not consider such conditions, as it was stated in the task that we have to start from such a setup.

References

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