Using Magnetic Fields to Direct Exhaust from Ion Thrusters

Question B

Team 609

November 13, 2017

Abstract

This paper considers the feasibility of using a magnetic field to confine the trajectories of dispelled ions from an electric ion thruster. We considered the physical demands of designing a magnetic plume collimator at two locations relative to the ion thruster: directly at the end of the grate accelerator, and at reasonable distances beyond the thruster. The demands were developed under the pretext that the first priority in these deliberations was to increase thrust to propellant efficiency. A second priority, which is not treated extensively in this paper due to its dependence on specific spacecraft designs, is to mitigate sputter effects associated with ware on the spacecraft.

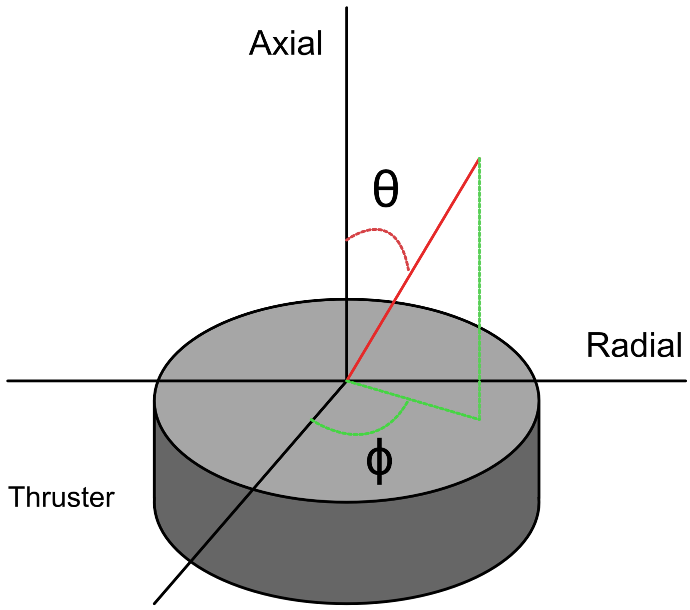
In lieu of laboratory data, this paper seeks to develop a rigorous physical understanding of the difficulties associated with creating such an apparatus, and discuss the issues one might face given specific design possibilities. We also develop a simple model of an ideal magnetic lens in order to estimate the strength, position, and size of such a lens.

The paper mathematically develops a set of physical constraints a magnetic collimator must abide by to be deemed a reasonable design option. The physical constraints we developed indicated magnetic fields were an impractical mechanism for reducing beam divergence in electric ion thrusters.

1 Nomenclature

Symbol	Description
I_{sp}	Specific Impulse of the ion thruster
T	Thrust of the Xenon ion thruster
M	Mass of the Xenon particles
P	Power of the Xenon ion thruster
η_m	propellant utilization efficiency.
α	Thrust correction factor for presence of doubly charged species in the thrust
I_b	Ion beam current
V_b	Voltage between the two grids
I^+	Ion beam current due to singly charged Xenon ion
I^{++}	Ion beam current due to doubly charged Xenon ion
$\dot{m}_i$	Mass flow rate
$qore$	Charge of the Xenon ions
η_e	Electrical efficiency of the thruster
η_T	Total efficiency of the thruster
T/P	Thrust to total input power ratio
K	Boltzmann's Constant
$\rho(r)$	Ion density function

Table 1: All important variables



The thruster axis, throughout this paper, is defined as in the axial direction in the figure above.

2 Introduction

Ion thrusters offer great advantages over chemical propulsion systems in many space travel scenarios. In recent decades, physicists have spent considerable time designing ion thrusters capable of acting as the primary propulsion mechanism on spacecraft. The high exhaust velocities achieved by these devices greatly enhance their fuel efficiency as compared to chemical alternatives, creating massive weight savings, and potentially opening the door to longer distance missions. Despite this promise, some pressing design concerns still remain.

Some of the challenges ion thrusters face involve the large divergence of their exhaust plume. This is primarily an issue for two reasons: firstly, the radial component of an ion's momentum does not contribute to the forward momentum of the payload, constituting a waste of energy, and secondly, the divergent plume, which is comprised of high energy ions, can bombard nearby components of the spacecraft and either corrode or bond with those surfaces, compromising the integrity of those components with time. In this paper we investigate whether an additional magnetic field could efficiently solve either of these problems associated with beam divergence.

We first consider whether a magnetic field can effectively improve the thrust to power ratio of a ion propulsion system, for if a magnetic field can be designed to collimate the beam and decrease the divergence half-angle, then, incidentally, the ion bombardment issue should also be alleviated. Our theoretical work follows a conservative line of thought; given the ideal magnetic field, what is the best job we could possibly do at collimating the beam, and is it good enough.

If it is determined that using magnetic fields to improve the thrust to power ratio is inefficient, then we shall proceed in determining whether an apparatus can be designed for the sole purpose of avoiding ion bombardment.

The structure of the paper proceeds as follows. First we provide some background information on ion thrusters and calculate some basic quantities and properties of the thruster in question. Then, we prescribe certain operational requirements for the distinct cases of a magnetic collimator placed at the thruster, and one placed beyond the thruster. Then it seeks to apply feasible apparatuses to these to scenarios.

3 Assumptions

In this section we will discuss the assumptions we made while modeling the motion of the particles in the Xenon ion thruster.

- We assume that the acceleration grids are flat. That is, the grid optics is such that the grids are neither concave nor convex in shape. This has an important implication in that the Xenon ions leaving the grids immediately after have velocities that are parallel to the thrust axis.[KR] Of course, this does not mean that the the beam will remain uniform in the direction of the thrust axis. The pressure gradient force, and the collisions between the particles cause the plume to diverge.

- We assume that the beam, upon leaving the final grid of the thruster, diverges uniformly. That is to say, the average half angle of the divergent beam remains constant. Specifically, for our case, we assume that the average half angle is $20°$. [Wan]

- We assume that the propellant utilization of Xenon ion thruster is 90% [HKMA]. This is a fairly good approximation in that most of the industrial standard Xenon thrusters operate on the same propellant utilization efficiency.

• Another assumption we make is that the fraction of doubly charged Xenon ion current in the beam is 0.1. That is, $\frac{I^{++}}{I^{+}} = 0.1$ and that presence of triply charged Xenon ions is negligible. This small fraction of doubly charged ion current will change T and I_{sp} by at most only 5%.[EB03]

• We also assume that the beam in our Xenon ion thruster is nearly mono-energetic. That is, the speed of every particle is nearly the same. This is not a bad assumption, since the fraction of doubly charged Xenon ion current is significantly less than that of singly charged ion current.

4 Spacecraft Thrust Fundamentals

4.1 Thrust and specific momentum

The thrust of the spacecraft, due to the ion beam current, is given by [GK08].(see appendix 8.1 for derivations):

$$T = \sqrt{\frac{2MV_b}{q}} I_b \qquad [N] \tag{1}$$

However, this equation is for a uniform, singly ionized beam leaving the thruster. We know that the beam does not just contain singly ionized Xe ions, and it also diverges. Only the velocity component that is parallel to the thruster axis contributes to the net momentum gained by the spacecraft. Therefore, we add a correction factor of $cos(\theta)$, to give us the velocity component parallel to the thrust axis, where θ is the average angle of velocity of all the particles.

There also exist differently charged Xenon ions in the ion thruster. Assuming that the Xenon ion thruster contains only two differently charged Xenon ions, that is, Xe^{+} and Xe^{++}, then we must introduce a thrust correction factor, α (See appendix 8.2 for details). After accounting for the divergent beam and the presence of differently charged particles, the thrust equation assumes the general form:

$$T = \alpha \; cos\theta \; \sqrt{\frac{2MV_b}{q}} \; I_b. \tag{2}$$

We now attempt to calculate the voltage difference between the two grids, as well as find out the beam current coming out of the thruster. The specific momentum of an electric thruster is defined by (see appendix 8.3 for details):

$$I_{sp} = \frac{\alpha \times cos\theta \times \eta_m}{g} \sqrt{\frac{2eV_b}{M}} \tag{3}$$

For a specific impulse of 5100s, Xenon ions with an atomic mass of 131.29, ion charge of 1.6×10^-19C, assuming the fraction of doubly charged Xenon ion current to be 0.1, the average divergent beam angle to be 20 °, and the propellant utilization efficiency to be 0.9, we have: [GK08]

$$I_{sp} = 123.6 \times cos(20) \times (0.9) \times \sqrt{V_b} \implies V_b \approx 2400V. \tag{4}$$

Using equation 2, we can also calculate the ion beam current, given the thrust of 350 mN:

$$T = 1.65 \times I_b \times \sqrt{2400} \implies I_b \approx 4.3A. \tag{5}$$

This ion beam current is the total amount of current being lost through the ejected ions. It is useful because we can find the total power used in accelerating the ions by multiplying it by the voltage of the accelerating mesh. We will discuss this further now.

4.2 Xenon Ion Thruster Efficiency

In this section we consider the efficiency of the Xenon ion thruster. The electrical efficiency of the thruster is given by [GK08] :

$$\eta_m = \frac{I_b V_b}{I_b V_b + P_o} \tag{6}$$

where η_e is the electrical efficiency and P_o is the other power input to the thruster that is needed for the formation of the thrust beam.

The total efficiency of the ion thruster, η_T, is given by [GK08] :

$$\eta_T = \alpha^2 \, cos^2\theta \, \eta_e \, \eta_m \tag{7}$$

And the thrust per unit input power is given by [GK08]:

$$\frac{T}{P} = \frac{2 \, \eta_T}{g \, Isp} \tag{8}$$

5 Design Criterion

In designing a magnetic beam collimator, there are a few basic criterion that we set out to meet. Firstly, because thrust follows a cos(θ) half-angle relation, we rule out apparatuses that only reduce the half angle slightly. 35cm diameter thrusters generally have a plume half angle between 15 and 20 degrees [Wan], meaning that thrust can only be increase by a maximum of approximately 6 percent if the plume is perfectly collimated (no radial velocity). As a crude, but reasonable approximation, we say that the cost, and power losses associated with a magnetic apparatus that only collimated the beam slightly would far outweigh any gains in thrust. Hence we only look for solutions that significantly increase thrust (meaning they collimate the beam significantly).

A fundamental question when designing such an apparatus is, "where will it be placed?" In such deliberations, one must ask: "How costly would it be to build an apparatus here?", "Does the apparatus require that components on the existing thruster be redesigned, and to what degree?", and "Where can the beam be most efficiently collimated?" A field placed right at the end of the thruster, in proximity to the plasma confinement chamber, might be the most efficient placement for increasing thrust, but could compromise the functioning of the magnetic fields used to confine the plasma, thereby requiring some significant redesign [GRF11]. These questions must be treated with care, and will be considered later when investigating feasible designs.

5.1 Optimizing Magnetic Collimator at Thruster

There are very few conceivable designs that would allow a magnetic collimator to be placed at the thruster without warranting serious redesign on the plasma confinement mechanism. Nonetheless, there are some ideas to be had here, so an analysis of the forces at the thruster mouth is still useful.

When considering a collimating beam placed at the thruster, we must consider the dispersion forces we must counter in order to keep the beam from diverging. There are two fluid-like interactions that cause the ion beam to disperse when it leaves the thruster. Firstly, a pressure gradient causes the ions to disperse radially and secondly, random particle collisions between the two species of ions scatter the beam. If we make the reasonable approximation that the ions have a uniform axial velocity when leaving the thruster, then they will be subject to a radial dispersion force due to the following radial dispersion forces:[Wie]

$$\begin{aligned} F_p &= -KT\frac{\nabla\rho(r)}{N} \\ F_c &= N_a\frac{m_a m_b}{m_a+m_b}v_{ab}(v_b-v_a) \end{aligned} \tag{9}$$

Where in the first expression, K is the Boltzmann constant, T is temperature, $\rho(r)$ is the ion density function, and N is the ion population, and in the second expression, F_c is the force on type a ions caused by type b ions where v_{ab} is the frequency of collisions between the two species. Thus, if we can find the gradient of the density function $\rho(r)$, we can calculate the necessary field B to perfectly collimate the ion plume. If we take a cross-sectional slice of the ion beam just as it leaves the thruster, we find it has an ion density given by the Gaussian distribution

$$\rho(r) = \frac{\rho_o}{h(t)}\exp(\frac{-r^2}{2R^2h(t)^2}) \tag{10}$$

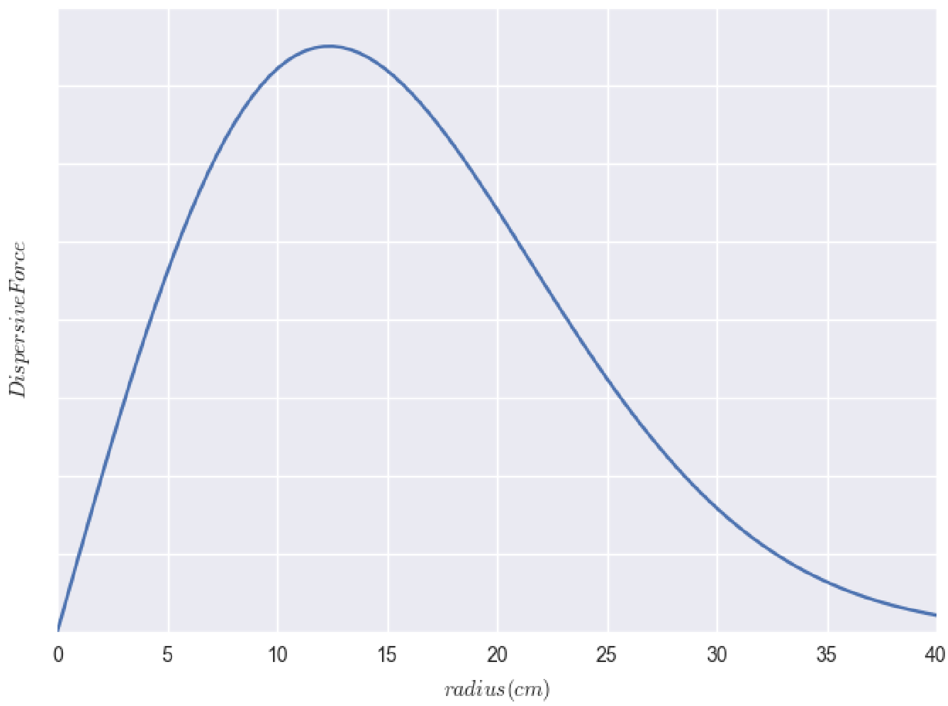
Where ρ_o is the ion density at the origin, R is the radius of the thruster, and $h(t)$ describes the plume's radial motion such that $r(r_o,t) = r_o h(t)$. The flattening effect of $h(t)$ is shown in the figure below. If we wish to apply a magnetic field such that the radial forces on each ion are net zero (i.e. $F_p + F_c - F_B = 0$) then it is to say that $h(t)$ becomes frozen at $h(0)$ which we define to be unity. Then we can take the gradient of the density function according equation (9) and set it equal to the Lorentz force as follows

$$-KT\frac{\nabla\rho(r)}{N} = \frac{KT\rho_o}{NR^2}r\exp(\frac{-r^2}{2R^2h(t)^2}) \tag{11}$$

Then equating our forces we get

$$\begin{aligned} \frac{KT\rho_o}{NR^2}r\exp(\frac{-r^2}{2R^2h(t)^2}) + N_a\frac{m_a m_b}{m_a+m_b}v_{ab}(v_b-v_a) &= q(v_a\times B) \\ B = \frac{1}{qv_a}[\frac{KT\rho_o}{NR^2}r\exp(\frac{-r^2}{2R^2h(t)^2}) + N_a\frac{m_a m_b}{m_a+m_b}v_{ab}(v_b-v_a)] \end{aligned} \tag{12}$$

Before we get too bogged down by the complexity of this expression, lets consider the graphical renderings below. The second term on the right is independent of radius, so we get a force profile like the one below, plus some constant.



This is the generic force profile of a B field that would perfectly counter the dispersive forces in the plume due to the pressure gradient and ion collisions. Note this approximation does not take into account the dispersive forces caused by the fringe field of the grates (for which we would see another spike in dispersion force around the radius ($\approx 17cm$). What this curve does tell us tough, is that ions very close to the center experience lower dispersion forces than those closer to the rim. It also tells us that the greatest dispersive forces (apart from fringe field forces) actually occur slightly inward from the outer radius of the thruster. This force profile can thus inform the design of a magnetic field both in terms of magnitude and position. An efficient magnetic collimating field ought to roughly match this dispersion force profile. Otherwise some ions would be deflected too much while others wouldn't be deflected enough. These are the constraints we place on a feasible apparatus located at the thruster.

5.2 Optimizing Magnetic Collimator Beyond Thruster

The overall goal of a magnetic lens some distance beyond the thruster would be to redirect stray ions so that they are launched directly behind the thruster, and not out radially at all. This is because the actual impulse that helps to accelerate the thruster only depends on the axial component of the ions' velocity. More exactly, if v_{ex} is the exhaust velocity, m_{xe} is the mass of a xenon atom, and θ is the angle of an ion's velocity in respect to the fore-aft axis, then the impulse to the thruster supplied by that ion is:

$$I_{i,axial} = m_{xe} v_{ex} \cos\theta \tag{13}$$

Our goal therefore is to increase I_{axial} (maximize $\Delta I_{axial} = m_{xe} v_{ex}(\cos\theta_2 - \cos\theta_1)$) by creating some magnetic field which will bend ions towards the fore-aft axis. Notice that the velocity of an ion is not changed by the magnetic field, since magnetic fields perform no work on particles. Therefore, taking all of the ions into consideration, we want to maximize:

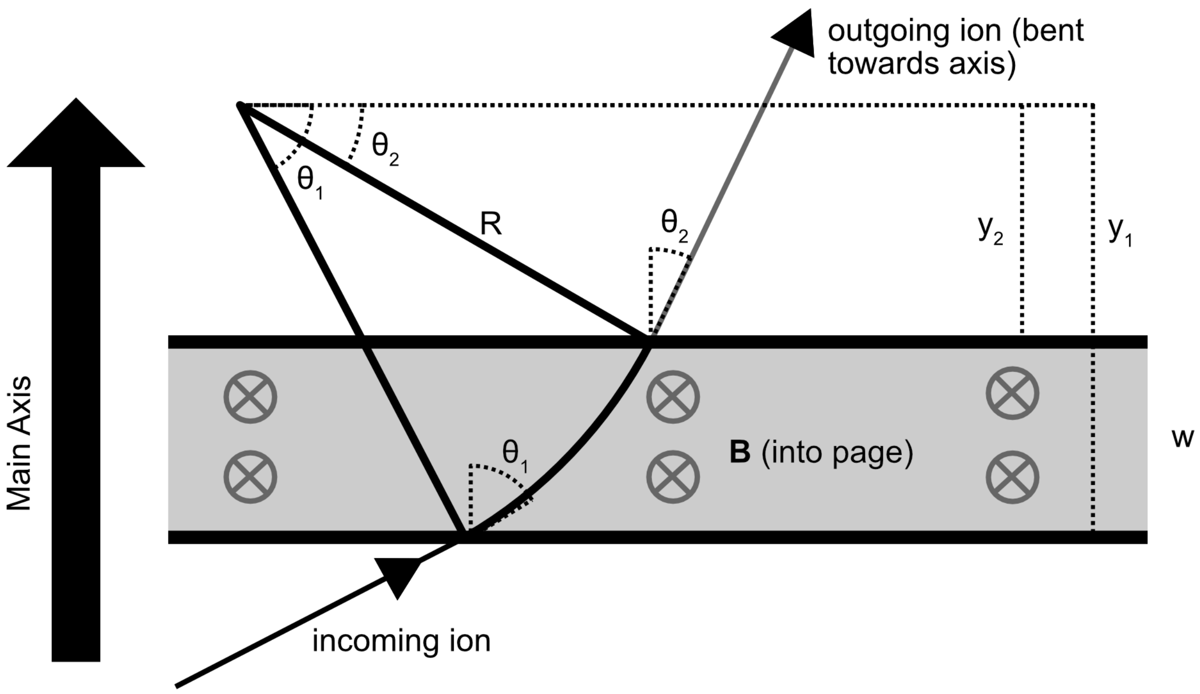


Figure 1: The trajectory of a xenon ion through a slab of azimuthal magnetic field. Note that this is the optimal direction of the magnetic field, which bends the ion towards the main axis as much as possible.

$$\Delta I_{axial} = m_{xe} v_{ex} \sum_{all\ ions} \cos\theta_{i2} - \cos\theta_{i1} \tag{14}$$

where θ_{i1} is the initial angle of the ith ion's velocity and θ_{i2} is the final angle of the ith ion's velocity after going through our magnetic field (both angles measured from the fore-aft axis).

What we will now show is that it is futile to try to design such a magnetic field: Even if we would be able to sculpt the "most efficient" field possible (one which causes the greatest ΔI_{axial} from a given volume and magnitude of $\vec{B}$) we will find that this will cause an insignificant increase in axial impulse.

The "most efficient" $\vec{B}$ is the one which increases the axial component of an ion's velocity the fastest, which means that $\vec{F} = q(\vec{v} \times \vec{B})$ from the biot-savart law should point as much axially as possible. $\vec{v}$ has no azimuthal component, since the ions never experience an azimuthal force since the thruster is radially symmetric. This means that $\vec{v}$ only has radial and axial components, and thus to convert that radial velocity to axial velocity the most efficiently, $\vec{B}$ should point in the azimuthal direction (Figure 1). Note that we are not saying that we should actually try to make an azimuthal $\vec{B}$: at any point in space there will be ions going in different directions at different times, and *any* $\vec{B}$ we define will help some ions, but might even harm others. Merely, we are saying that if we could define a different $\vec{B}$ for each ion, and that ion was off-axis, this is what it would be.

We will show that it is impossible to meaningfully impact v_{axial} with a reasonably strong and reasonably sized $\vec{B}$. Imagine that we could create the $\vec{B}$ in Figure 1, with a finite thickness w in the axial direction, and infinite extent in the radial direction. Say an ion approaches this $\vec{B}$ at an off-axis angle of θ_1. What is the ions Δv_{axial} from interacting with the $\vec{B}$?

First let's think about this qualitatively. The stronger the $\vec{B}$ and the thicker the slab of it (larger w), the more the ion will be curved. Also significant, but out of our control, is e and m_{xe}, the charge and mass of an ion. If e were larger and m_{xe} smaller, the more the the particle would be curved as well. What about the velocity of the incoming particle? You might think that this would not be a factor in the curvature, since the strength of the Lorentz force ($\vec{F} = q(\vec{v} \times \vec{B})$) is proportional to v, but the amount of time that this force is impacting the particle ($t \approx \frac{w}{v}$) proportional to inverse v. However, this is not the case: To maintain a particular curvature of radius R, the particle needs to feel a force of magnitude $\frac{m_{xe}v^2}{R}$ in order to receive enough centripetal acceleration. Thus, as v increases, the radius of curvature also increases.

Now, for a given curvature, determined by $\vec{B}, w, e, m_{ex}$, and v, how does θ_1 actually affect the Δv_{axial}? As you can imagine from Figure 1, as θ_1 gets bigger, Δv_{axial} also increases, since the particle will spend a longer time in the $\vec{B}$, and during that time it will feel a force more aligned with the axis.

OK, now let's do some math to actually quantify Δv_{axial} given a certain $\vec{B}, w, e, m_{ex}, v$, and θ_1. The following math is only valid when $0° < \theta_1, \theta_2 < 90°$. As you can see from Figure 1,

$$\Delta v_{axial} = v\cos\theta_2 - v\cos\theta_1 = v(\cos\theta_2 - \cos\theta_1) \tag{15}$$

Now let's find θ_2 in terms of the other terms we know. First , using some similar angles (drawn in the Figure), we can get it in terms of the radius of curvature and θ_1.

$$w = y_1 - y_2 = R\cos\theta_1 - R\cos\theta_2 = R(\sin\theta_1 - \sin\theta_2) \tag{16}$$

From here we can find R using the Lorentz force and centripetal acceleration. We drop the vectors and just use magnitudes halfway through because $\vec{v}$ is perpendicular to $\vec{B}$

$$\vec{F} = m\vec{a} \tag{17}$$
$$e(\vec{v} \times \vec{B}) = m\frac{v^2}{R}\hat{in} \tag{18}$$
$$evB = \frac{mv^2}{R} \tag{19}$$
$$qeB = \frac{mv}{R} \tag{20}$$
$$\Rightarrow R = \frac{mv}{eB} \tag{21}$$

Combining this with (16), we can find an expression for θ_2:

$$w = R(\sin\theta_1 - \sin\theta_2) \tag{22}$$
$$\Rightarrow \theta_2 = \sin^{-1}(\sin\theta_1 - \frac{w}{R}) \tag{23}$$
$$\theta_2 = \sin^{-1}(\sin\theta_1 - \frac{weB}{mv}) \tag{24}$$

plugging this back into (15), we can find the fractional increase in impulse $\frac{\Delta v_{axial}}{v}$. The last step uses the trig identity $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$:

$$\Delta v_{axial} = v(\cos\theta_2 - \cos\theta_1) \tag{25}$$
$$\frac{\Delta v_{axial}}{v} = \cos(\sin^{-1}(\sin\theta_1 - \frac{weB}{mv})) - \cos\theta_1 \tag{26}$$
$$= \sqrt{1 - (\sin\theta_1 - \frac{weB}{mv})^2} - \cos\theta_1 \tag{27}$$

Let's analyze this expression a bit. The $\frac{weB}{mv}$ term is related to the rate at which the particle curves. As it gets bigger, the particle curls faster, and $\frac{\Delta v_{axial}}{v}$ increases. When the radius of curvature is the same as the width of the slab ($R = w$), then $\frac{weB}{mv} = 1$. In this situation, if we shot the particle into the bottom of the slab at an angle of $\theta_1 = 90°$ (parallel to the surface of the slab), then the particle would curl around a quarter of a circle and exit perfectly vertically, and all of it's velocity

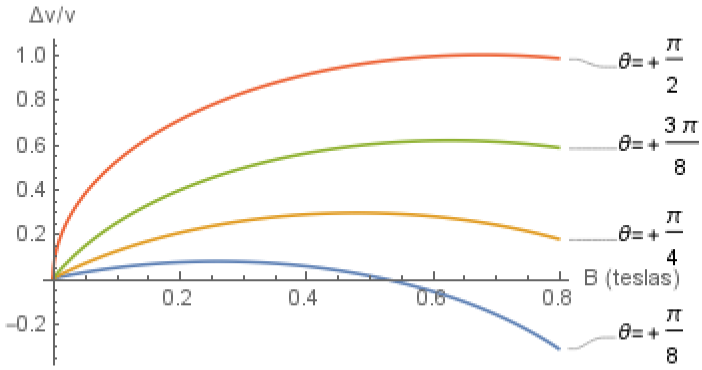


Figure 2: The increase in impulse from a particle traveling through a slab of uniform slab of azimuthal magnetic field at several different incident angles. The width of the slab is assumed to be 10cm and the velocity of the xenon ions 50 km/s

would be converted from horizontal to vertical, so $\frac{\Delta v_{axial}}{v} = 1$. We don't want to make $\frac{weB}{mv}$ any larger than 1, or we will be turning the trajectory back to the side. The extreme other end of this is when $\frac{weB}{mv} = 0$, which happens when there is no magnetic field or the width of the slab is 0. Then, the particle isn't curled at all, and the entire expression simplifies to 0 (using $1 - \sin^2 x = \cos^2 x$).

What values of $\frac{weB}{mv}$ could we reasonably expect to create? Let's say that our slab is 10 cm thick, which is already a fairly thick $\vec{B}$ to create. Then

$$\frac{weB}{mv} = B\ (6.979 \times 10^8\ teslas^{-1}) \tag{28}$$

With the radius of curvature caused by this $\frac{weB}{mv}$, in Figure 5.2 we can see the impulse boost caused by a $\vec{B}$ of varying strength at a few different incident angles. As expected, $\frac{\Delta v_{axial}}{v}$ is larger for a given B at larger angles. Also, the maximum amount of impulse boost is at these large angles. What this means in a practical sense is that if we are actually trying to implement a magnetic lens, and want to get the best bang for our buck (largest $\frac{\Delta v_{axial}}{v}$ for a given strength of $\vec{B}$), we should place it in the locations within the exhaust plume that contain ions traveling as much off-axis as possible.

Where in the plasma plume does this occur? Near the outlet of the thruster it is possible to model the particles as traveling in a straight line from a point source a little ways behind the thruster. If you take a cross section radially at any axial distance from the thruster, the axial component of the current density is a normal distribution. [HSH+12]. This is plotted in Figure 5.2 . Since we are assuming that every particle has the same charge and that they are all traveling at the same speed, the current density is proportional to the the actual density of particles. Thus, when choosing the best location for our magnetic lens, we should try to place it in a location that has the best combination of high density and high angle.

6 Engineering electromagnets.

6.1 Proposal: Implementing Toroid coil at thruster to focus divergent ion beam

Upon initial inspection, a Toroid apparatus seems capable of roughly fitting the force profile described in section 5.1. Consider the arrangement below:

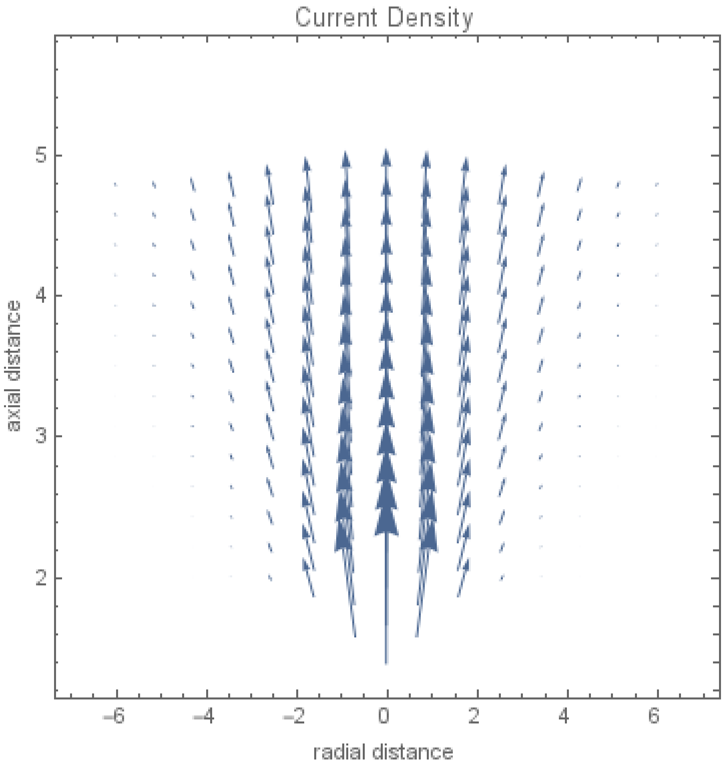


Figure 3: The current density at different positions within the plasma plume. Generated from the assumption that the current density in the axial direction forms a gaussian distribution over a radial cross-section, and that particles travel in a straight line from a point source.

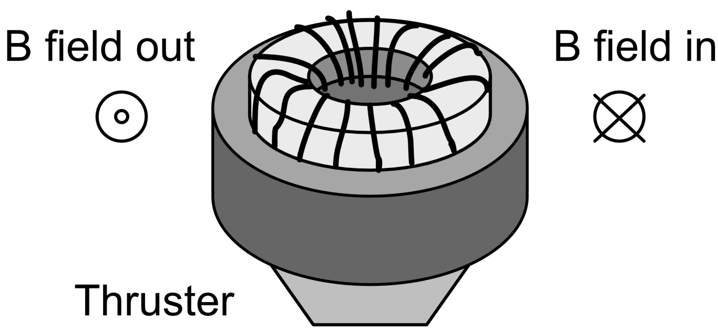


Figure 4: Ion thruster with hollow toroid at surface.

The immediate advantage of using a toroid is that the magnetic field is almost entirely contained within the coils so interference with the plasma confining fields in the ionization chamber is unlikely. Notice that the Toroid only needs to cover sections of the beam where we found the dispersion forces were high, leaving the densely populated center unobstructed by any new material. Additionally, we know the field in a Toroid is given by:

$$\frac{\mu_o N I}{2\pi r}\hat{\phi} \tag{29}$$

This would seem to be a useful relation because, if we adjust the number of coils and current properly, we can roughly map the $1/r$ magnetic field to the $r\exp(-r^2/a)$ force profile we prescribed in section 5.1.

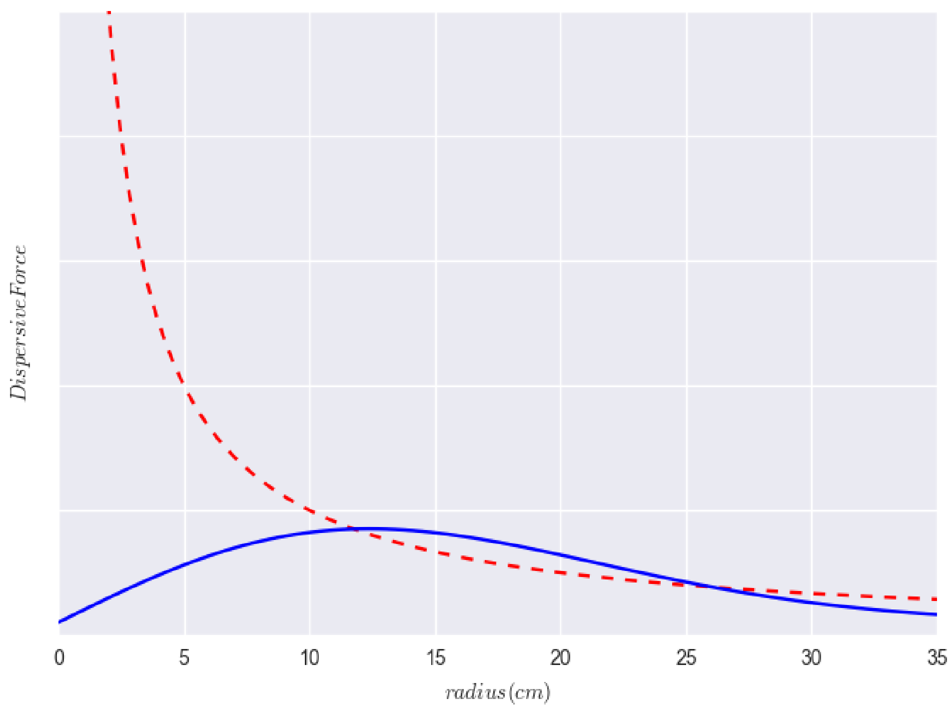


Figure 5: toroid field strength compared to force profile

Keep in mind that we only care to put the toroid over a particular interval (perhaps from 10cm to 15cm), so we only care about part of the fit over a particular interval. In theory, two or three toroids of different strength could be put at incremental radii to better fit the force profile. The toroid seems among one of the best options for an 'at the thruster' magnetic collimator, but its shortcomings likely prevent it from being of much practical use. The most obvious shortcoming of the toroid is that it is a physical obstacle for the ions to collide with as they leave the acceleration grate. Collision rates with the toroid can be mitigated by reducing the number of coils so as to decrease exposed surface area, but only with great losses to the electromagnet's efficiency. Whatever field strength is lost in reducing the coil numbers must be made up for by increasing the current which in turn will result in heat losses.

Beside from the difficult balancing between toroid efficiency and ion-toroid collisions lies the issue of toroid corrosion. With such high exposure to the high energy ions coming from the thruster, the lifetime of such a device likely disqualifies it from being implemented in a thruster design. It would be a difficult task to shield the toroid from these ions in a way that didn't further obstruct the ion's path or hurt the magnitude of the toroid's field.

6.2 Proposal: Implementing plasma lens to focus the divergent ion beam beyond thruster.

For the case of a magnetic collimator beyond the thruster, one possible solution is to use a plasma lens to reduce the beam divergence. An electrostatic plasma lens is placed at a certain distance outside the grids, in order to focus the ion beam, so that the Gaussian profile of the beam becomes narrower after passing through the lens. That is, the angle from the centerline of the normal distribution of the current density decreases. This packs or focuses the ions into a more tightly Gaussian packet, and also increases the current density in the center of the profile. The net effect of this is to increase the momentum of the ions in the direction of the thrust axis, so that the thrust increases. [GRF11].

The plasma lens operates on the principle of creating a region of crossed electric and magnetic fields using a magnetic coils and electrodes [spa14]. As the ion beam passes through this region, the ions are accelerated by the electric field so that they are focused to a point. [GRF11]. Figure 3 [GRF11] shows the setup:

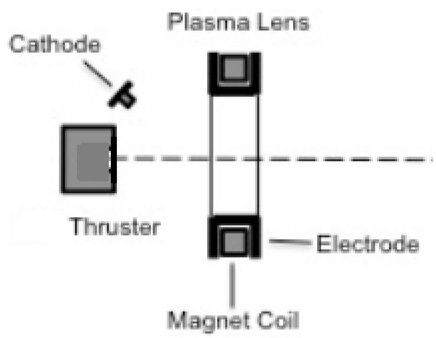


Figure 6: set up to use plasma lens to collimate and/or focus the ion beam

The plasma lens generates a magnetic field as a protruding "outward spiral", as shown in figure 4.[GRF11]

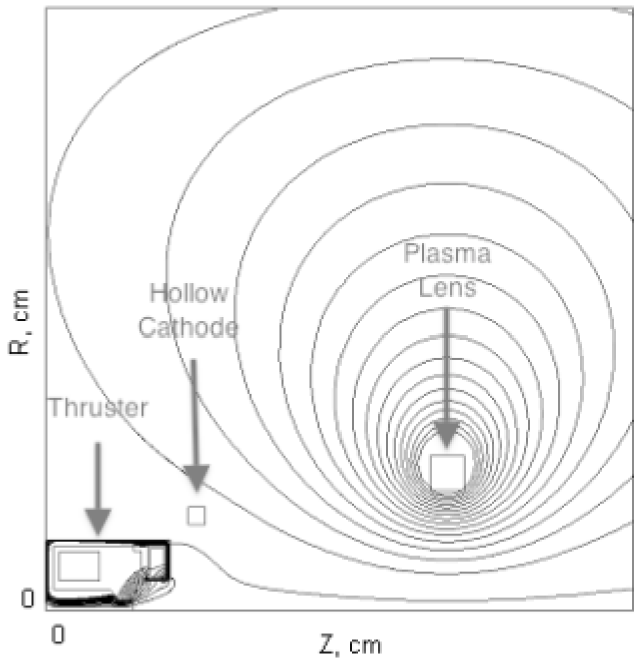


Figure 7: magnetic field line of the plasma lens. Z is the distance along the thruster axis. R is the radial position at which the ion trajectory intersects the plane of the Plasma lens.

The magnetic field of the plasma will lens cause the ions to focus, by the Lorentz force law. This in turn will decrease the average half angle divergent beam. Although it seems that this implementation might be beneficial in generating more thrust, the system is of less practical value due to some reasons:

• The plasma lens must not be placed immediately above the thruster, and at some distance away from the thruster. This is because the magnetic fields from the plasma lens mess up the magnetic fields inside the thruster. As a result, the peak ion current density, and the discharge current density will decrease. But when we place it at some radii away from the thruster, the drawback of that is, it will not be able to enclose all of the plume, and only a portion of the ion current will be able to pass through the plasma lens and get focused. It might be possible to increase the field of the plasma lens, and placing them close to the grids, but the thruster channel configurations would have to be changed, so that the magnetic field from the plasma lens does not affect the fields inside the channel.[GRF11]

• The decrease in the half angle for the beam that goes through the plasma lens might not be significant. This could also be due to the fact that the Plasma Lens electrode depletes the electrons in the plume. This will, then, cause the plume to diverge after the plasma because of space charge repulsion. We might need to add another cathode, after the plasma lens, to solve this problem.[GRF11]

• Although the power consumption of this setup could be decreased by collecting less current, which could be implemented by reducing the size of the electrode, or by increasing the radius of the plasma lens, the power consumption of the electrodes will outweigh the increase in the thrust. This makes this design practically less feasible.[GRF11]

7 Discussion

After Developing spacecraft rocket equations, we come to the conclusion that it is the $\cos\theta$ term in the thrust that actually will increase the thrust of the spacecraft. We then came up with engineering solutions that would generate magnetic fields that will decrease theta, and hence increase the thrust. However, after analyzing different ways to generate a magnetic fields outside the thrust chamber, in order to make the outgoing divergent beam more uniform, we conclude that it is not practical to generate such a magnetic field. This is because we found, through our research and analysis, that the thrust to power ratio will not increase as much and, in some cases, may even decrease. This limits the practicality of using magnetic fields to collimate the ion beam.

• We proposed a solution that if we use plasma lens outside the thruster, we might be able to increase the thrust by reducing the angular width of the Gaussian ion beam. However, we were not able to carry out a quantitative discussion, as to how much the lens will be able to focus the ion beam, and how much additional power is required to operate the plasma lens. This is primarily due to the fact that we do not know how far away to put the lens from the thruster, so that it will not mess up with the magnetic configurations of the inside of the thruster chamber. Generally, we found through our research that the electrodes consume more power than the plasma lens can increase thrust. Power losses outweigh thrust increase. One of the main reasons is that the current that electrode draws is not just due to the ion current, but some other phenomenon, too. [spa14]. Therefore, we cannot accurately determine how much power the electrodes will draw. It will,nevertheless, be a good idea to optimize the distance the plasma lens is placed from the thruster (without messing the inside configurations), figure out exactly how much of the ion beam passes through the lens, (while the Gaussian ion beam profile is still intact and has not totally spread, using the current density profile calculations), and determine either experimentally or through a numerical simulation as to how much the beam's angular width decreases, after passing through the lens, by changing the magnetic fields and the electrode potential. And then finally compute the thrust to power ratio of the system to determine whether or not it is practically feasible.

• Similarly, the toroid also presents physical challenges because it is a physical obstruction in the way of the ion beam current. This makes the toroid geometry actually not practical because, even though it is collimating the ion beam, it is also decreasing the thrust, which is exactly the opposite of what we want in the end.

• Our paper focused on increasing the thrust, by decreasing the divergence of the beam. But that is not the only advantage it offers. Depending on the structure of the spacecraft, reducing beam divergence will decrease the sputtering affect, that will in the long run increase the lifetime of the grids and the spacecraft in general.

8 Appendix

8.1 Thrust equation

Consider how the thrust of the spacecraft is related to the exhaust velocity.The thrust, T, is given by the rate of change of momentum. [GK08]

$$T = \dot{m} v_{ex} \tag{30}$$

Where m is the mass of the spacecraft at any time t, and v_{ex} is the exhaust velocity of the ions coming out of the final grid. For Xenon ion thruster, the ions are accelerated through a high electric field between the grids. Therefore, the velocity of these ions is much greater than the velocity of any neutral Xenon particle that might escape out of the grids. Hence,

$$T \approx \dot{m}_i v_i. \tag{31}$$

where $\dot{m}_i$ is the mass flow rate and v_i is the ion velocity. By the law of conservation of energy, the work done by the field on the Xenon ions must be converted into the kinetic energy of the ions. Therefore,

$$\begin{aligned} \frac{M v_i^2}{2} &= q V_b. \\ v_i &= \sqrt{\frac{2 q V_b}{M}} \end{aligned} \tag{32}$$

where V_b is the potential difference between the plates and q is the charge on the Xenon ions.Also, the mass flow rate is related to the ion beam current, by [GK08]

$$\dot{m}_i = \frac{I_b M}{q} \tag{33}$$

where I_b is the beam current, M is the mass of the ions, and q is their charge. Now, if we substitute equation (32) and equation (33) into equation (30), we get equation (1).

8.2 Correction factor for multiply charged species

If the beam contains only singly and doubly charged ions, then, the total beam current is given by: [GK08]

$$I_b = I^{++} + I^{+} \tag{34}$$

where I^{+} is the singly ionized, and I^{++} is the doubly ionized species. We define the correction factor, α as:

$$\alpha = \frac{1 + \frac{I^{++}}{\sqrt{2} I^{+}}}{1 + \frac{I^{++}}{I^{+}}} \tag{35}$$

where $\frac{I^{++}}{I^{+}}$ is the fraction of the doubly charged Xenon ion current in the beam.

8.3 Specific impulse

The specific impulse for any thruster is given by: [GK08]

$$I_{sp} = \frac{v_{ex}}{g} \tag{36}$$

Similarly as for thrust, substituting in the ion velocities, and incorporating angle correction, presence of doubly charged species, and propellant utilization efficiency, we are able to get equation (3).

8.4 Code to generate figures

8.4.1 Figure 5.2

Mathematica

```
v = Quantity[50, "KilometersPerSecond"];
q = Quantity["ElementaryCharge"];
m =ElementData["Xenon", "AtomicMass"];
k := w q B / (m * v)
k/. w-> Quantity[10, "Centimeters"]
dvOverV = Sqrt[1 - (Sin[theta] - k)^2] - Cos[theta];
dvOverVProp = dv /. B->Quantity[B, "Teslas"];
thetas = {Pi/8,Pi/4,3Pi/8, Pi/2};
func = dvOverVProp/.{theta->thetas, w->Quantity[10, "Centimeters"]};
p = Plot[func, {B, 0, .8}, PlotLabels->"\[Theta]="+ thetas, AxesLabel->{"B (teslas)",
```

8.4.2 Figure 5.2

Mathematica

```
axialComp:= PDF[NormalDistribution[0, y*m], x];
radialComp = axialComp * (x/y);
vel = {radialComp,axialComp};
VectorPlot[vel/.m->.5, {x,-6,6}, {y,2,5}, FrameLabel->{"radial distance", "axial dista
```

8.4.3 Figures 1

Python:

```
import matplotlib.pyplot as plt
import numpy as np
import seaborn

a = 1
b = (35/2)**2
c = 1
P = 100

r = np.linspace(0,35,100)
rho = a*r*np.exp(-r**2/b) + c
B = P/r

fig,ax = plt.subplots(1)
ax.plot(r,B, '--r')
ax.plot(r,rho, '-b')
ax.set_ylabel(r'$Dispersive␣Force$')
ax.set_xlabel(r'$radius(cm)$')
```

```
ax.set_ylim(0, 50)
ax.set_yticklabels([])
#ax.set_xticklabels([])

plt.show()
```

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