# University Physics Competition 2017

Problem A Solar Sailing to Mars

# Optimization of the solar sail's voyage from Earth to Mars

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## Abstract

This paper analyzed the voyage of the solar sail from Earth to Mars. Three partial models were established to describe the process: Solar Sail Thrust Model, Ideal Solar Sail Time Optimal Transfer Orbit Model and Orbital Transfer Optimization Model Based on PSO.

In the Solar Sail Thrust Model, the formula ( $F = P_0 \cdot A \cdot 2\cos^2\alpha \cdot L^2 / r_1^2$ ) was established to describe the sun pressure of the solar sail.

In the Ideal Solar Sail Time Optimal Transfer Orbit Model, the differential equations were set up to describe the motion of the spacecraft. And the initial constrains and final constrains were provided. Adopting Hamiltonian operator, the covariate variable $\lambda = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4]^T$ were built. Applying the pontryagin minimum principle to find optimal control input that minimizes Hamilton's function, the optimal angle $\alpha^*$, which can be expressed by covariate variables, can be calculated. The orbital transfer problem of solar sail can be converted into an optimization problem of multi constrains and multivariable.

In the Orbital Transfer Optimization Model Based on PSO Model, particle swarm optimization algorithm was adopted to calculate a more accurate answer. Also, dynamic penalty function constraint was added and the penalty function of orbital transfer was defined as $F = J + M \cdot S$.

By changing the area of the solar sail ( $10000m^2$ $50000m^2$ $100000m^2$ $200000m^2$ respectively), graphs of orbital transfer, direction adjustment and velocity were drawn to satisfy the problem's demand. Finally, When $A = 100000m^2, m_s = 700kg$ , payload reaches a relatively large value of 1300kg and transfer time reduces to 504d(about 116 days less than the flight plan in reference [8]). And the final velocity ( $8.3\ km/s$ ) smaller than $9\ km/s$ is within the safe landing velocity range.

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# 1. Introduction

The success of JAXA's IKAROS and NASA's Nanosail-D has demonstrated the technological capability of deploying and actively controlling a photonic solar sail in the interplanetary space. The solar sail is a thruster that obtains the thrust by reflecting photons from its ultralight sail and through the momentum exchange between photons and the sail surface. Since solar sail spacecraft are free from carrying fuels and have low cost, they are superior in long cycle missions like landing on Mars.

According to the assignment, our work begins at the point when the solar sail escapes the earth and ends when it can land on Mars with a safe relative velocity. As for solar-powered spacecraft, the acceleration comes from the solar sail (In fact, the solar attraction also contributes to the acceleration) and is proportional to the size of the sail. The bigger the sail is, the faster the spacecraft can speed and thus the shorter the flight time will be. However, since the total mass is limited and the final speed must be less than or equal to $9\ km/s$, adding the area of the sail means reducing the payload and increasing the final speed. So to optimize the flight plan, we find it necessary to obtain a balance between payload and the size of the sail.

To realize the voyage from Earth to Mars, we established three partial models to describe the process and they are: Solar Sail Thrust Model, Ideal Solar Sail Time Optimal Transfer Orbit and Orbital Transfer Optimization Model Based on PSO. With the combination of the three partial models, the optimal flight plan can hopefully be found after calculation run on computers.

# 2. Assumptions and Notations

## 2.1 Basic Assumptions for Our Models.

- Assume that there is only specular reflection of sunlight on the solar sail. On the one hand, the model can be simplified because specular reflection ensures that the value of the tangential acceleration of light pressure is always zero. On the other hand, by choosing proper material, we can indeed approximate this effect.
- Suppose the light pressure acts on the spacecraft's center of mass. If it does not

act on the center of mass, there will be additional angular acceleration, and it is not conducive to the stability of the spacecraft, also the simulation of flight state. Of course, it can simplify the model.

- Assume that the spacecraft is only exposed to the sun's gravity and light pressure during the flight. Because after the spacecraft is free from earth bondage and before it is captured by Mars, the gravities from both the earth and Mars are very small. And ignoring the two gravities can simplify the calculation and facilitate the model establishment.
- Suppose the solar sail does not require time to adjust. In another word, the change of the acceleration is abrupt. We know the effect of this adjustment is weakness. So, comparing with the whole process we can absolutely ignore this influence. If we do not deal with it in this way, the possibility of solving problems will be reduced.
- Assume that the size of the solar sail makes no difference to the shape of it, especially during the process of acceleration. And its shape can be kept forever. After disregarding the shape changes due to acceleration, then, the value of the optical force is only related to the distance between the sun and the spacecraft and the angle between the normal and the light path.
- Assume that when the spacecraft's speed drops to $9\mathrm{km/s}$, it just enters the landing orbit of Mars. Because in this model, we don't care the way how the spacecraft land on Mars.

## 2.2 Notations

Table 1 Notations

| Variable | Explanations |
|---|---|
| $\mathrm{P}(r)$ | The sun pressure when the distance from Sun is r ($N/m^2$) |
| $R_\delta$ | The radius of Sun (m) |
| $I_0$ | The specific intensity of the integrated frequency($I_0 = 2.04\times10^7 W/m$) |
| $F_i$ | The light pressure(N) |

| | |
|---|---|
| $A_\varepsilon$ | The effective area of sail( $m^2$ ) |
| $L$ | The distance between Sun and Earth( $AU$ ) |
| $\mathrm{P}_0$ | The light pressure at 1AU( $P_0 = 4.56\times10^{-6} N / m^2$ ) |
| $\sigma_s$ | The mass per unit area of the sail( $kg / m^2$ ) |
| $a_c$ | Characteristic acceleration( $m / s^2$ ) |
| $P_{eff,0}(P)$ | The effective light pressure acting on a non-ideal solar sail from 1AU( $P_{eff,0}(P) = 8.288\mu N / m^2$ ) |
| $a_0$ | $a_0 = 5.95 m / s^2 \ at\ 1\ AU$ |
| $\beta$ | Light pressure factor, the ratio of solar pressure acceleration to the solar gravitational acceleration $\beta = \frac{a_c}{a_0}$ |
| $\lambda$ | The collaborative variable |

# 3. Physical Analysis of Model

To realize the voyage from Earth to Mars, we established three partial models to describe the process and they are: Solar Sail Thrust Model, Ideal Solar Sail Time Optimal Transfer Orbit and Orbital Transfer Optimization Model Based on PSO.

## 3.1 Solar Sail Thrust Model

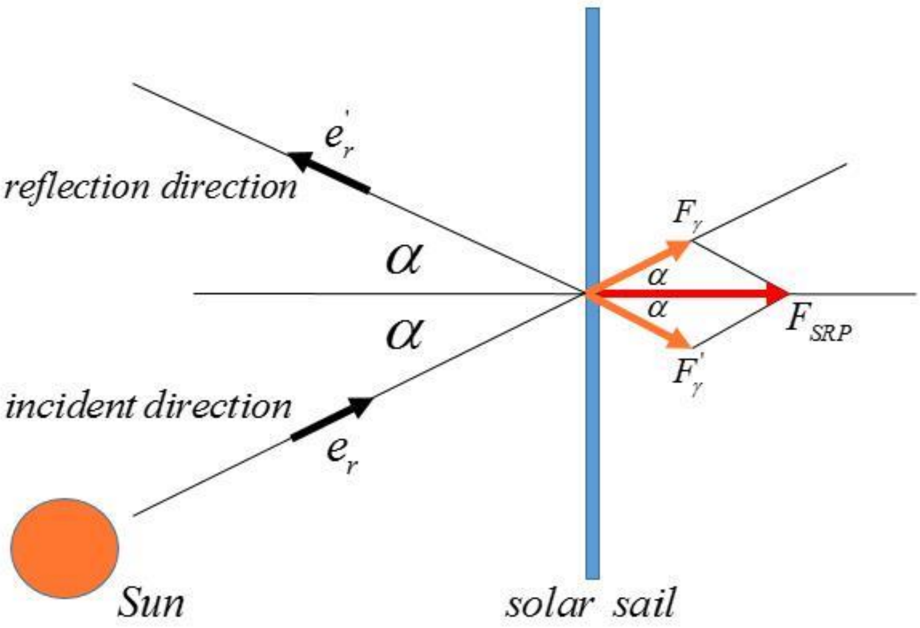


Fig 1. A picture showing the process of reflection of light on an ideal solar sail, and the mechanical effect. It can tell us that the light pressure is always perpendicular to the surface of the solar sail, and the tangential force disappears.

It is assumed that the solar sail is ideal, that is, the solar sail is a perfect reflector and does not absorb any photons.

Sun pressure is expressed as [1]:

$$\mathrm{P}(r) = \frac{2\pi I_0}{3c}\left\{1-[1-(\frac{R_\delta}{r})^2]^{\frac{3}{2}}\right\} \tag{1}$$

$R_\delta$ refers to the radius of Sun.

The light pressure generated by incident photons is:

$$F_i = \mathrm{P}(r)A_\varepsilon(\cos\alpha\vec{n} - \sin\alpha\vec{t}) \tag{2}$$

$A_\varepsilon$ is the effective area of sail, $\alpha$ is the angle between the incident ray and the unit normal line of the solar sail $\vec{n}$. The normal line of the solar sail $\vec{n}$ is perpendicular to the sail and away from the sun. $\vec{t}$ is the unit vector perpendicular to the normal $\vec{n}$, and its counterclockwise direction is positive.

Then the light pressure generated by reflection photons is:

$$F_i = \mathrm{P}(r) A_\varepsilon (\cos\alpha \vec{n} + \sin\alpha \vec{t}) \tag{3}$$

The effective area is where the surface of the sail is projected on a plane perpendicular to the sun's rays. $A_\varepsilon = A\cos\alpha$ So the values of optical force acting on the ideal sail is

$$F_i + F_t = 2P(r)A\cos^2\alpha \vec{n} \tag{4}$$

That is why the optical force always has the same direction with $\vec{n}$, both of them are away from the sun. From the equation, we can also find that Light exerts a pressure equal to twice its energy density when reflected from a surface.

The power $F$ obtained by the solar sail is always along the normal direction, its size is

$$F = \frac{P_0 L^2}{r_1^2} \cdot \frac{2m}{\sigma} \cdot \cos^2\alpha = \frac{P_0 L^2}{r_1^2} \cdot 2A \cdot \cos^2\alpha = P_0 \cdot A \cdot \frac{2\cos^2\alpha}{r_1^2 / L^2} \tag{5}$$

$$P_0 = 4.56 \times 10^{-6} N / m^2$$

Among them, $L$ is the distance between Sun and Earth, $r_1$ is the distance from the sun to the solar sail, $\mathrm{P}_0$ is the sun pressure at 1 $AU$.

### 3.1.1 Analysis of Solar Sail Thrust Model

Solar-powered spacecraft, powered solely by reflected solar photons, has a thrust range that is only a "quasi-circular" small area located on the surface of the sail in Figure 2 and always pointing away from the sun. In addition, it can be seen from the analysis that by changing the orientation of the sail, the orbital angular momentum of the solar sail spacecraft can be reduced so as to make it move relative to the inner helix of the sun, and the orbital angular momentum of the solar sail spacecraft can also be increased, making external spiral movement relative to the sun.

The question shows solar sailing from Earth to Mars, thus this is a process away from the sun. What we need to do is to reduce the orbital angular momentum.

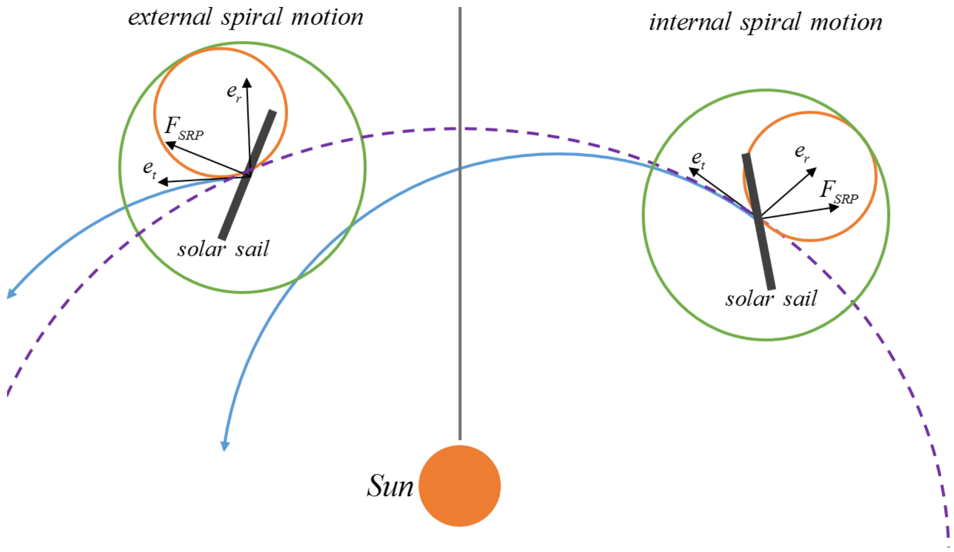


Fig 2. A picture showing two different kinds of motion depending on the angle of the sail. The external spiral motion can generate smaller resistance, while the internal spiral motion makes the resistance larger.

### 3.1.2 The Main Parameters of Solar Sail

The main parameters of solar sail are: total solar sail load, solar sail payload, characteristic acceleration, light pressure factor.

**(1) Total solar sail load**

Defined as the mass per unit area of a sail (including the sail surface and the mass of structure required to deploy, stretch and sail the sail)

$$\sigma_s = \frac{m_s}{A} \tag{6}$$

$\sigma_s$ is an important parameter to show the effectiveness of solar sail structure design.

**(2) Solar sail payload**

$$\sigma = \frac{m}{A} = \frac{m_s + m_p}{A} = \sigma_s + \frac{m_p}{A} \tag{7}$$

It is an important parameter that indicates the pressure factor of the sail, defined as the mass per unit area of the solar sail spacecraft, "P" means the entire spacecraft mass except the sail mass.

**(3) Characteristic acceleration**

It is defined as the maximum acceleration of solar sail obtained at 1AU from the sun:

$$a_c = \frac{2S_0}{c} q_\perp(0,P)\frac{A}{m} = P_{eff,0}(P)\frac{A}{m} = \frac{P_{eff,0}(P)}{\sigma_s + \frac{m_p}{A}} \tag{8}$$

$P_{eff,0}(P)$ is the effective light pressure acting on a non-ideal solar sail from 1AU from the sun[2].

**(4) Light pressure factor**

It represents the ratio of the solar pressure acceleration of a solar sail perpendicular to the sun's rays to the solar gravitational acceleration of the solar sail[3] ( $a_0 = 5.95 m/s^2\ at\ 1\ AU$ )

$$\beta = \frac{a_c}{a_0} \tag{9}$$

Light pressure factor $\beta$ is an important parameter of solar sail. Since both solar pressure and solar gravitation are inversely proportional to the square of the distance, the pressure factor has nothing to do with the distance but only relates to the solar sail spacecraft load. For the ideal solar sail, for

$$\beta = \frac{1.53}{\sigma(g/m^2)} \tag{10}$$

The ideal solar sail light pressure type can also be written

$$\overrightarrow{F_{SRP}} = \beta\frac{\mu m}{r}\cos^2\alpha\vec{n} \tag{11}$$

Where $\mu = GM_{sun}$, G is the Gravitational constant, $M_{sun}$ is the weight of the Sun, m is the whole weight of the sun sail, because of $m \ll M_{sun}$, $\mu = GM_{sun}$.

## 3.2 Ideal Solar Sail Time Optimal Transfer Orbit

### 3.2.1 Equation of motion

The system model is shown in figure 3, where the x-axis points in the vernal equinox direction. The spacecraft is modeled as a perfectly flat solar sail with mass m and area A and can be considered as a particle on the dynamics. Define solar sail coordinate system $s - x_b y_b$. The $s - x_b y_b$ axis defines the normal direction of the solar sail. The solar sail $\alpha$ $(-\pi \le 2\alpha \le \pi)$ is defined as the angle between the $x_b$ axis and the sun's rays striking the sail. The solar sail's initial and target orbits are the orbits of the solar system's planets (Earth and Mars), modeled here as a heliocentric coplanar orbit.

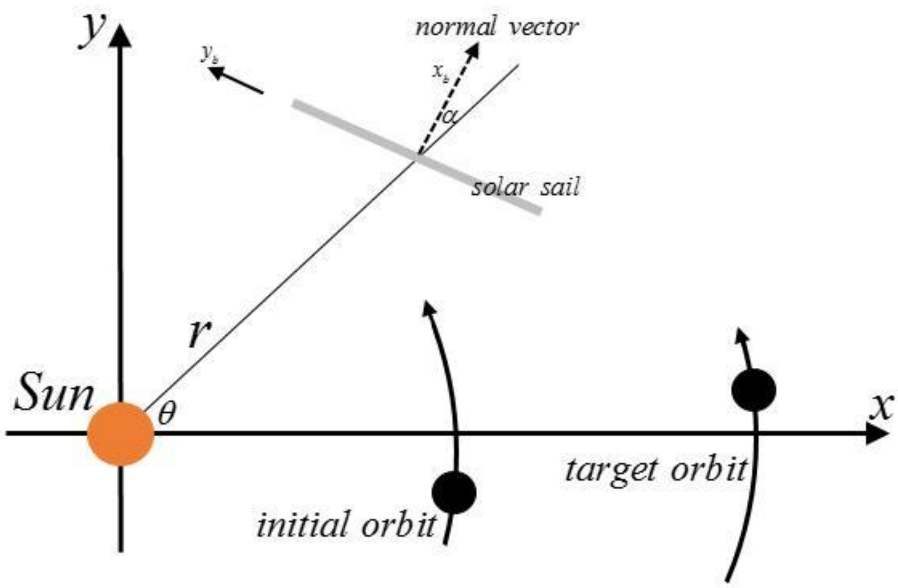


Fig 3. A picture showing the coordinate and the meaning of parameters

Given the polar form of the solar sail equation of motion, the position vector is $r \triangleq (r,\theta)$, the velocity vector $v \triangleq (v_r, v_\theta)$, where $v_r = \frac{\partial r}{\partial t}, v_\theta = r\frac{\partial \theta}{\partial t}$, then the solar sail two-dimensional heliocentric orbital equation of motion is as follows[4]:

$$
\begin{aligned}
&\frac{\partial r}{\partial t} = v_r && (12.1)\\
&\frac{\partial \theta}{\partial t} = \frac{v_\theta}{r} && (12.2)\\
&\frac{\partial v_r}{\partial t} = \frac{\beta\mu\cos^3\alpha}{r^2} + \frac{{v_\theta}^2}{r} - \frac{\mu}{r^2} && (12.3)\\
&\frac{\partial v_\theta}{\partial t} = \frac{\beta\sin\alpha\cos^2\alpha}{r^2} - \frac{v_r v_\theta}{r} && (12.4)
\end{aligned}
\qquad (12)
$$

where $\beta$ is the light pressure factor, $-\frac{\mu}{r^2}$ is the contribution of the sun’s the universal gravitation to the solar sail’s acceleration.

### 3.2.2 Restrictions

The above equations of motion can be written in the form of $\frac{\partial x}{\partial t} = f(x,u,t)$, where the state vector and the control input are respectively defined as $x = [r \quad \theta \quad v_r \quad v_\theta]^T$

and $u=\alpha$.

The initial conditions of the solar sail transfer orbit are $r(t_0)=r_0, \theta(t_0)=\theta_0$

$v_r(t_0)=11.2km/s, v_\theta(t_0)=\frac{1}{\sqrt{r_0}}$.

Terminal constraints after entering the target orbit:

$$r(t_f)=r_f, \theta(t_f)=\theta_f, v_r(t_f)=0, v_\theta(t_f)=\frac{1}{\sqrt{r_f}}$$

Where, $t_0$ and $t_f$, respectively, departure time and arrival time, $r_0$ And $r_f$ are the orbital radii of the initial planet and the target planet, respectively. The mission of departing from Earth to Mars requires $\theta(t_0)=\theta_0$ and $\theta(t_f)=\theta_f$.

### 3.2.3 Optimal Conditions

The objective function of optimal control of solar sail time is

$$J=\int_{t_0}^{t_f} dt \tag{13}$$

The Hamilton function of the system is [5]:

$$\begin{aligned} &H(x,u,\lambda,t)=1+\lambda^T f(x,u,t) \\ &=1+\lambda_1 v_r+\frac{\lambda_2 v_\theta}{r}+\lambda_3(\frac{\beta\mu\cos^3\alpha}{r^2}+\frac{{v_\theta}^2}{r}-\frac{\mu}{r^2})+\lambda_4(\frac{\beta\sin\alpha\cos^2\alpha}{r^2}-\frac{v_r v_\theta}{r}) \end{aligned} \tag{14}$$

Where $\lambda=[\lambda_1,\lambda_2,\lambda_3,\lambda_4]^T$, the collaborative variables.

The corresponding co-equation equation can be obtained by solving $\frac{\partial\lambda}{\partial t}=-\frac{\partial H}{\partial x}$.

$$\frac{\partial\lambda_1}{\partial t}=\frac{\lambda_2 v_\theta}{r^2}+\lambda_3(\frac{2\beta\mu\cos^3\alpha}{r^3}+\frac{{v_\theta}^2}{r^2}-\frac{2\mu}{r^3})+\lambda_4(\frac{2\beta\sin\alpha\cos^2\alpha}{r^3}-\frac{v_r {v_\theta}^2}{r}) \tag{15}$$

$$\frac{\partial\lambda_2}{\partial t}=0, \text{If and only if } \theta(t_f)=\theta_f \tag{16}$$

$$\frac{\partial\lambda_3}{\partial t}=-\lambda_1+\frac{\lambda_4 v_\theta}{r} \tag{17}$$

$$\frac{\partial\lambda_4}{\partial t}=-\frac{\lambda_2}{r}-2\frac{\lambda_3 v_\theta}{r}+\frac{\lambda_4 v_r}{r} \tag{18}$$

According to **Pontryagin minimum principle**, the optimal control input $u^*\equiv\alpha^*$, minimizing Hamilton function:

$$u^*=\arg\min_{u\in U} H(x^*,u,\lambda^*,t), \forall t\ge 0 \tag{19}$$

Where $x^*$ and $\lambda^*$ represent the optimal state and the co-ordination vector. That is, the optimal direction angle $\alpha^*$ can be obtained from $\frac{\partial H}{\partial \alpha} = 0$.

$$\alpha^* = \begin{cases} \arctan(\frac{-3\lambda_3 - \sqrt{9\lambda_3^2 + 8\lambda_4^2}}{4\lambda_4}) & while \ \lambda_4 \neq 0 \\ 0 & while \ \lambda_4 = 0, \lambda_3 \leq 0 \\ \pm\frac{\pi}{2} & while \ \lambda_4 = 0, \lambda_3 > 0 \end{cases} \tag{20}$$

From the co-equation equation (15) ~ (19) can be seen, Optimal control input can be completely determined by the initial value of co-variant. Therefore, the initial value of co-kinetic variables can be used as the optimal variable, so the solar-transfer orbit optimization problem can be transformed into a multi-constrained multivariable optimization problem with optimal parameters:

$$Y = [t_0 \quad t_f \quad \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4]^T \tag{21}$$

## 3.3 Orbital Transfer Optimization Model Based on PSO

Since we have converted the optimal control problem into parameter optimization problem, particle swarm optimization can be used to optimize the parameters and get optimal transfer trajectory.

Particle Swarm Optimization (PSO) is an evolution computing technology based on theory of swarm intelligence [6]. PSO optimizes the search intelligently by the group generating from the cooperation and competition among particles. PSO has a strong versatility and the characteristic of global optimization.

PSO is a population evolutionary algorithm proposed by simulating the predatory behavior of birds in nature. It has the mechanism of information sharing among particles, and the ability to remember the best position of particles. It is based on unique search mechanism. In the feasible solution space and velocity space, the particle swarm can be initialized randomly. That is to determine the initial position and velocity of particles. The position is used to represent the solution of the problem.

The velocity and position of each particle are updated according to the following two formulas [7]:

$$v_{i,j}(t+1) = wv_{i,j}(t) + c_1 r_1 [p_{i,j} - x_{i,j}(t)] + c_2 r_2 [p_{g,j} + x_{i,j}(t)] \tag{22}$$

$$x_{i,j}(t+1) = x_{i,j}(t+1),\ j = 1, 2, \cdots, d \tag{23}$$

The graph below vividly shows how the particle position is updated:

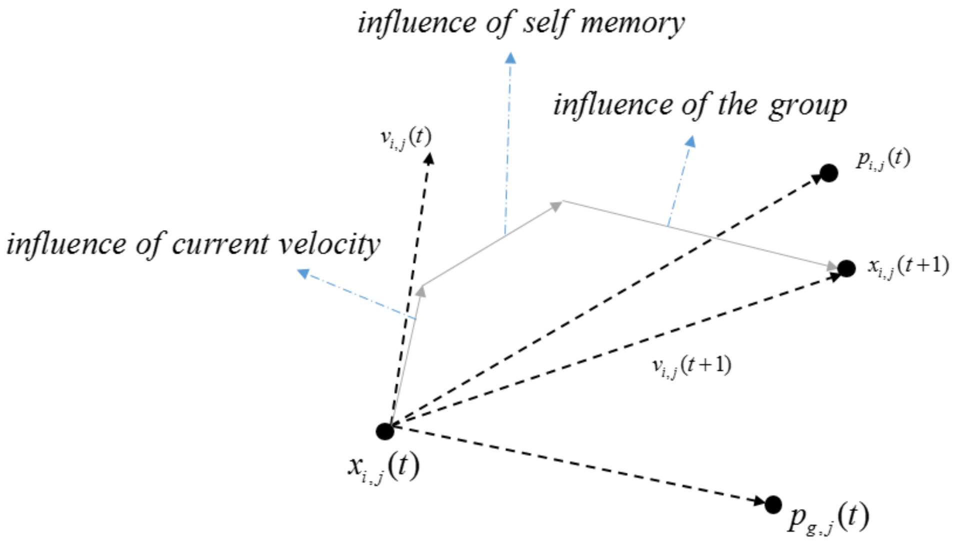


Fig 4. A picture showing the influence of self-memory, the group, and current velocity to a new position

The detailed flow chart of the basic particle swarm algorithm is as follows:

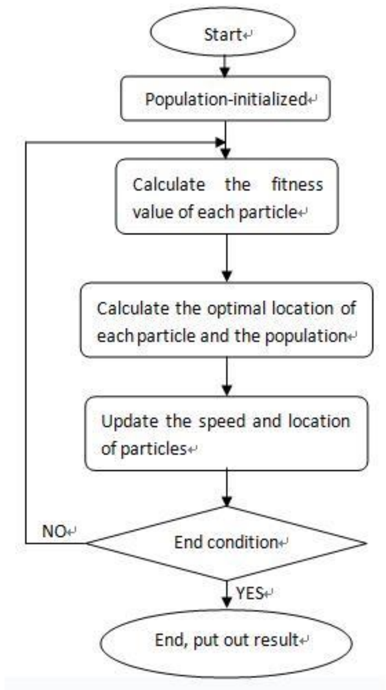


Fig 5. A flow chart of the basic particle swarm algorithm

### 3.3.1 Algorithm Parameters Analysis

The parameters of the standard PSO algorithm include: inertia weight factor, positive acceleration constant $c_1, c_2$, maximum velocity $v_{\max}$, population size $n$, the maximum number of iterations $T_{\max}$.

**(1) Inertia weight factor**[8] （$w$）

$w$ is used to control the effect of current particle velocity on the next generation's velocity. It can affect the capabilities of the global and local search.

**(2) The positive acceleration constant** （$c_1, c_2$）

The acceleration constants $c_1, c_2$ respectively represent the weight of the statistical acceleration item of each particle toward the position of pbest and gbest. Their reasonable settings can improve the convergence speed and avoid local minima.

**(3) Maximum velocity (** $v_{\max}$ **)**

It decides the maximum distance the particle can reach in one flight

**(4) Population size (** $n$ **)**

The larger the number of particles is, the larger the search space will be and the easier it is to find the global optimal solution. However, the elapsed time is increased accordingly.

**(5) The maximum number of iterations (** $T_{\max}$ **)**

Usually, it is the condition of stopping algorithm.

### 3.3.2 Disposal of The Condition of Constraint:

Constraints optimization problems can be expressed as follows [9]:

$$\min f(x) \tag{24}$$

$$s.t. g_i(x) \le 0 (i = 1, 2, \cdots, \mathrm{n}) \tag{25}$$

$$h_j(x) = 0 (j = 1, 2, \cdots, p) \tag{26}$$

$$l_k \le x_k \le u_k (k = 1, 2, \cdots, d) \tag{27}$$

Where, x is the decision vector $x = [\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_d]^T$, d is the dimension of decision vector.

When solving the constraint optimization problem, it is necessary to satisfy the inequality constraint condition and the equality constraint condition. In the range of values, the optimal decision variable is searched to minimize the target function value.

### 3.3.3 Constraints Processing Method Based on Penalty Function

Aiming at the terminal constraints in the solar sail trajectory optimization problem, a constraint processing technique is adopted, that is, the trajectory optimization problem is equivalently transformed into a constrained optimization problem. Our model mainly uses penalty function method

Using the penalty function, map the original problem as:

$$\min fitness(x) = f(x) + \varphi(x)$$

$f(x)$ is the objective function value. $\varphi(x)$ is the penalty function. $fitness(x)$ is the fitness value function.

In this model, we define penalty function of the transfer orbit as:

$$fitness(x) = J + MS \tag{28}$$

*$J$ is the index function(13). $M(>0)$ is the penalty factor. $S = \left\| x(t_f) - x_f \right\|$ is the constraint condition of the solar sail transfer orbit.*

When the condition meets its constraints, S=0, then, no matter how much M is, it is always true that $fitness(x) \equiv J$ . That means there is no penalty. However, when it doesn't satisfy the condition, S>0, and the more serious this kind of damage is, the larger the value of S is. Also, $fitness(x) = J + MS$ is larger. It means damaging the condition is also a kind of penalty. The penalty is increased with the growing of M. According to the experience, it is proper to make $M_{i+1} = cM_i, c \in [10,1000]$ .

# 4. Results and Discussion

During the process of the transition from Earth to Mars, the solar sail transforms from Earth' orbit to Mars' orbit in the sun-centered ecliptic. Because the distance from

Mars to Sun is further than that from the Earth to Sun. The solar sail needs to spiral out to escape the sun (left part in fig 2).

The terminal constraint condition is reflected in the performance indicator function by using the orbit transfer algorithm based on the minimum value of Pontryagin, and the penalty function. Then we use PSO to search and optimize the optimal solution, find out the minimum transfer time. We find the optimal orbit according to the kinematic relations.

The terminal constraint condition is :

$$r(t_f) = 1.524AU, \theta(t_f) = \theta_f, v_\theta(t_f) = 1/\sqrt{1.524}, v_r(t_f) = 0 .$$

Choosing an ideal solar sail whose specific acceleration is $k = 1mm/s^2$ . The simulation is based on PSO. The radiuses of earth and Mars are 1AU and 1.524AU respectively. The optimization parameter is $Y = [t_0, t_f, \lambda_1, \lambda_2, \lambda_3, \lambda_4]^T$ . As for the calculating the time interval, it is necessary to reduce one optimal variable, $t_0$ ,because it is no relation with the launch time.

Although the upper and lower bounds of the covariate variable initial value can take positive and negative infinity, the actual calculation time is generally given reasonably, taking ± 5 in our model. After repeated debugging, the running time and convergence of the algorithm are balanced. The parameters of PSO are eventually set as: $w = 0.5, c_1 = c_2 = 2, v_{max} = 3, n = 200, T_{max} = 1000$ .

The question gives that the sail is made of material of mass 7 $g/m^2$ and the mass of the sail plus payload is 2,000 kg.

So, $\sigma_s = \frac{m_s}{A} = 7g/m^2 \quad \sigma = \sigma_s + \frac{m_p}{A} = 7 + \frac{m_p}{A} \quad \beta = \frac{1.53}{\sigma(g/m^{-2})} = \frac{1.53}{7 + \frac{m_p}{A}}$

According to the differential equations (12), $\beta$ will affect the acceleration in the radial direction and the angular acceleration. Therefore, if we choose different $A$, different optimization results can be got from PSO simulation.

Through the access to information [10], we found that the general area of the solar sail is about $70000m^2$ (The radius of the circle is 150m). Therefore, we chose A as $10000m^2$ $50000m^2$ $100000m^2$ $200000m^2$.

Here are our simulation results:

1. When A=$10000m^2$.

After the simulation, the solar sail's flight time is 1600d, and the initial value of the co-variant is

$$\lambda_0 = [-1.799\ 1.224\ 2.652\ \ -1.953], \alpha^*_0 = \arctan(\frac{-3\lambda_3 - \sqrt{9\lambda_3^{\ 2} + 8\lambda_4^{\ 2}}}{4\lambda_4}) = 66.1^\circ$$

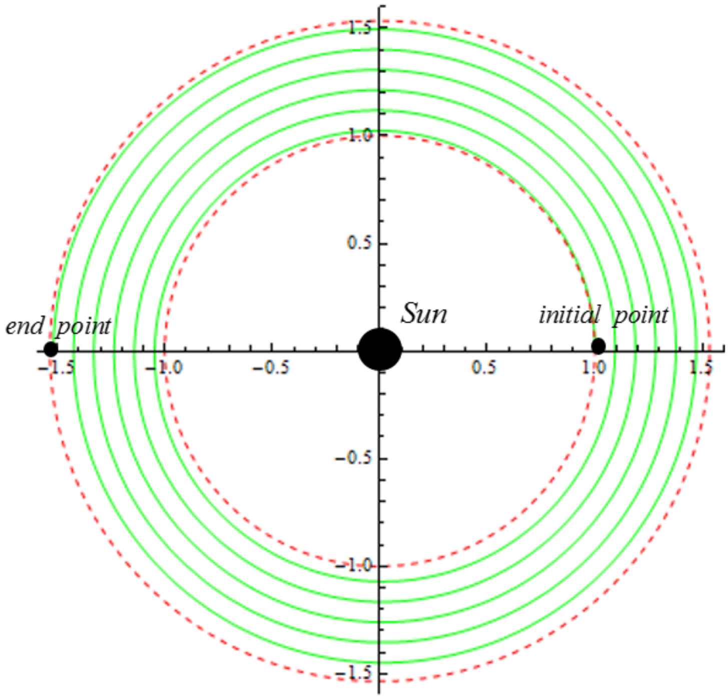


Fig 6. The voyage of the solar sail when A=$10000m^2$ The voyage time is so long, because the solar sail has to run a lot of circles. And after each circle, the radial distance from the solar sail to Mars is changed inappearantly.

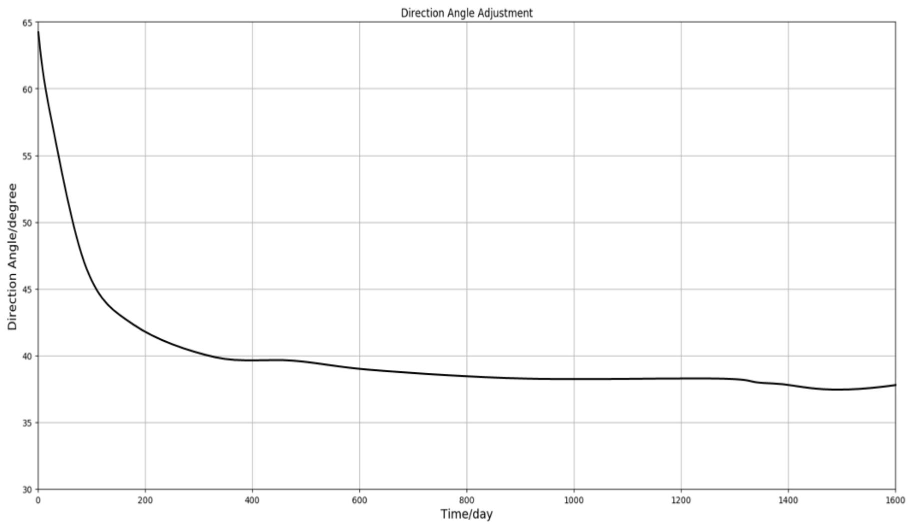

Fig 7. The adjustment of the solar sail's angle following the change of time (A=$10000m^2$) The direction angle is gradually declining. It declined very fast in the beginning and became almost horizontal after fast decline.

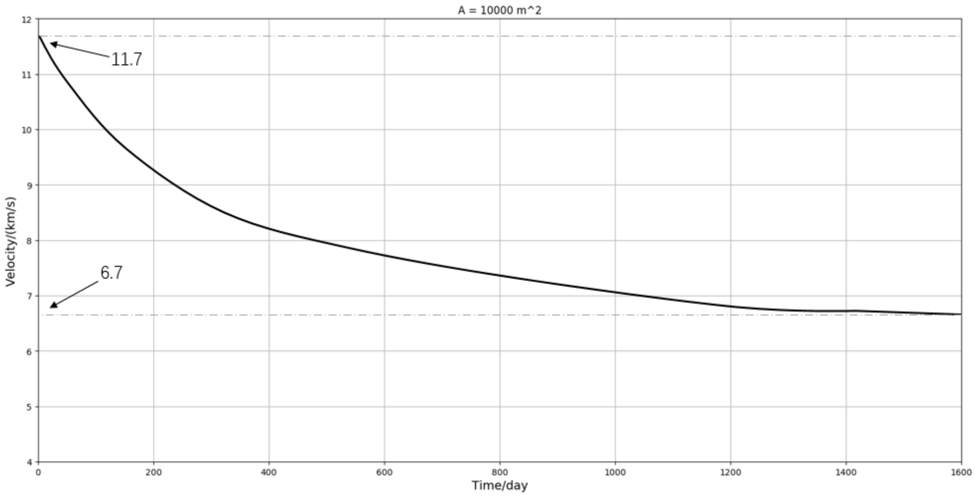


Fig 8. The change of the solar sail's velocity following the change of time (A=$10000m^2$) This voyage is a process of slowing down. And the acceleration is smaller, when the distance from the solar sail to the sun is further. When it reaches the destination, the speed is about 6.7 km/s (lower than 9 km/s).

2. When A=$50000m^2$.

After the simulation, the solar sail's flight time is 782d, and the initial value of the co-variant is

$$\lambda_0 = [-1.572\ 1.304\ 2.325\ \ -1.904], \alpha^*_0 = \arctan\left(\frac{-3\lambda_3 - \sqrt{9\lambda_3^2 + 8\lambda_4^2}}{4\lambda_4}\right) = 64.2^\circ$$

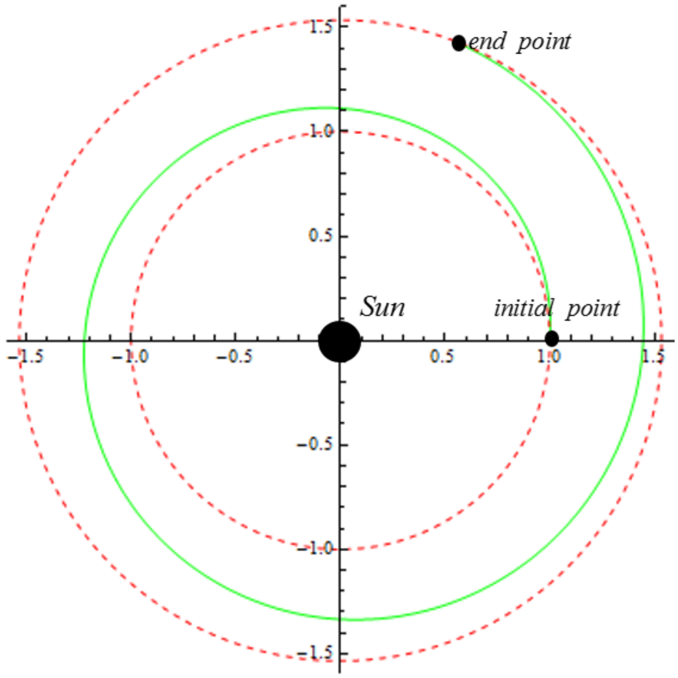


Fig 9. The voyage of the solar sail when A= $50000m^2$ Comparing with Fig 6, the time is less. And after each circle, the change of distance is obvious. It is better than the first scheme.

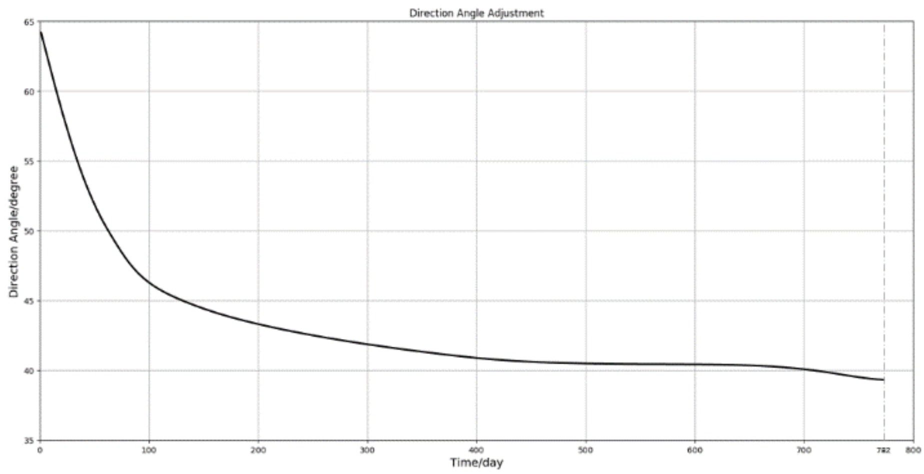


Fig 10. The adjustment of the solar sail's angle following the change of time (A= $50000m^2$ ) The direction angle is gradually declining. It declined very fast in the beginning and became almost horizontal after fast decline.

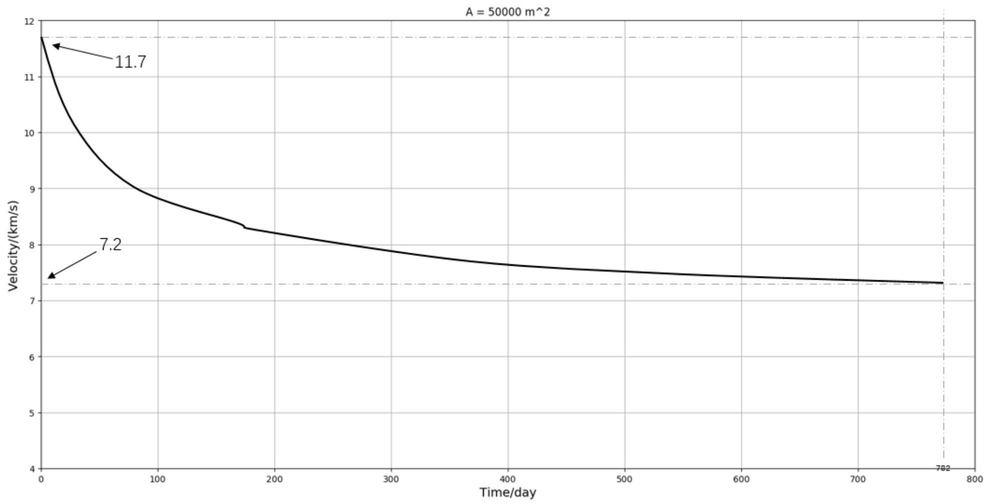


Fig 11. The change of the solar sail's velocity following the change of time (A= $50000m^2$) When it reaches the destination, the speed is about 7.2 km/s (lower than 9 km/s).

3. When A=$100000m^2$.

After the simulation, the solar sail's flight time is 504d, and the initial value of the co-variant is

$$\lambda_0 = [-1.609\ 0.042\ \ -0.160\ \ -1.597], \alpha^*_0 = \arctan(\frac{-3\lambda_3 - \sqrt{9\lambda_3^{\ 2} + 8\lambda_4^{\ 2}}}{4\lambda_4}) = 32.5^\circ$$

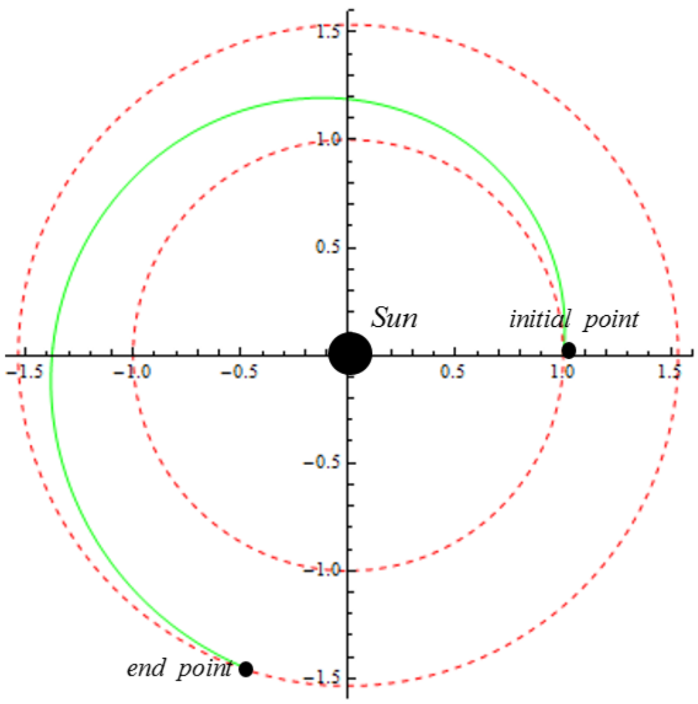


Fig 12. The voyage of the solar sail when A=$100000m^2$ Comparing with Fig 6 and Fig 9, the

time is less. And after each circle, the change of distance is more obvious. It is better than the first scheme, also the second one.

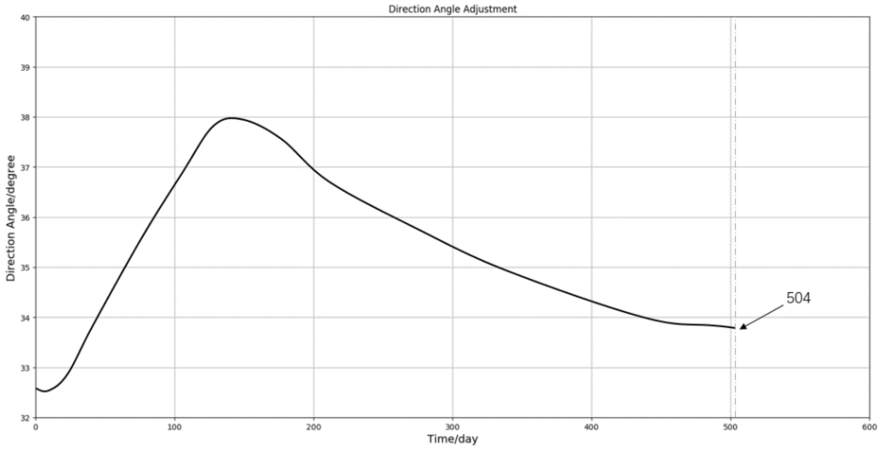


Fig 13. The adjustment of the solar sail's angle following the change of time (A=$100000m^2$)

The direction angle increased first and then decreased, and the rate of rise is faster than the rate of descent

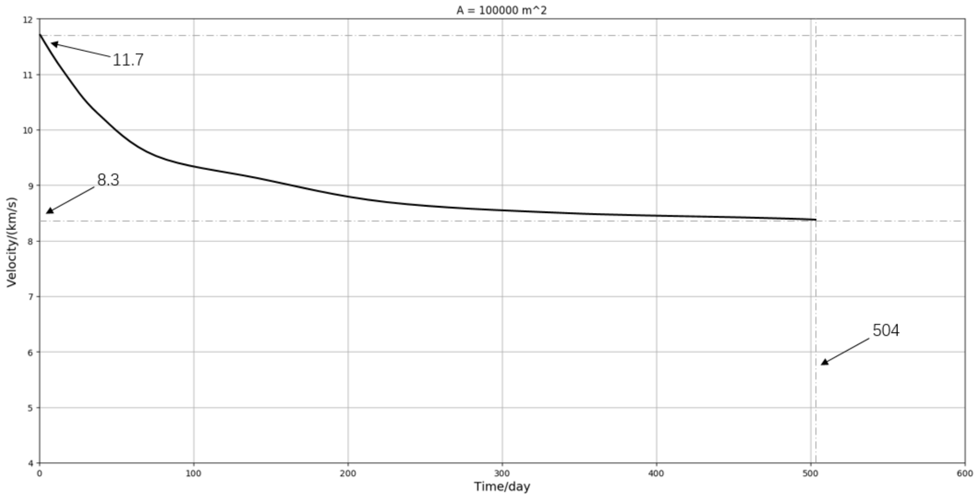


Fig 14. The change of the solar sail's velocity following the change of time (A=$100000m^2$)

When it reaches the destination, the speed is about 8.3 km/s (lower than 9 km/s).

4. When A= $200000m^2$ .

After the simulation, the solar sail's flight time is 480d, and the initial value of the co-variant is

$$\lambda_0 = [-1.564\ 0.012\ \ -0.154\ \ -1.632], \alpha^*_0 = \arctan(\frac{-3\lambda_3 - \sqrt{9\lambda_3^{\ 2} + 8\lambda_4^{\ 2}}}{4\lambda_4}) = 32.3^\circ$$

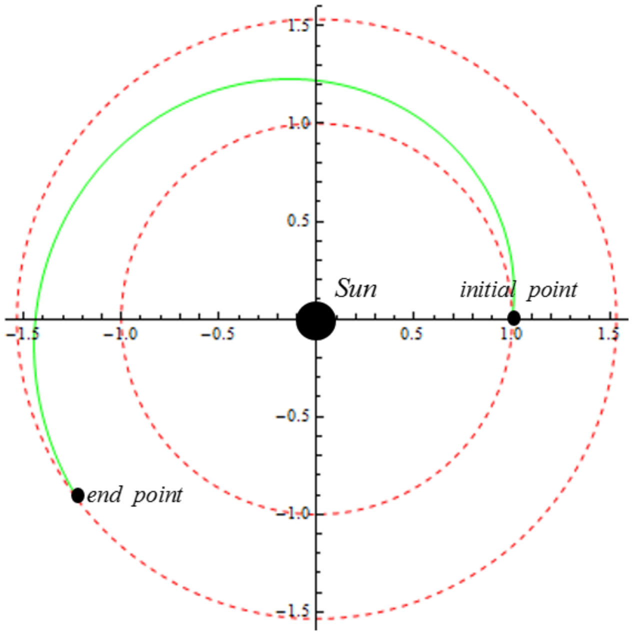


Fig 15. The voyage of the solar sail when A= $200000m^2$ Comparing with Fig 6, Fig 9 and Fig 12, the time is least. And after each circle, the change of distance is much more obvious. Seeing in this way, it is better than all the other schemes.

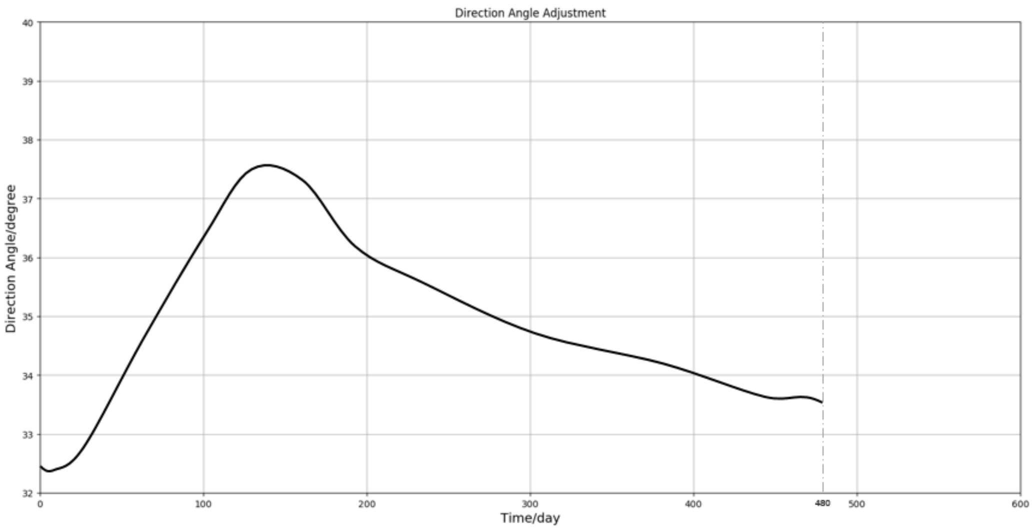


Fig 16. The adjustment of the solar sail's angle following the change of time (A= $200000m^2$ ) The direction angle increased first and then decreased, and the rate of rise is faster than the rate of descent

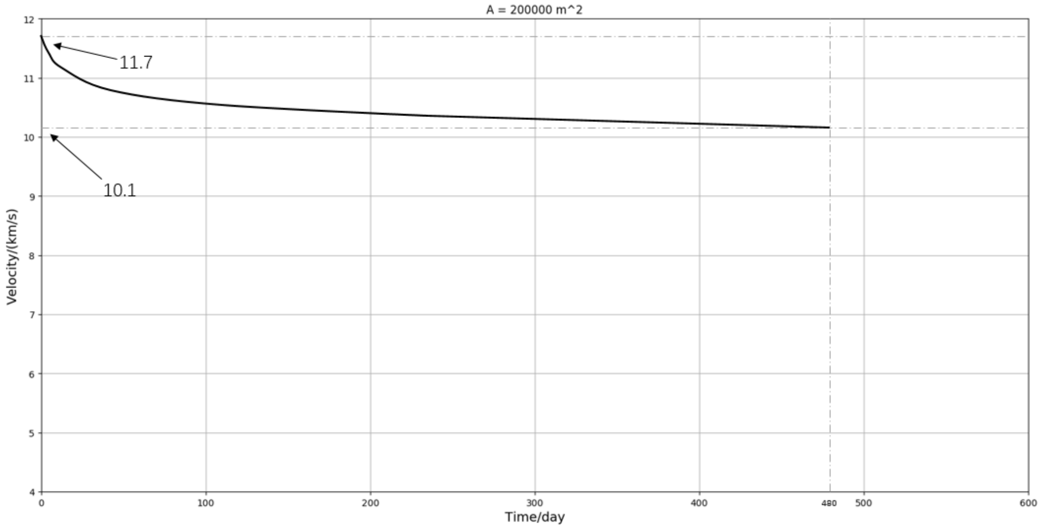


Fig 17. The change of the solar sail's velocity following the change of time (A= $200000m^2$ )

The figure tell us that when A= $200000m^2$ or larger, the solar sail cannot land on the Mars safely. So, though the time spent is least, it is infeasible.

# 5. Strengths and Weaknesses

## 5.1 Strengths

1. In this model, we think the solar sail is ideal and can reflect the light completely. By ignoring these parts difficult to predict and calculate, to a large extent, the model is simplified. Because, sometimes, these parts are unexpected and random. Then we can solve the problem efficiently.
2. By using Hamilton's equation to transform the orbit optimization problem into a multi-restriction problem, the abstract problem goes specific. After this transformation, we can use mathematical method to deal with it.
3. Using Hamilton's equation makes the result more accurate. Because Hamilton's equation uses the integration principle and differential stationary point to make the change of time clear. Then we can find the orbit spending the shortest time.

## 5.2 Weaknesses

1. When the size of the solar sail is increasing, its shape is inevitably following to

change. Then the light pressure acting on the solar sail is also changed a lot, especially when there is acceleration. So it will make the result of this model different from the fact.

2. This is only an ideal model. The reflection is regarded to be absolutely specular reflection. And there is no absorption of the sunlight. But we know that there is no material can realize this point. So error is larger after dealing with the model in this way.

3. In the process of establishing our model, we think the orbits of Earth and Mars in the shape of circle. In fact, there are many differences between our model and the fact. First, the velocity of Earth and Mars is always changing, but it is invariable in the model. Second， the direction of the sunlight is changing asymmetrically, while in our model it is symmetrical.

# 6. Conclusion

We analyzed the process of the voyage from earth to Mars. There are two main forces acting on the solar sail, the sunlight pressure and solar gravity. And based on our assumption, the light pressure is always perpendicular to the surface of the solar sail. That is our model about light pressure. The feasible method to arrive at Mars is using external spiral motion. Then in order to solve the problem about the shortest time, we use PSO to optimize the orbit. The result we should get is the size of the solar sail which could optimize the time of the voyage and the orbits. By using Hamilton's equation and the particle swarm optimization, optimize and solve the problem.

In our result, the spacecraft should have the ability to maintain the original speed. In another word, the speed should slow down slowly. Also, the final velocity should be lower than 9km/s, and the closer will be better. Finally, after comparing different process with different parameters, we get an optimized result, when $A = 100000m^2, m_s = 700kg$, payload is relatively large 1300kg, transfer time is 504d. At the beginning of the voyage, the angle of the solar sail is 32.5° . Then adjust the angle following the time as **Fig 13** shows, and the solar sail can reach Mars in 504 days. It spends 116 days less than the plan in reference [8]. And when it reaches Mars, the speed will slow down at 8.3km/s.

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# Appendix I

Code written in python

```
# coding: utf-8
import numpy as np
import random
import matplotlib.pyplot as plt
from math import pi,sqrt,cos,sin

#-------------------------parameters---------------------------
class PSO():
    def __init__(self,pN,dim,max_iter):
        self.w = 0.5                     #Inertia weight factor， [0.1]
        self.c1 = 2                      #positive acceleration constant
        self.c2 = 2                      #positive acceleration constant
        self.r1= 0.6                     #random constant
        self.r2= 0.3                     #random constant
        self.pN = pN                     #number of particle
        self.dim = dim                   #searching dimensionality
        self.max_iter = max_iter         #number of iterations
        self.X = np.zeros((self.pN,self.dim))          #location
        self.V = np.zeros((self.pN,self.dim))          #velocity
        self.pbest = np.zeros((self.pN,self.dim))      #personal optimal
        self.gbest = np.zeros((1,self.dim))            #total optimal
        self.p_fit = np.zeros(self.pN)                 #personal historical optiaml
        self.fit = 1e10                  #total best finess
        self.r0 = 1.0
        self.u0 = 0.0
        self.v0 = 1/sqrt(self.r0)

        self.rs = 1.524
        self.us = 0.0
        self.vs = 1/sqrt(self.rs)

        self.rf = self.r0
        self.uf = self.u0
        self.vf = self.v0

    def getparameters(self):
        return self.gbest


    def getrf(self):
        self.rf = self.rf + self.uf
```

```
        return self.rf

    def getuf(self,x):
        uu = 6.67*1.99e10
        k = 2
##          q = x[random.randint(1,self.dim-1)]
        q = x[0]
        self.uf = self.vf*self.vf/self.rf-uu/self.rf/self.rf+k*cos(q)*\
                    cos(q)*cos(q)/self.rf/self.rf
        return self.uf

    def getvf(self,x):
        k = 2
##          q = x[random.randint(1,self.dim-1)]
        q = x[0]
        self.vf = -self.uf*self.vf/self.rf+k*sin(q)*cos(q)*cos(q)/self.rf/self.rf
        return self.vf

#----------------------------fitness-----------------------------
    def getfit(self,x):
        q1 = 12.5
        q2 = 15
        q3 = 40
        cfit = q1*abs(self.rs-self.getrf()) + q2*abs(self.vs-self.getvf(x)) + \
               q3*abs(self.us-self.getuf(x)) + x[self.dim-1]

        return cfit
#---------------------initialzation---------------------------------
    def init_Population(self):
        for i in range(self.pN):
            for j in range(self.dim):
                self.X[i][j] = random.uniform(0,1)
                self.V[i][j] = random.uniform(0,1)
            self.pbest[i] = self.X[i]
            tmp = self.getfit(self.X[i])
            self.p_fit[i] = tmp
            if(tmp < self.fit):
                self.fit = tmp
                self.gbest = self.X[i]

#---------------------refresh---------------------------------
    def iterator(self):
        fitness = []
        for t in range(self.max_iter):
```

```
            for i in range(self.pN):                    #refresh gbest\pbest
                temp = self.getfit(self.X[i])
                if(temp<self.p_fit[i]):                 #refresh personal optimal
                    self.p_fit[i] = temp
                    self.pbest[i] = self.X[i]
                    if(self.p_fit[i] < self.fit):       #refresh total optimal
                        self.gbest = self.X[i]
                        self.fit = self.p_fit[i]
            for i in range(self.pN):
                self.V[i] = self.w*self.V[i] +\
                            self.c1*random.uniform(0,1)*(self.pbest[i] - self.X[i]) +\
                            self.c2*random.uniform(0,1)*(self.gbest - self.X[i])
                self.X[i] = self.X[i] + self.V[i]
            fitness.append(self.fit)
            print(self.fit)                             #display
        return fitness

#----------------------simulation-----------------------
run_sim=1000
run_dim=6
run_n=200
my_pso = PSO(pN=run_n,dim=run_dim,max_iter=run_sim)
my_pso.init_Population()
fitness = my_pso.iterator()
#-------------------draw--------------------
plt.figure(1)
t = np.array([t for t in range(0,run_sim)])
fitness = np.array(fitness)
plt.plot(t,fitness, color='b',linewidth=3)
plt.title("Figure1")
plt.xlabel("iterators", size=14)
plt.ylabel("fitness", size=14)
plt.show()
```