Problem A: Sending a Light Sail Propelled Nanocraft to Alpha Centauri

Team 258

Abstract

The stability of the sail, the precision of the initial trajectory, and the materials necessary for the design are all analyzed in an attempt to model a nanocraft's journey to Proxima Centauri B. Approximations and assumptions are made to simplify the issue of sail stability, which is the most difficult part of the analysis. Materials that are suitable for the sail – at least partially – are currently available. The requirements on the sail are that its emissivity, ϵ, reflectance, η, and absorptance, α, meet the following condition: $\frac{\epsilon\eta}{\alpha} \geq 6.6 \cdot 10^3$. The sail is assumed to be a rigid body with high reflectance, so that Newton's Laws and Euler's equation can be used to describe its motion. Assuming a constant laser beam shape over the area of the sail, these differential equations can be found analytically. Numerical simulations are performed to analyze the stability of the craft, suggesting stability for angular perturbations on the order of $2 \cdot 10^{-6}$ rad. The maximum initial transverse perturbation is 10 μm. If Proxima Centauri b is to be reached within the Earth-Moon distance, a 10 nrad maximum angular perturbation is needed. The laser beam would have to be aligned with the sail according to these specifications. Although these results suggest that highly accurate trajectories are necessary, the model proposed is conservative, in the sense that it does not allow the laser beam to stabilize the sail.

Contents

1. Introduction

1.1. Approach

A model will be developed for the motion of a light sail accelerated by a high power, 50 GW laser. The data given in the problem statement will then be taken as the initial conditons for this initial value problem. Whatever parameters characterize a light sail are analyzed in an attempt to answer the main problem: Is reaching Proxima Centauri b feasible with reasonable accuracy and 10-20 minutes of acceleration time? If so, what tolerances exist for the accuracy of the laser beam and the design of the sail in order to achieve this?

1.2. General assumptions

Since [1], in 1984, attempts have been made to determine ideal sail materials for laser sailing. An ideal material would have a combination of properties that does not exist in any available materials. Therefore, the material of the sail was assumed to be heat resistant, ultra-thin – on the order of 100 nm, and light – weighing 0.5 g for an area of 10 m^2. This material was assumed to be perfectly reflecting. Due to the large amount of energy imparted to the sail, imperfect reflectors would not allow the craft to accelerate to the target speed.

Furthermore, the sail is assumed to be a rigid body that retains its shape. The nanocraft, also weighing 0.5 g, is connected to the sail by rigid, strong, light connectors. In reality, these connectors – and their respective moments of inertia – would have to be accounted for to produce an accurate model of the acceleration of the craft.

The differential equations that are derived assume that the sail is solely under the influence of the laser's radiation, ignoring gravity of the Moon, Earth, and Sun. The sail is to be released above the Earth's atmosphere, however, so escape velocity would not have to be taken into account.

Although any diffraction-limited, coherently combined laser beam will be a gaussian – to achieve the highest power per unit area, a flat-topped beam shape has been used for the analysis in this paper. If a smaller width than the FWHM value actually hits the sail, then this is not a bad approximation; however, in practice, the beam will be hitting the light sail up to its first airy disk for the highest possible power efficiency.

2. The physics of light sailing

2.1. About laser beams

In order to achieve laser powers of 50 GW, coherent combination of mode-locked, pulsed laser beams is necessary. This is usually done – and has been achieved – with lasers of wavelength $\lambda = 1064\,\text{nm}$ [2]. However, as will be shown, this wavelength makes no difference on calculations of acceleration.

According to DeBroglie, the momentum imparted by a photon is $p = \frac{h}{\lambda}$ and its energy follows $E = pc$. Likewise, a laser beam with power P is emitting P/pc photons per unit time. That means that the momentum of a laser beam per unit time is simply P/c. When this laser beam hits a surface the momentum of the reflected photons should also be considered. Thus, the momentum per unit time exerted on this surface is $2P/c$.

2.2. About light sails

This section will mostly follow [1]. If one considers the results of the previous section and applies Newton's second law, then it is straight-forward to find the following expression for the acceleration of a light sail:

$$a = \frac{2\eta P}{mc} \tag{1}$$

where P is the power that arrives to the sail, η is the reflectance of the material and m is the total mass of the spacecraft. Although mass distribution will be discussed later, the system will consist of a nanocraft (ideally a light, wafer-scale chip) fixed by four connectors to a light sail. A representation of this design can be seen in Figure 1. The sail is assumed to be spherical in this report (unlike in Figure 1), with a large radius R. By taking the limit $R \longrightarrow \infty$, the flat sail case is recovered, which has been proved to be unstable under realistic conditions. Literature on this topic has details about conical and hyperbolic sails, which will not be covered here [5][?].

Writing $m = m_{ls} + m_{sc}$ as a sum of the masses of the sail and the craft, and now $m_{ls} = \rho A d$, where ρ is

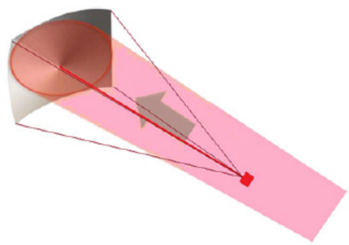

Figure 1: Schematic representation of the nanocraft and laser beam direction. [3]

the density of the sail material and d is its thickness, the acceleration is:

$$a = \frac{2\eta P}{(m_{sc} + \rho A d)c} \tag{2}$$

Some considerations may be derived from this formula. First, this formula puts a limit on the maximum acceleration one could achieve with such a system. Until now, loss of power due to heating of the material has not been considered. Also, according to [1] if the goal is to optimize the sail film thickness, the payload and the structure mass in the effective sail material density could be included, so that Equation 2 becomes:

$$a = \frac{2P}{Ac}\frac{\eta}{\rho d} \tag{3}$$

where the last factor only depends on the parameters of the sail material.

According to the above equation, in order to maximize a, a thin sail from a low density material is needed. Note, however, that for a very thin material most of the power passes through the sail and is then wasted. Additionally, some power will be absorbed, and, if the absorptance of the material is too high, power will also be lost here. Assuming that this material does not have to be a perfect black body, the absorbed power αP has to be equal to a fraction of the radiated heat $2\sigma A \varepsilon T^4$, where α is the absorptance, ε is the emissivity, σ is the Stefan-Boltzmann constant, and T is the operating temperature of the sail. Equation 3 becomes:

$$a = \frac{4\sigma}{c}\frac{\varepsilon \eta T^4}{\alpha \rho d} \tag{4}$$

Equation 4 can be used to compute the acceleration of a light sail given the power of a laser beam array. This will give a first estimation of the possible velocities, which will be of the order of $0.2c$. Moreover, Equation 4 can be used to discuss the materials required to build a suitable light sail.

2.3. Relativistic velocities

According to Equation 2 and considering the relativistic case where $p = mc\gamma\beta$, one can write:

$$\frac{2\eta P}{mc} = \frac{dp}{dt} = mc\beta\frac{d\gamma}{dt} + mc\gamma\frac{d\beta}{dt} \tag{5}$$

following [4]. Considering the definition of the gamma factor one derives the differential equation:

$$\frac{2\eta P}{mc^2} = \frac{d\beta}{dt}\frac{1}{(1-\beta^2)^2} \tag{6}$$

Integration yields an analytical solution for t:

$$t = \frac{mc^2}{4P\eta}\Big[\frac{\beta}{1-\beta^2} + \frac{1}{2}\ln\Big(\frac{1+\beta}{1-\beta}\Big)\Big] \tag{7}$$

which can be inverted to find β. Despite other intuitive considerations, this will be the main evidence that a large error is not made by not considering special relativity.

2.4. Light sailing dynamics

One of the main problems is to achieve a sail with a beam-riding stability without need of active-feedback of the spacecraft. In this section,[3] and [5] will be followed to determine the equations of motions of the spacecraft. A Cartesian reference frame (X, Y, Z) with its origin at the center of mass of the spacecraft and its Z-axis in the direction of the cylindrical symmetry axis of the sail will be used. This choice of Z is important, as it makes the used reference frame (RF) a non-inertial RF moving along the axis of the laser beam with the acceleration given by the laser-imparted force. One of the assumptions made is that the sail behaves like a rigid body, so that its motion can be described with both Newton's second law (Equation 8) and Euler's equation (Equation 9):

$$\mathbf{F} = m\ddot{\boldsymbol{x}} \tag{8}$$

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} = \boldsymbol{\tau} \quad (9)$$

where ω is the angular velocity and $\mathbf{I}$ the inertia tensor of the sail. First, $I_{xx} = I_{yy}$ because of the cylindrical symmetry of the problem and also $I_{ij} = 0 \; if \; i \neq j$, since the principal axes are being used. Then a simple calculation gives,

$$I_{xx} = I_{yy} = I = m_{ls}L_c^2 + m_{sc}(L - L_c)^2 \quad (10)$$

where L is the distance between the nanocraft and the sail and L_c, between the center of mass of the sail and the center of the nanocraft.

By applying Equation 2 – supposing an ideal reflectance – to an element of the sail surface, the force applied by the laser beam can be determined:

$$\mathbf{F} = \int_S \frac{2\boldsymbol{p}(\boldsymbol{x}) \cdot \hat{\mathbf{n}}(\boldsymbol{x})}{c} \hat{\boldsymbol{n}}(\boldsymbol{x}) dS \quad (11)$$

with $\boldsymbol{p}(\boldsymbol{x})$ being the bean power flux at a point $\boldsymbol{x}$ in the surface of the sail, S the light sail area, and $\boldsymbol{n}(\boldsymbol{x})$ a normal unit vector to this surface at $\boldsymbol{x}$. Considering the constant direction of $\boldsymbol{p}(\boldsymbol{x})$: parallel to the z-axis and perpendicular to the Earth surface where the laser is assumed to be installed, this implies $\boldsymbol{p}(\boldsymbol{x}) = p(\boldsymbol{x})\hat{\boldsymbol{z}}$.

Similarly the expression for the torque applied to the beam can be found using $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$:

$$\boldsymbol{\tau} = \int_S \frac{2\boldsymbol{p}(\boldsymbol{x}) \cdot \hat{\mathbf{n}}(\boldsymbol{x})}{c} (\boldsymbol{r}(\boldsymbol{x}) \times \hat{\boldsymbol{n}}(\boldsymbol{x})) dS \quad (12)$$

Here, $\boldsymbol{r}(\boldsymbol{x})$ corresponds to the distance between the center of the sail and a point $\boldsymbol{x}$ on the sail's surface. From basic geometry, one can see that it can be computed as $\boldsymbol{r} = (x, y, L_c - R + \sqrt{R^2 - x^2 - y^2})$ The main purpose of the dynamical analysis is to study small perturbations in the x and y axis. As a consequence of these perturbations, two rotations of the center of mass defined by the angles θ_x and θ_y, which correspond to rotations about the x- and y-axes respectively, are caused. Using the gradient on the surface of a sphere, $\hat{\mathbf{n}} = (\frac{x}{R}, \frac{y}{R}, \frac{\sqrt{R^2-x^2-y^2}}{R})$ can be found, and so $\boldsymbol{p}(\boldsymbol{x}) \cdot \hat{\mathbf{n}}(\boldsymbol{x}) = p(\boldsymbol{x})\frac{\sqrt{R^2-x^2-y^2}}{R}$ where R is the radius of curvature of the sail. It is also easy to see that a small displacement Δ in the X-direction will trigger a rotation about the Y-axis

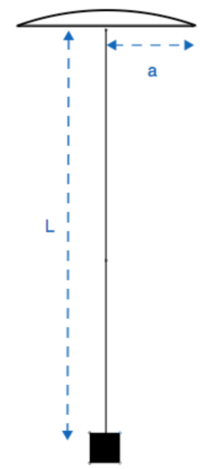

Figure 2: Schematic representation of the sail and the nanocraft. The light sail is spherical with a large radius R. In this scheme, the direction of Z is taken in the direction of L. X and Y are perpendicular to Z.

(θ_y). This is equivalent to motion in the Y-direction and rotation about the X-axis because of the symmetry of the sail and the coupling between X and θ_y in the equations of motion for F_x is the same as the coupling between Y and θ_x in the equations of motion for F_y. Only one displacement (in the X-direction) will be solved for, since a displacement in Y will be equivalent. Solving for motion along the Z-axis is trivial because the torque is zero about Z. Thus, Z is governed by a constant acceleration which makes it depend only on t^2.

Using the approximation for small angles, $\sin\theta_y \approx \theta_y$, $\Delta = X + \theta_y L_r$ where $L_r = \sqrt{(\sqrt{R^2 - a^2} + L_c - R)^2 + a^2}$ is the distance between the rim and the center of mass of the sail. A schema of L and a can be found in Figure 2. Now, the force in the X-direction can be computed, using the flux of the beam. As it has been mentioned, a gaussian beam model for the laser will not be used, with a constant one in its place (Equation 13). This allows the solution of the integrals analitically and hence to make explicit the set of differential equations of motion. The power of the beam can be assumed to be

$$p(\boldsymbol{x}) = \begin{cases} p_0 & \text{if} \quad 0 \leq x^2 + y^2 \leq a \\ \\ 0 & \qquad \text{otherwise} \end{cases} \tag{13}$$

Such an illumination model allows $dS = \Delta dy \frac{R}{\sqrt{R^2 - x^2 - y^2}}$. This is because there is a constant power flux over all the area except the rim of the sail. Then, the force is simply:

$$F_x = \int_{-a}^{+a} \frac{p_0 \Delta}{R} x dy = \frac{p_0 \Delta}{R} \int_{-a}^{+a} \sqrt{a^2 - y^2} dy$$
$$= -\frac{1}{2} F_{rad} \frac{X + L_r \theta_y}{R} \tag{14}$$

where the definition $F_{rad} = 2 p_0 \pi a^2$ is used. Note that this is precisely the force coming from the laser beam that was computed in the last section, which is assumed to cause all the acceleration. The same can be done for the torque:

$$\tau_y = 2 p_0 \Delta \Big(\frac{L_c}{R} - 1 \Big) \int_{-a}^{+a} x dy$$
$$= -\frac{1}{2} F_{rad} \Big(\frac{L_c}{R} - 1 \Big) \Big(X + L_r \theta_y \Big) \tag{15}$$

Then, Newton's second law and Euler's equation can be written as:

$$m \ddot{X} = F_{rad} \theta_y - \frac{1}{2} F_{rad} \frac{X + L_r \theta_y}{R} \tag{16}$$

$$I \ddot{\theta}_y = -\frac{1}{2} F_{rad} \Big(\frac{L_c}{R} - 1 \Big) \Big(X + L_r \theta_y \Big) \tag{17}$$

In the first one, an extra term has been added which represents the additional force created due to the angle θ_y. If a^2 is assumed to be much less than R^2, $L_r = L_c$.

Taking the limit $R \longrightarrow \infty$ the flat sail model is recovered, governed by equations:

$$m \ddot{X} = F_{rad} \theta_y \tag{18}$$

$$I \ddot{\theta}_y = \frac{1}{2} F_{rad} \Big(X + L_r \theta_y \Big) \tag{19}$$

2.5. From equations of motion to Alpha Centauri

If the goal is to arrive at Proxima B in Alpha Centauri at a closer distance than that between the Earth and the Moon, motion in the x and y directions once acceleration stops should be considered. If in this instant of time the angles θ_x and θ_y are too large, a close enough approach to Proxima B cannot be guaranteed.

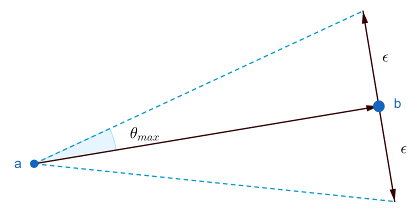

Figure 3: Schematic view of the range of allowed trajectories to Proxima B, located at (b). Earth is located at point (a).

Figure 3 can be used to better understand this problem. In it, θ_{max} represents the maximum angle the craft can have with respect to the x or y axis when the acceleration is stopped after $t \sim 10$ min. If the distance between the Earth and the Moon is $\epsilon \sim 400,000$ km the condition for achieving the goal is:

$$\theta_{max} \approx \sin \theta_{max} \leq \frac{\epsilon}{4.25 \text{ years} \cdot c} \approx 1 \cdot 10^{-8} \text{ rad} \tag{20}$$

Thus, considering the perturbations in the light sail, the maximum error in the laser beam can be characterized. In order to satisfy this constraint, a great accuracy in the laser beam is required.

3. Model analysis and results

3.1. Feasibility of light sailing

A first estimate will show that nearly c velocities are possible with such a laser beam (ignoring relativity). As discussed in previous sections, the momentum imparted by a 50 GW beam each second is

$$\frac{5 \cdot 10^{10}}{c} \cdot 2 \ J = \frac{1}{3} \cdot 10^3 \ \text{kg} \frac{\text{m}}{\text{s}} \tag{21}$$

where η, the reflectance, is assumed to be 1. Assuming the sail is perfectly aligned with the laser beam (i.e. with no perturbations), the sail will accelerate at a constant rate. This acceleration will be given by

$$\frac{\frac{1}{3}\cdot 10^3 \ \ \mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}}{0.001 \ \mathrm{kg}} = \frac{1}{3}\cdot 10^6 \ \frac{\mathrm{m}}{\mathrm{s}^2} \tag{22}$$

If this acceleration is maintained for 10 minutes, the craft will achieve a speed of

$$\frac{1}{3}\cdot 10^6 \ \frac{\mathrm{m}}{\mathrm{s}^2}\cdot 600 \ s = 2\cdot 10^8 \ \frac{\mathrm{m}}{\mathrm{s}} \tag{23}$$

without taking into account relativity. The gamma factor, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, is only a 0.5% correction for $v = 0.1c$, where c is the speed of light. Therefore, up to $0.1c$, relativity does not need to be taken into account to still retain 99% accuracy. Past this speed, γ increases to 1.02, requiring a 2% correction. Due to the other assumptions made, this is not considered to affect the results of this paper significantly, so it has been ignored. Since the craft needs to reach a target speed of $0.2c$, any acceleration past this (which will be greatly abated when $v \to 0.5c$) is inconsequential, as this is an ideal case.

The non-ideal case is much more complex, including many parameters. According to [1] the upper limit of acceleration of an object can be derived from Equation 4. In [1], Forward claims that aluminum is the best known material for the sail; however, its maximum operating temperature of 600 K limits its uses. If aluminum ($\epsilon = 0.06$, $\eta = 0.82$, $\alpha = 0.135$, $\rho d = 2.71 \ \frac{g}{cm^3}$) was used for the design proposed in this paper, it would limit the acceleration to $1.32 \ \frac{m}{s^2}$. This is unacceptable, as the craft would be unable to reach relativistic speeds in a short time. An upper limit on the desired acceleration – to reach $0.2c$ in 10-20 minutes – would be $a = \frac{0.2c}{600 \ s}$, $a = 1\cdot 10^5 \ \frac{m}{s^2}$. The issue with aluminum is its low emissivity, causing it to heat up – and therefore reach its temperature limit – too quickly. The following analysis will attempt to determine an ideal material for the sail, according to the constraints that the design in this paper requires.

The sail design proposed here requires a low surface density. Since a total sail weight of 0.5 g is required for a surface area of 10 m^2 the total mass per unit area is $\rho d = 5\cdot 10^{-5} \ \frac{\mathrm{kg}}{\mathrm{m}^2}$. The density of such a material would then be $\rho = 5\cdot 10^3 \ \frac{\mathrm{kg}}{\mathrm{m}^3}$. This is reasonable, as the material would be only 5 times as dense as water (but would need to be thinned to 10 nm). Materials that are less dense could be used as a thicker sail, as long as they meet the other requirements. Assuming an operating temperature of 1000 K, a model can be built for the constraints on the emissivity, reflectance, and absorbace:

$$\frac{\epsilon\eta}{\alpha} \geq 6.6\cdot 10^3 \tag{24}$$

These requirements are not out of reach. According to [6], multi-layer dielectric materials currently exist that can be coated to result in 99.995-9% reflectance for a specific laser line (in this case 1064 nm). This is two orders of magnitude better than required by the above analysis, allowing the constraints on absorptance and emissivity to not be as stringent. Of course, designing a material that has all these properties – high reflectance, low absorptance, high operating temperature, and high emissivity – is difficult.

3.2. Accelerating to relativistic limit

Using Equation 7, it can be shown that all the equations used are not within the relativistic limit. In Figure 4 this equation is plotted.

It can be seen that for the considered mass of 1 g, speeds up to $0.2c$ are achieved in a reasonable amount of time. Obviously, as m increases – still under a few grams, more time is needed to achieve higher speeds, so spacecrafts are non-relativistic for longer periods of time.

As shown for $\beta \ll 1$, the constant acceleration behavior is recovered, as treated earlier. This can also be shown from Equation 7. When $\beta \to 1$:

$$\frac{\beta}{1-\beta^2} \to \beta, \ \ \ln\left(\frac{1-\beta}{1+\beta}\right) \to 2\beta \tag{25}$$

thus recovering Equation 2.

3.3. Stability analysis

In this section the system of ordinary differential equations (Equation 16 and Equation 17) that determines the dynamics of the spacecraft has been solved

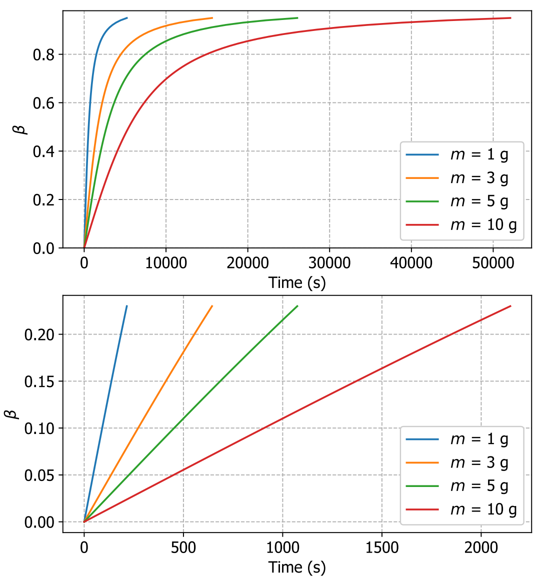

Figure 4: β versus time, considering relativity. Different masses have been plotted. Lower figure shows constant acceleration until $\beta < 0.2$.

numerically. The parameters used in the simulation are $a = 200$ cm and $L = 2000$ cm; both masses m_{ls}, m_{sc} have been taken to be the same – according to [6] this is the optimal configuration – so $L_c = L/2$. Also, if a^2 is much smaller than R^2, then $L_c = L_r$. In an idealized case, it has been shown that it is possible to reach velocities of $v_{max} = 0.2c$. Now, one might wonder what force the spacecraft needs to achieve such velocities. The assumption that $\eta = 1$ when deriving the equations of motion still holds. Simple calculations for $\Delta t = 600$ s give an approximate value of $F_{rad} = 10^2$ N, an upper limit on the force required.

3.3.1. Flat sail

If $R \to \infty$, the sail becomes flat. The intuitive idea behind the flat sail instability is that any non-uniformity in the beam intensity or alignment of the sail will cause a torque with no restoring force; hence, it will not be at all stable. The plot for this simulation is not shown, because the velocities in the x-axis exceed the relativistic limit. Therefore, the derivations made are not valid anymore. Regardless, the simulations still suggest an unstable behavior of a flat sail.

3.3.2. Spherical Sail

Figure 5 shows the components x and z of the motion of the light sail for a spherical sail with radius of curvature $R = 1000$ cm. There is no torque on the z-axis, so the shape of the function of motion of the sail is going to be well approximated by Equation 2 (with $\eta = 1$). This is reasonable, since the kinetic energy associated to x and y axes is not comparable with the total kinetic energy, as will be shown. Note that z has a parabolic behavior.

The simulation has been run for two different perturbations, first for $\delta_x = 0.001$ cm and $\theta_y = 0.001$ degrees, getting an amplitude of 2 cm and then; for $\delta_x = 0.0001$ cm and $\theta_y = 0.0001$ degrees, getting an amplitude of approximately 0.2 cm. Note that as one may expect, bigger initial perturbations imply bigger amplitudes. This means that the more perturbed the system is initially, the greater the variations of the coordinates on the x-axis (and similarly on the y-axis). These variations are reflected in the value of the amplitude of the oscillations. Simulations also show that for larger initial perturbations no acceptable amplitudes are obtained, showing an unstable behavior for the spacecraft at those initial conditions.

The energy absorbed by these perturbations is related to the kinetic energy of the oscillatory motion according to $\langle T \rangle_x = \frac{1}{2} I \omega^2$. However, there are oscillations in both the x-axis and y-axis. Thus, an estimation for the kinetic energy of one oscillation is $\langle T \rangle \approx 600J$, and the kinetic energy per second is around $7000\,W$, which is negligible in comparison with the power of the laser beam (50 GW). This allows the consideration of Equation 2 as an equation of motion for z.

3.4. Accuracy of the system

Once the craft stops accelerating after a time of 10 min, it has traveled a distance $\sim 10^7$ km, as shown in Figure 5. This is negligible compared to the distance between the Earth and Alpha Centauri, so it can be assumed that the sail is still at Earth (though out of its gravitational field). According to Figure 3, a

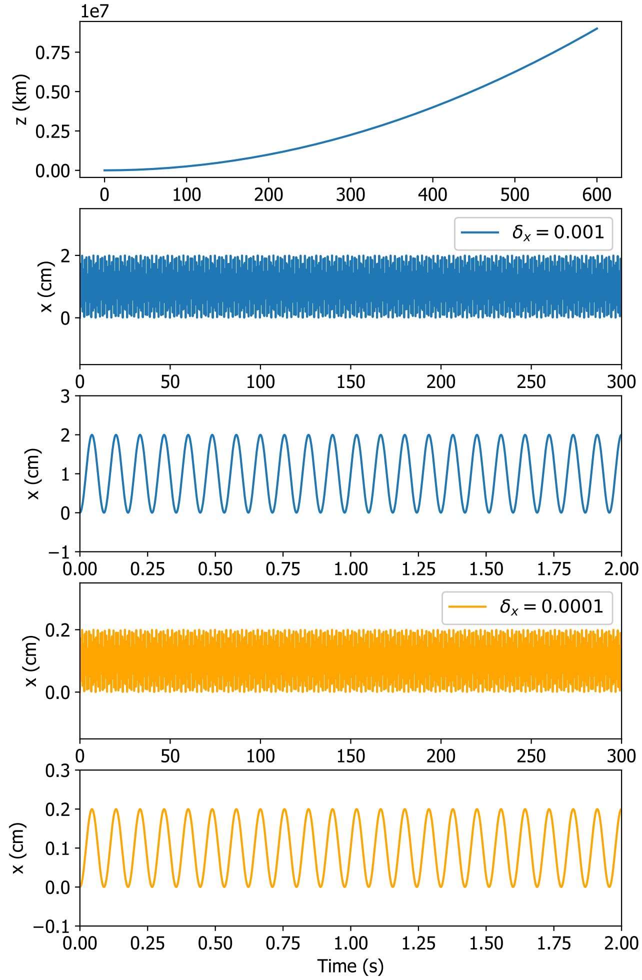

Figure 5: Spacecraft position for small initial perturbations in X and θ_y. Simulation results.

really small angle in the x or y axis is needed in order to achieve the goal of flying by Proxima Centauri B within the desired distance. This has been found to be of a great complexity.

The simulation gives a value equal to the initial perturbation for the maximum deviation of the x-axis – i.e 0.0001 or 0.001 degrees. These angles correspond in the best case to an angle of the order of $2 \cdot 10^{-6}$ rad.

As seen in the introduction, the needed accuracy is on the order of nrad. Simulations show that this could be achieved by reducing the initial perturbations in θ_x by two orders of magnitude, but leaving constant the initial perturbations in x. A laser beam capable of this would be able to propel the craft to Proxima B. Greater errors do not guarantee the success of the trip. Simulations also show that the amplitude of the oscillations are small enough to keep the spacecraft stable; hence, the needed accuracy is about $1 - 10$ μm for perturbations in x and of the order of nrad for perturbations in θ_x.

Some considerations can be made regarding the power of the laser beam. It has been assumed that in order to achieve the desired velocities, the system should receive a force in the z-direction of around 100 N. This leaves unused more than half of the available power. Thus, such an ideal system where $\eta = 1$, and none of the power passes through the sail or is otherwise lost has significant margin for error. Many papers argue that the beam would not be able to be completely enclosed by the sail for the entire duration of the acceleration [6][4]. These results show that this is not a significant issue — as long as half the power hits the sail, on average.

4. Conclusions

4.1. Discusion of the results

The ability of such a system to achieve nearly c velocities in a reasonable amount of time was one of the primary concerns. At a first estimate of $\eta = 1$ and a perfectly aligned laser with no perturbations, results show that up to $0.2c$ speeds can be easily achieved. Another important issue was the effects of relativity when reaching velocities on the order of the speed of light. However, it has been shown that the case studied here can be assumed to be non-relativistic. If the mass of the craft is increased, then the spacecraft is only more of a non-relativistic system.

The material of the light sail was also considered. Due to an idea purported by [1], the properties of aluminum were analyzed, but it was determined to not be a suitable candidate due to its emissivity and operating temperature. The acceleration necessary to reach $v = 0.2c$ was determined and used to set constraints on an ideal material for the sail. The ideal material for a sail should have low surface mass density. Moreover, by assuming an operating temperature of 1000K, a simple relation that this material must satisfy can be obtained: Equation 24. In words, the material needs to have high reflectance, low absorptance, high operating temperature and high emissivity. Even though such a material could seem hard to manufacture, materials that satisfy these requirements at least partially are already available.

Finally, stability of the light sail was considered. Two models have been considered: a flat sail which is not stable at all under small perturbations, and a spherical sail which is stable. For the latter, calculations were performed approximating the intensity of the laser beam with a piece-wise function. These calculations suggest that the amplitude of the oscillatory motion in both the x and y axes increases when the initial perturbation is increased. In order to plan such an interstellar flight, a system with accuracy of nrad in θ_x and of μm in both x and y axes is necessary.

4.2. Strengths and weaknesses

- Weaknesses

  1. In order to achieve maximum power in the spot generated by a laser beam, a gaussian beam profile would be used [6][**?**]. This profile would change the calculations for the stability of the sail considerably. The calculations performed in this paper would no longer be accurate.
  2. The connectors of the sail to the craft are difficult to account for. If they are to be rigid and strong, then their mass would be a factor in the calculations of the sail dynamics, something which this paper does not account for.
  3. The necessary precision is calculated to be a less than nrad angular perturbation, which is most likely not achievable.
  4. Uniformity – or lack thereof – of the laser beam has not been taken into account.
  5. Dust impacts during flight will greatly alter the course taken by the spacecraft. According to [6], a craft on the order of grams will be perturbed by dust impacts. Although these impacts will not damage the spacecraft itself, they will cause it to slowly drift off-course [6]. If this nanocraft is around 1 g, as assumed by Lubin, angular acceleration caused by dust impacts will be on the order of $10^-5 \ \frac{rad}{sec}$, with the direction depending on the direction of impact of the dust. This angular acceleration will destabilize the craft and lead to a drift from the trajectory. In his paper, Lubin suggests photon thrusters on the craft which will counteract this small acceleration. If it is not counteracted, it will lead to approximately a complete rotation in just one day. This completely eliminates the possibility of the spacecraft staying on course for an order of decades without some kind of on-board correction mechanism, such as thrusters. Since such a mechanism would cause a (significant) increase in mass, this is likely the most detrimental weakness to this model.

- Strengths

  1. The analysis provided through this model suggests that it is possible for a craft to achieve speeds on the order of 0.2 c with the given laser power of 50 GW, and – with sufficient precision – to reach Proxima Centauri b at a closer distance than that between the Earth and Moon.
  2. As discussed previously, less than half the total power of the laser is needed to be used for acceleration. If 50% of the power is transformed into kinetic energy of the

spacecraft, then the target of 0.2 c is still achieved.

3. The materials that are required for the construction of the sail are not readily available but can be theoretically created and fabricated in the foreseeable future.
4. The control of the laser profile through a phased laser array could allow it to be used as a source of extra stability for the sail [6]. This suggests that this analysis was a conservative one.
5. The suggested sail design has been proven to be stable under laser beam conditions which are not favorable and can be made to be more stable with manipulation of the laser beam, as is possible with modern phase-locked lasers [6][5].

4.3. Future work

More analysis needs to be performed on the ideal shapes for a laser-propelled sail. Several designs have been proposed – such as hyperboloids, cones, spheres, and others, but – due to the possibility of laser beam shape modification – which of these designs is ideal is unclear. Moreover, there needs to be a determined ideal balance between accelerating power and stability of the sail.

The analysis of laser beam shapes for ideal stability is crucial to the success of this kind of mission. [6] suggests that a minimum be created in the center of the beam to promote stability, while [5] argues that four gaussian beams on a spherical sail can keep it stable with initial perturbations of several centimeters, which is promising. In either case, more analysis and modeling is necessary in order to assess what the best shape is for stability of the craft. Assuming an amount of power as given in this problem is achievable, then perhaps losing some of that power to a less-favorable (in terms of power efficiency) beam shape in order to achieve a more stable craft is necessary.

Due to the high precision required in aiming the spacecraft towards Alpha Centauri as well as the problem of dust impacts discussed above, methods of correcting the craft's trajectory and unwanted acceleration need to be analyzed. Ideally, some kind of mechanism would be on-board which would be able to correct the trajectory, suggesting that the weight of the craft would have to be increased. Detailed analysis of ultra-light correction systems should be performed.

References

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A. Python code for numerical simulation

```
# Python modules
from numpy import linspace
import random as random
from scipy.integrate import odeint
from math import sqrt, pi, log
import matplotlib.pyplot as plt
from matplotlib import rcParams
rcParams.update({'font.size': 13})
rcParams['font.family'] = 'sans-serif'
rcParams['font.sans-serif'] = ['tahoma']

# Equations of motion spherical sail
def light_spherical_sail(y,t,Frad,Lc,R,m,I):
    x, vx, thetay, omegay = y
    dydt = [vx, Frad(t)/m*thetay-1/2/m*Frad(t)*(x+Lc*thetay)/R,
            omegay, -1/2*Frad(t)/I*(x+Lc*thetay)*(Lc/R-1)]
    return dydt

# Equations of motion flat sail
def light_flat_sail(y,t,Frad,Lc,m,I):
    x, vx, thetay, omegay = y
    dydt = [vx, Frad(t)/m*thetay,
            omegay, 1/2*Frad(t)/I*(x+Lc*thetay)]
    return dydt

        # Constants
a = 200 #cm
R = 1000 #cm
Lc = 1000 #cm
m1 = 1/2 #g
m2 = 1/2 #g
m = (m1+m2) #g
L = 2000 #cm
I = m1*Lc**2+m2*(L-Lc)**2 #g*cm^2
F_rad_0 = 10**7 #cm*g/s^2
tim = 10*60 #s
P = 50*10**9 #W
c = 3*10**8 #m/s
eta = 1 #reflectance
def F_rad(x): #Force from the laser beam
    if x<=10*60:
        return F_rad_0  #+normal(F_rad_0/100,F_rad_0/100) #adding noise
    if x>10*60:
        return 0

y0 = [0.001,0,0.001,0] #Initial cond set 1
y1 = [0.0001,0,0.0001,0] #Initial cond set 2
yr = [0.0000001,0,0.00001,0] #Initial cond set 3

sol1 = odeint(light_spherical_sail, y0, t, args = (F_rad,Lc,R,m,I))
sol2 = odeint(light_spherical_sail, y0, t2, args = (F_rad,Lc,R,m,I))
sol12 = odeint(light_spherical_sail, y1, t, args = (F_rad,Lc,R,m,I))
sol22 = odeint(light_spherical_sail, y1, t2, args = (F_rad,Lc,R,m,I))
# solving for different conditions

#Plotting results from ODEs
```

```
fig,ax = plt.subplots(5,1,figsize=(7,12))

ax[0].plot(t3,z)
ax[0].set_xlabel('Time (s)')
ax[0].set_ylabel('z (km)')

ax[1].plot(t,sol1[:,0],label=r'$\delta_{x} = 0.001$')
ax[1].legend()
ax[1].set_xlabel('Time (s)')
ax[1].set_ylabel('x (cm)')
ax[1].set_xlim(0,tim/2)
ax[1].set_ylim(-1.5,3.5)

ax[2].plot(t2,sol2[:,0])
ax[2].set_xlabel('Time (s)')
ax[2].set_ylabel('x (cm)')
ax[2].set_xlim(0,tim/300)
ax[2].set_ylim(-1,3)

ax[3].plot(t,sol12[:,0],label=r'$\delta_{x} = 0.0001$',color='orange')
ax[3].legend()
ax[3].set_xlabel('Time (s)')
ax[3].set_ylabel('x (cm)')
ax[3].set_xlim(0,tim/2)
ax[3].set_ylim(-0.15,0.35)

ax[4].plot(t2,sol22[:,0],color='orange')
ax[4].set_xlabel('Time (s)')
ax[4].set_ylabel('x (cm)')
ax[4].set_xlim(0,tim/300)
ax[4].set_ylim(-0.1,0.3)

plt.savefig('dxbis.eps')
plt.show()

#Solving for flat sail
sol_flat = odeint(light_flat_sail,y1,t4,args = (F_rad,Lc,m,I))
fig,ax = plt.subplots(figsize=(7,4.5))
ax.plot(t4,sol_flat[:,1])
plt.show()
```