Problem A: Analysis of the influencing factors during the sail acceleration

Team 295
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Abstract

This paper studied the influencing factors during the process that a light sail propelled nanocraft fly the Proxima Centauri b at a distance closer than Earth-Moon distance. The dynamics analysis was given in the paper. Four partial models were established to describe such analysis: Laser Transmission Model, Acceleration Dynamics Model, Stability Analysis Model and Track Model.

In the Laser Transmission Model, the function $P = f(\beta_m, \beta_p)$ was given to calculate the laser intensity after crossing the atmosphere.

In the Acceleration Dynamics Model, based on the assumption that sail was a rigid plate, we discretized the sail into 10000 area elements whose attribute could be shown by dA, dm, dP, $\vec{r}_i$. And each element corresponds to a transmitter. In the model, a fixed coordinate system and a Lysol coordinate system was built. First we got the center of mass of the sail by summing all elements up. Then the torque matrix and inertia tensor matrix around the center of mass was given. Followed was the calculation of angular acceleration through the theorem of moment of momentum. Finally, Euler dynamical equations were applied to solve out the angular velocity and thus the angle shift could be described.

In the Stability Analysis Model, accuracy of the beam from the laser array was shown as $\delta P/P$, margin of error in the fabrication of the light sail was described by δm and the uniformity of the laser beam was given by $\delta \mathrm{d}$. Apply these small perturbations into the dynamics model. These small perturbations were uneven and in random. As a consequence, it was nearly impossible to get an analytical solution of the Euler equations.

In the Track Model, under the consideration of the gravitation of the Proxima Centauri and Proxima Centauri b, we calculated two possible maximum angles deviation and found $\alpha_{\max 2} = arc\sin(LD/D_{pe}) + arc\sin(R_{\text{orbit}}/D_{pe})$ fit the actual situation better. This angle was the critical condition of the dynamics model.

Then through the numerical simulation, we got preliminary solutions that the margin laser accuracy should be $10^{-8.306}$, fabrication error should be $10^{-5.391}$ and uniformity of the laser beam should be $10^{-4.314}$. In the process of simulation, we found the number of element had influence on the results. So in the sensitivity analysis, when the element was respectively 45*45, 35*35, and 40*40, the effect of intensity, fabrication and uniformity tended to be stable.

Results above considered one factor while the results caused by another two factors were ignored. But in fact, with overall consideration of them, an optimal solution could be searched out. And the optimal solution was respectively $10^{-8.769}$, $10^{-6.032}$ and $10^{-4.938}$.

Contents

1. Introduction

Space travelling was always a quite popular exploring activity. And Alpha Centauri b was such a planet that scientist paid much attention to because it was only 4.2 light years away from the Earth. Years ago, it would spend about 30 thousand years traveling to Alpha Centauri b which was the nearest galaxy from our planet. With the development of nanotechnology and laser technology, a new method was proposed that we could make a laser array to drive a nanocraft with light sail to travel to Alpha Centauri b. The nanocraft's speed was supposed to accelerate to 20 percent of light velocity in 10 to 20 minutes by a laser array whose power was about 50 gigawatts. It made the traveling to Alpha Centauri b possible. However, in order to get there accurately, it had a high command on the precision of laser array, the margin of error in the fabrication of the light sail and the uniformity of the laser beam. As a result, we ought to control those influential factors before the launch.

2. Restatement and Analysis of Problems

Now there is a plan to send a light sail propelled Nanocraft to Alpha Centauri through a ground-based array of lasers. This plan is proposed that a few grams of ultralight spacecraft with a light sail area of approximately 10 square meters is accelerated under array of lasers whose power is 50 gigawatts, and its velocity can reach 0.2 times velocity of light within 10-20 minutes. This spacecraft's aim is to flyby Proxima Centauri b at closer than the Earth-Moon distance. It was asked to make quantitative analysis to the impact of three factors to this plan; the factors are the accuracy and precision of the beam from the laser array, the margin of error in the fabrication of the light sail and the uniformity of the laser beam.

This problem requires the spacecraft flyby Proxima Centauri b at closer than the Earth-Moon distance. The whole process can be divided into three stages. First stage, laser generated by laser array passes though the atmosphere and then perpendicular rays to the light sail which can produce light pressure and provide power. Second stage, the light sail accelerates under the light pressure. Third stage, the light sail fly to Proxima Centauri b at the speed of 0.2 times velocity of light. In the ideal case, light sail should fly along the lines connecting the direction of light sail and Proxima Centauri b. However, because of the laser array and light sail's technological limitation, there exits an angle between actual trajectory and virtual desired trajectory which can not exceed a certain range.

Therefore, finding out the relationship between drift angle and three factors becomes the key to solving the problem.

The necessary accuracy and precision of the beam from the laser array can be understood as the deviation of laser power, the margin of error in the fabrication of the light sail can be understood as uneven distribution of light sail mass, and the uniformity of the laser beam can be understood as laser position deviation.

In order to quantitative calculation of the light sail's velocity direction facilitately, it is needed to apply the discretization method and regard the plane as a dot matrix.

The deviation of laser power and laser position deviation will lead to the uneven distribution of pressure; uneven distribution of light sail mass will lead to the change of inertia tensor. If consider relativity, the mass of light sail will also change. All the factors make effect on drift angle.

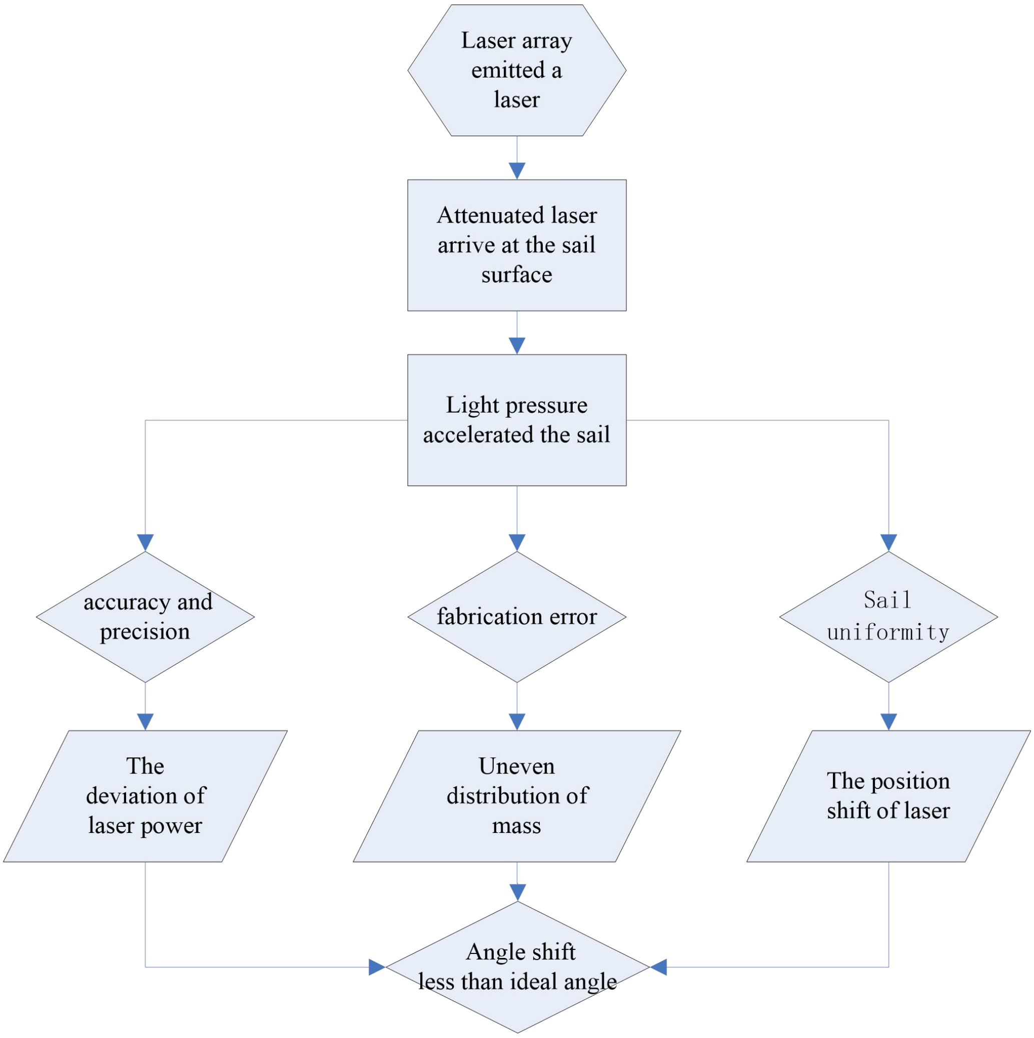

Figure 1 Mind map in the whole process

3. Assumptions and Notations

3.1. Assumptions

1. Light sail was a rigid plate for considering no deformation.
2. Light sail was launched from upper atmosphere and was not influenced by external factors in universe.
3. Due to the short moment of acceleration, the direction of laser that laser gun emitted laser in would not be changed by rotation of the Earth.
4. The relative distance between the position where sail was launched and the position of Proxima was constant.
5. The distribution of particulate matter in the vertical direction of the atmosphere did not change。
6. The influence on the area of sail caused by relativistic effect was overlooked.
7. The accuracy and precision of the beam from the laser array, the margin of error in the fabrication of the light sail and the uniformity of the laser beam were completely independent of one another and occur in random.

3.2. Notations

Table 1 Notations

Variables	Explanations
$\beta(h)$	dissipation coefficient when altitude is h (Mm^{-1})
P	light intensity(W)
I_C	Inertial tensor
$\vec{M}$	External torque(N • m)
$\alpha_x, \alpha_y, \alpha_z$	Drift angle(rad)
LD	The Moon's average distance to Earth(384403.9 km)
R_{orbit}	The orbital radius of Proxima Centauri b(7500000 km)
D_{pe}	The distance of earth to Proxima Centauri b(4.2 light year)

4. Model

4.1. Laser Transmission Model

The energy loss of the laser beam in the atmosphere is represented by the extinction coefficient [1]. The extinction coefficient is equal to the sum of the absorption coefficient and scattering coefficient. The formula is:

$$\mu = \alpha + \beta = \alpha + \beta_m + \beta_p$$

Where μ represents the extinction coefficient, α represents the absorption coefficient, β represents scattering coefficient, β_m represents molecular scattering coefficient and β_p represents particle scattering coefficient.

The scattering of suspended particles is the main cause of transmission loss, considering the attenuation of atmospheric energy transfer over a single vertical path.

$$\ln P = -\int (\beta_p + \beta_m) \mathrm{d}h$$

Where P represents light intensity and h represents height.

β_m and β_p decreases exponentially as height increases. The formula is:

$$\beta_p(h) = k_1 e^{-k_2 h}, \beta_p(h) = k_1 e^{-k_2 h}$$

Laser energy in the atmosphere after decay can be obtained through integrating.

$$\frac{P}{P_0} = \exp(-(k_1 + k_2)(1 - e^{k_2 h}) - (k_3 + k_4)(1 - e^{k_4 h}))$$

4.2. Acceleration Dynamics Model

The acceleration process of the sail was a three-dimensional dynamic model. In the case of error, the direction of the sail would produce a deflection with the ideal direction. The complete dynamic model should be established to get the angle calculation. In order to make the model clearer, we built basic model as below in ideal condition.

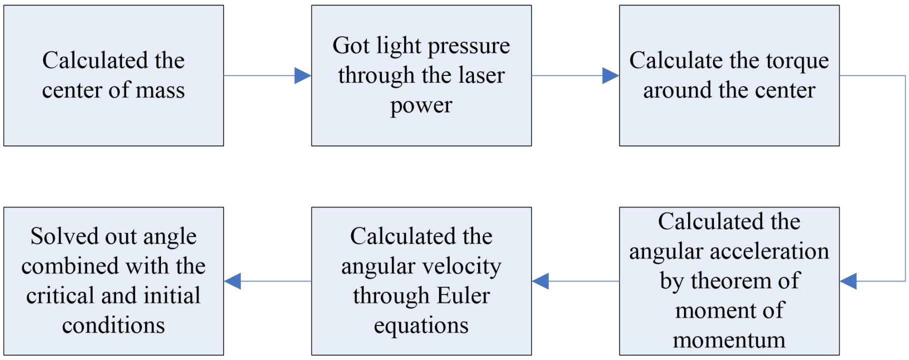

Figure 2 Mind map

4. 2. 1 Calculation of the Center of Mass and the inertia tensor

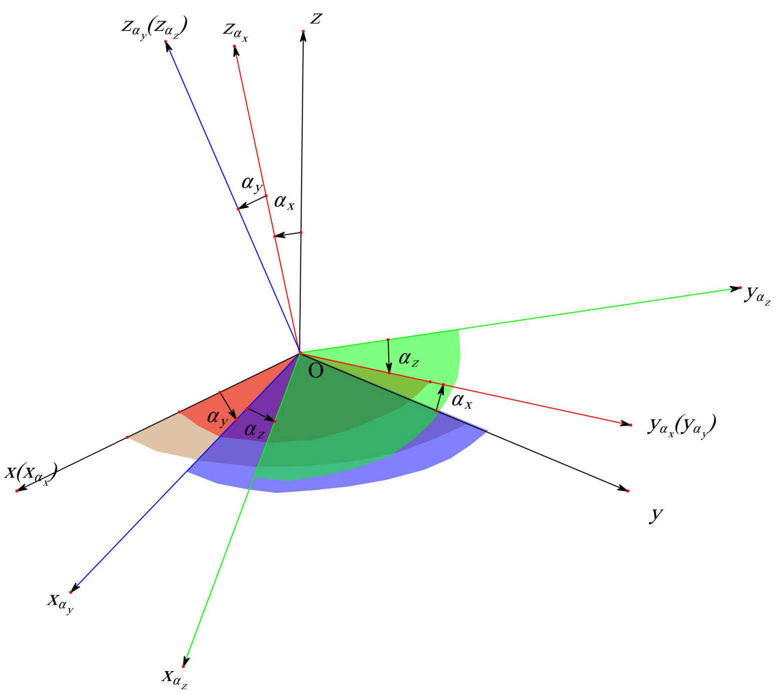

Figure 3 Euler transformation schematic diagram

In the figure, a fixed coordinate system $0xyz$ was established and the rotation of the sail was shown in different colors. Grey plate was the initial plane. Red plate was the plane that initial plane rotated an angle α_x around x axis. Blue plate was the plane that red plate rotated an angle α_y around y axis. Green plate was the plane that blue plate rotated an angle α_z around z axis.

The total mass of the light sail was described by m. Then the microelement analysis of the sail was done. As for each area element dA, its mass could be given by:

$$dm = \frac{m}{A} dA$$

The Lysol coordinate system was named $\xi 0 \eta$.

$$\xi_C = \frac{\sum m_i \xi_i}{m}, \eta_C = \frac{\sum m_i \eta_i}{m},$$

$$m_i = dm$$

The two coordinate systems were connected by Euler transformation.

The inertia tensor around the center of mass was I_C described as:

$$I_C = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix}$$

I_x was the moment of inertia of rotation about x axis; I_y was the moment of inertia of rotation about y axis; I_z was the moment of inertia of rotation about z axis. I_{xy}, I_{yz} and I_{xz} were the product of inertia.

Considering the effect of relativity on mass, mass was a value that varied over time:

$$m = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} m_0$$

4. 2. 2 Light Pressure Calculation Through Laser Energy

In the case where the laser energy at each element was known, the force of the element could be solved out. A strong laser irradiated light sail during the whole acceleration process. Then combined with the relativistic effect, the light pressure was shown as [2]:

$$F = \frac{2P}{c+v}$$

Think about what it did on an area element:

$$\mathrm{d}F_i = \frac{2P_i}{c+v}$$

Among which P was the average intensity without error and v was the magnitude of velocity.

4. 2. 3 **Obtaining the Angular Velocity by Euler Dynamics Equation**

The total torque can be obtained by integrating.

$$\mathrm{d}\vec{\mathrm{M}}_{\mathrm{i}} = \vec{r}_i \times \mathrm{d}\vec{F}_i, \vec{M} = \int \mathrm{d}\vec{M}_i = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Where $\vec{r}_i$ represents the direction vector from the center of mass to the operating point，

$\vec{M}_i$ represents every particle's torque on the center of mass, $\vec{M}$ represents the total torque.

According to the moment of momentum theorem:

$$I_C\ddot{\alpha} = M \ , \ddot{\alpha} = I_C^{-1}M = \begin{bmatrix} \ddot{\alpha}_x \\ \ddot{\alpha}_y \\ \ddot{\alpha}_z \end{bmatrix}$$

Where α represents angle of rotation.

According to the Euler kinetic equation

$$\begin{cases} I_x\ddot{\alpha}_x + (I_z - I_x)\dot{\alpha}_y\dot{\alpha}_z = M_x \\ I_y\ddot{\alpha}_y + (I_y - I_z)\dot{\alpha}_x\dot{\alpha}_z = M_y \\ I_z\ddot{\alpha}_z = M_z \end{cases}$$

$\dot{\alpha}$ can be obtained by solving the equations. The curve of the Angle changes with time can be obtained by integrating against t, combining with the initial conditions.

4.3. **Stability Analysis Model**

Since the distance between the target star and our transmitting point is incredible, a small error will lead to a large error in the final results. Then the laser beam accuracy model, fabrication model of the light sail and the laser beam uniformity model were established to evaluate the stability of the optical sail independently.

4. 3. 1 **The Laser Beam Accuracy Model**

The surface area element of the sail was analyzed accurately. It was assumed that there was a small disturbance in the laser intensity on each element which could be quantized as δP . And the intensity on each element was given by:

$$P_i = P + \delta P$$

Then this expression was substituted into the dynamics model above.

4. 3. 2 Fabrication Error Model of the Light Sail

When the sail was made, due to the manufacturing technique limit, the fabrication error was shown that the mass distribution was uneven. Assumed that there was a fabrication error at the area element:

$$dm_i = dm + \delta m$$

Such an error leads to an initial angle shift of light sail.

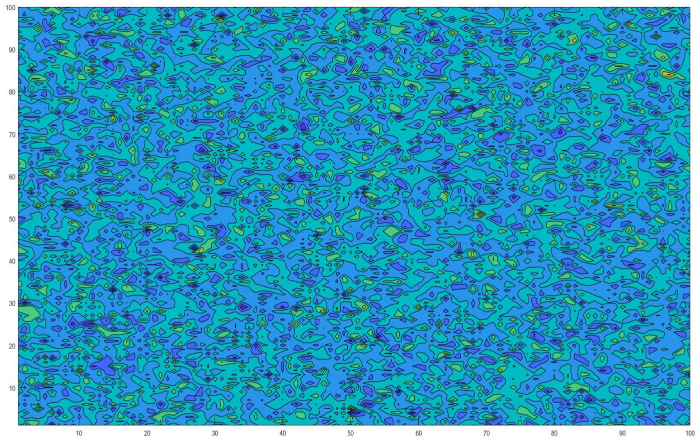

Figure 4 uneven distribution of mass

The darker the color, the greater the mass.

4. 3. 3 Laser Beam Uniformity Model

There may be an error of distribution during the manufacture of the laser array. In a word, it means uniformity. In the ideal situation, the uniformity is shown that the distance between each laser emitter is equal. If some laser had a position deviation described as δd , a torque was also generated. The torque's effect caused an angle shift of the light sail. And in such situation, the position of element could be given by:

$$x^2 + y^2 = \delta d^2$$

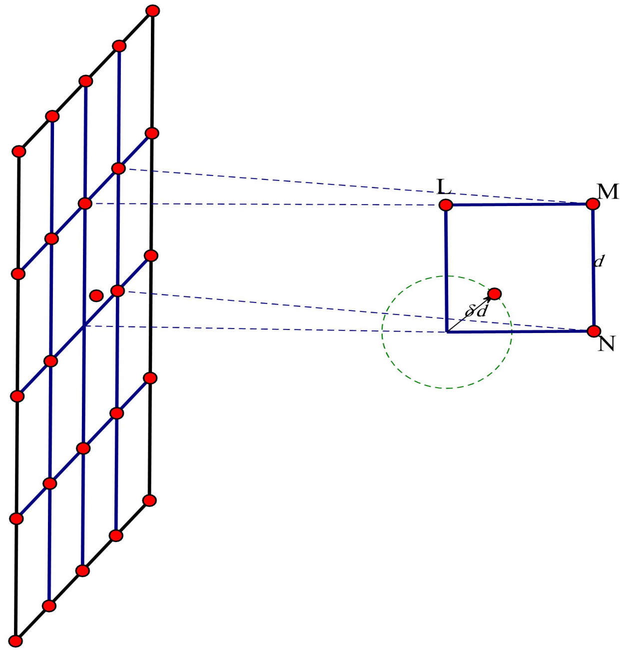

Figure 5 Schematic diagram of laser array position excursion

4.4. Track Model

4. 4. 1 Gravity Effect on the Path

As the sail approached the Proxima Centauri b， the track of the light sail would had a deviation because of the gravity from the Proxima Centauri and Proxima Centauri b, which made the sail easier to arrive in the distance closer than Earth-Moon distance. The effect of gravitation on track can be expressed by differential equations：

$$\ddot{\vec{r}}_1 = -\frac{\mu_p}{r_1^2}, \ddot{\vec{r}}_2 = -\frac{\mu_s}{r_2^2}$$

Among which $\vec{r}_1$ and $\vec{r}_2$ was respectively the distance between the Proxima Centauri b and sail and the between the Proxima Centauri and the sail. μ_p and μ_s was respectively the gravitation constant of the Proxima Centauri b and the Proxima Centauri.

The effect of gravitation on track could be calculated combined with the initial condition $\dot{\vec{r}}(0)$. $\dot{\vec{r}}(0)$ could be given by yaw angle.

Through the calculation, the effect of gravitation on track could be overlooked. Therefore, the track could still be regarded as a straight line when calculating the maximum allowable deviation angle.

4. 4. 2 Maximum Angles Deviation

Light sail typically travels along a straight line and arrive Centauri b, which requires a suitable

Angle between the sail and Centauri b. Actually, Centauri b is a planet revolving around the Proxima Centauri. The sail should approach the planet within a sphere with a radius of the moon. Since the planet orbits is a plane rather than a sphere and the deflection of the sail due to error is a solid angle. In order to maximize the probability of the sail skimming over Centauri b, it is assumed that the launch point has been selected when the solar sail is launched, which makes the solar sail plane parallel to the plane of revolution

There are two strategies of launching: flying toward the planet and flying towards the star. The two strategies obtain different maximum angles deviation.

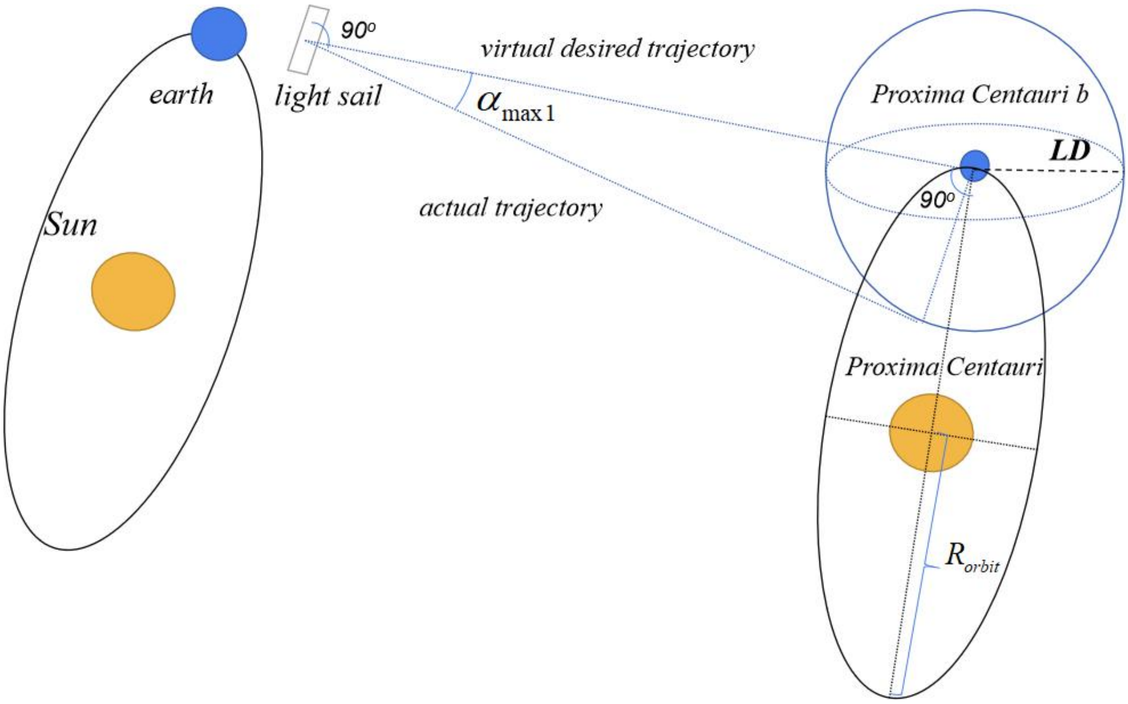

Figure 6 The first case of maximum permissible deflection angle

$$\alpha_{\max 1} = \arctan(\frac{LD}{D_{pe}})$$

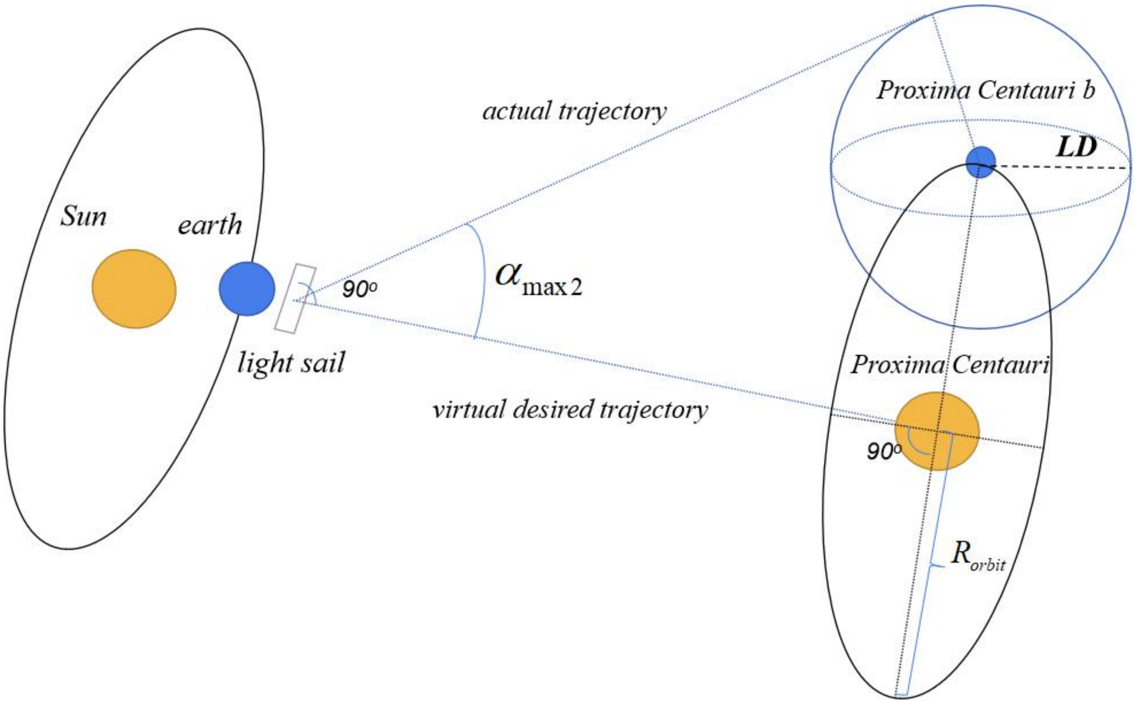

Figure 7 The second case of maximum permissible deflection angle

$$\alpha_{\max 2} = arc\sin(\frac{LD}{D_{pe}}) + arc\sin(\frac{R_{\text{orbit}}}{D_{pe}})$$

Considering the period of revolution of Alpha Centauri b $T_{plant} = 11.2\text{day}$,while the time it takes for the sail to reach the target $T_{sailing} = \dfrac{D_{pe}}{v_{sail}} \approx 21\text{year}$, $T_{sailing} >> T_{plant}$ .Considering the uncertainty of sail flight time, when the sail reaches Centauri, the exact position of Centauri b cannot be determined. Therefore, it may be assumed that the sail can be adjusted so that when the sail reaches its intended position, Centauri b will also revolve to the same position.Consequently, the second strategy should be applied.

5. Result and Discussion

5.1. Results Calculation

5. 1. 1 Calculation of Attenuation of Laser Intensity

1. Molecular scattering coefficient β_m

Based on our assumptions, the gas molecules are much smaller than the wavelength of light waves. So in the transmission process, there mainly existed Rayleigh scattering. And there is a linear relation between the molecular scattering coefficient and the density of air.

Molecular scattering coefficient could be given by:

$$\beta_m = \beta_0 \lambda \rho$$

Among which β_0 was the molecular scattering coefficient on the surface of the Earth.

Air density could be calculated according to a molar form of the ideal gas law:

$$\rho = \frac{pM}{RT}$$

Among which p could be given by:

$$p = p_0 (1 - \frac{Lh}{T_0})^{\frac{gM}{RL}}$$

According to the calculation above, when height was lager than 44330.76, the pressure could be regarded as 0.

L was temperature lapse rate. T_0 was the sea level standard atmospheric. g was the earth-surface gravitational acceleration. M was the molar mass of dry air. p_0 was the sea level standard atmospheric pressure. R was the ideal gas constant.

Table 2 parameters of air [3]

parameter	values
L (temperature lapse rate)	0.0065 K / m
T_0 (the sea level standard atmospheric)	288.15 K
g (the earth-surface gravitational acceleration)	9.80665 m / s^2
M (the molar mass of dry air)	0.0289644 kg / mol
p_0 (the sea level standard atmospheric pressure)	101.325 kPa
R (the ideal gas constant)	8.31447 $J / (mol \cdot K)$

After consulting data, β_0 was equal to 200.32 Mm^{-1}

Then $\beta_m = 200.32 e^{-0.00083h}\ Mm^{-1}$

2. Particle scattering coefficient β_p

Particle scattering coefficient was mainly decided by the aerosol molecular density. Based on our assumptions, the aerosol molecular density is approximately equal to the density of air. Thus particle scattering coefficient was approximately regarded as below:

$$\beta_p = 200.32e^{-0.00013h}$$

3. Light intensity calculation

The light intensity after decay could be solved out through integral method:

$$P = P_0 \exp(-(0.000200 + 0.00083)(1 - e^{0.00083h}) - (0.000200 + 0.00013)(1 - e^{0.00013h}))$$

Substituted $h = 44330.77$ into it and got intensity as below:

$$P = 0.723P_0 = 36.15 \times 10^9 W$$

5.1.2 Calculation of Sail's Angle Shift

The attenuated light intensity was used as the initial light intensity emitted to the sail. And under the consideration of the fluctuations of intensity, the intensity on each element could be described as below:

$$P_i = P + \delta P$$

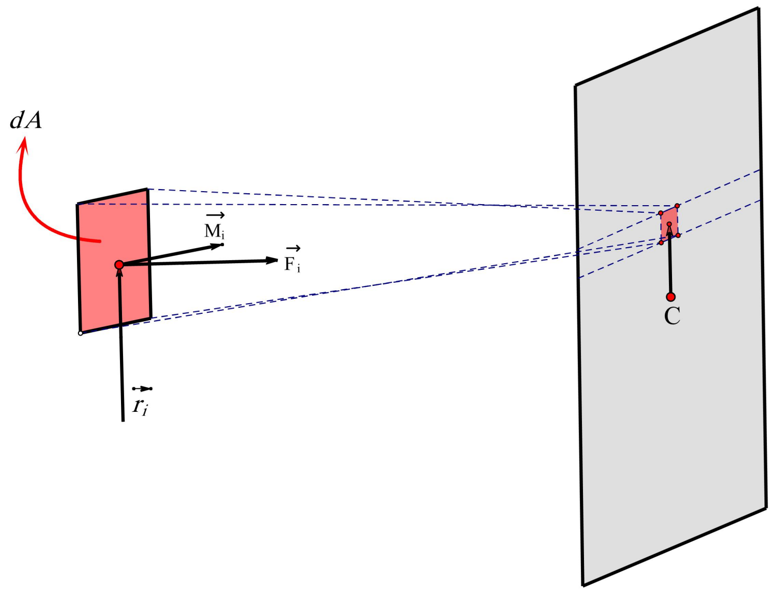

Figure 8 force analysis diagram on an element

It could be seen from the stress analysis diagram of the area element that $F_i = \dfrac{2(P + \delta P)}{c + v}$,

the initial force $F_i = \dfrac{2(P + \delta P)}{c}$.

The acceleration process could be viewed as a combination of translational and fixed-axis rotation of the plate. Translation was an accelerated process. And the change in speed caused a change in the force. Consequently the torque around the center of mass also variedwith the velocity; Small perturbations of laser beam uniformity were considered in the process which generated an extra torque due to the δd . In a sentence, the torque vector around the center of mass was a vector that varied with velocity and δd .

During the calculation of inertia tensor, the small perturbation of the mass was considered. At the same time, the average mass of the area element varied with the velocity because of relativistic effect. So the inertia tensor was a vector that varied with velocity and δm .

5. 1. 3 **Stability Calculation Based on Gradient Descent Method**

In the process of solving the above numerical simulation, The small perturbations of the mass, the intensity of the laser beam and the uniformity of the laser beam were random and did not affect each other. Thus they could all described by the Gaussian distribution under independent homogeneity as below:

$$\begin{cases} P_i \sim N(P_0(t), \delta P^2) \\ m_i \sim N(m_{0i}, \delta m^2) \\ d_i \sim N(0, \delta d^2) \end{cases}$$

Among which m_{0i} was the average mass without error; $P_0(t)$ was a random process that changed over time.

During the process of numerical simulation, we found that the angle of rotation and small disturbance had a monotonic positive correlation. Then we used the gradient descent method to simulate and analyze the relationship between small disturbances and small angle drift.

5.2. Analysis Results

According to the calculation result, the angle between the true direction of velocity and the ideal direction should not be greater than $\alpha_{\max} = 1.9842\times10^{-7} rad$. We match the measurement indexes of three influencing factors with the deviation angle generated. Then we obtain the graph of the variation of deviation angle with three factors. We take the logarithm of the factors to show the line tendency clearly.

5. 2. 1 **Calculation Results of the Effect of Laser Precision on Velocity Direction**

Laser precision is considered as laser intensity deviation, whose index is $\frac{\delta P}{P}$. The variation curve of $\Delta\alpha$ with $-\lg\frac{\delta P}{P}$ is shown in the figure:

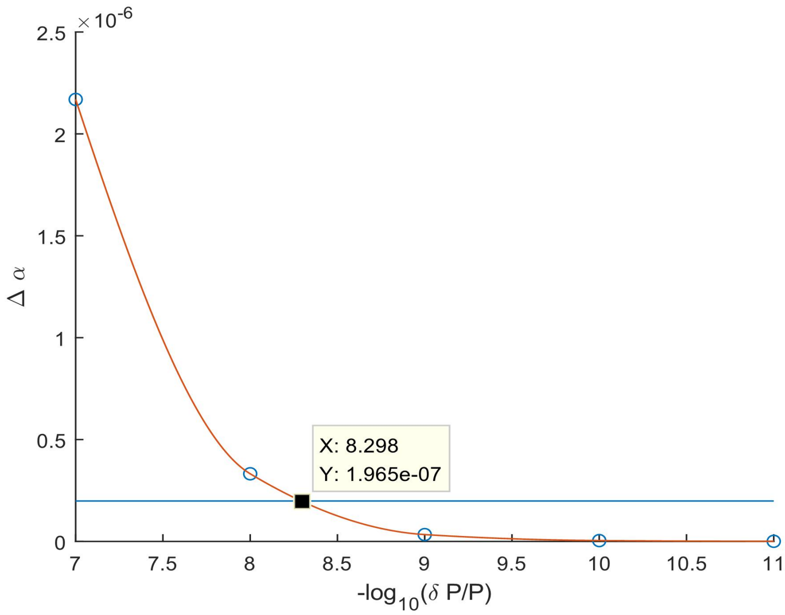

Figure 9 The effect of intensity deviation on angle drift

5.2.2 Calculation Results of the Effect of Sail Error on Velocity Direction

Sail error is considered as the nonuniform distribution of sail mass, whose index is δm . The variation curve of $\Delta\alpha$ with δm is shown in the figure:

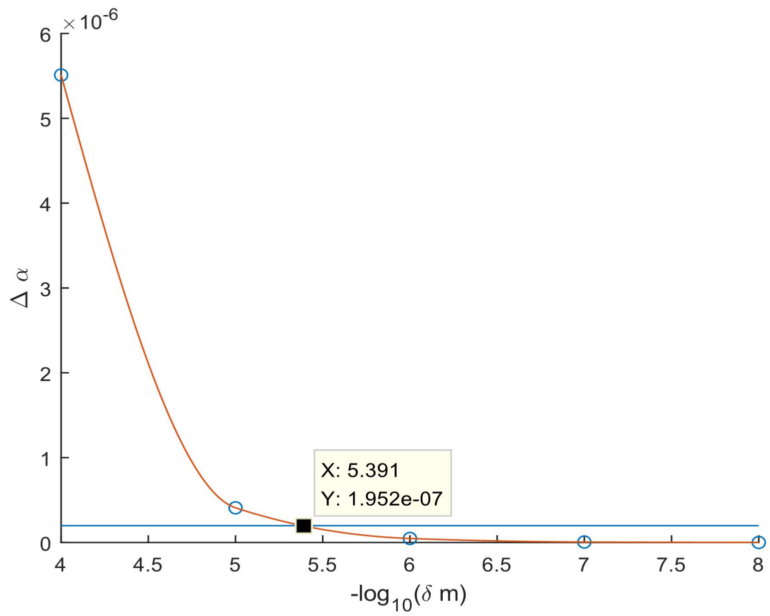

Figure 10 The effect of mass error on angle drift

5. 2. 3 Calculation Results of the Effect of Laser Beam Uniformity on Velocity Direction

Laser beam uniformity is considered as the nonuniform distribution of laser beam energy, whose index is. The variation curve of δd is shown in the figure.

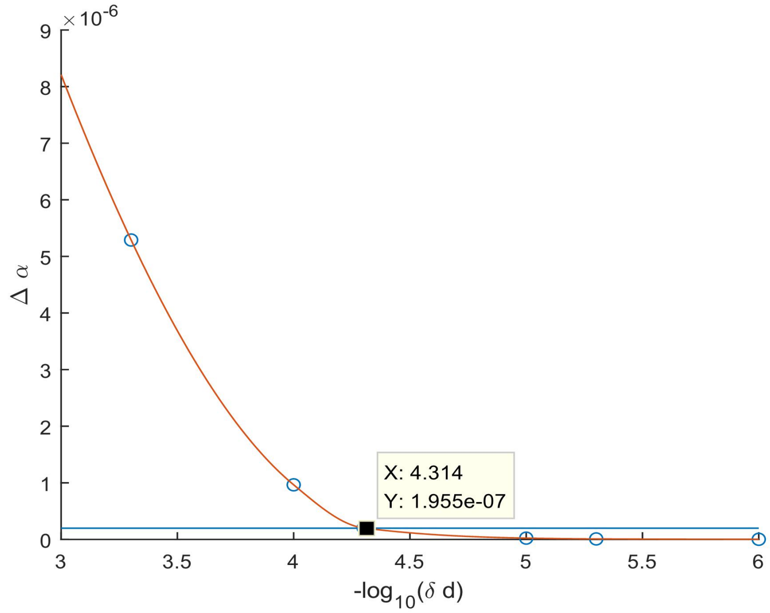

Figure 11 The effect of uniformity on angle drift

6. Sensitivity Analysis

In the previous solving process, we assumed that the size of the laser array $N*N$ is $100*100$. Now we analysis the sensitivity of $\Delta\alpha$ to N. N ranges from 10 to 100. We calculate the delta alpha once as soon as N increased by 5.

According to the former research, we know there are three factors influencing the delta alpha. So, we make three experiments where we only consider one of factors. Then we observe the changing of delta alpha in different N.

6.1. Experiment 1

We set δP as 100 w, analysis the sensitive of Δα to N, only considering error of P the uniform mass, neglecting the uniform distribution of m and d.

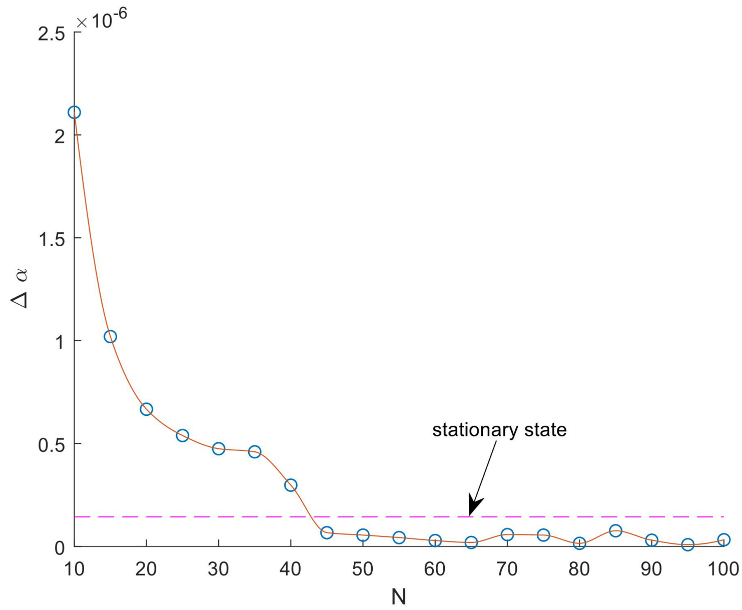

Figure 12 Sensitivity analysis of the number of microelements in intensity

The graph shows that when N<45, $\Delta\alpha$ falls sharply as N increases. So $\Delta\alpha$ is sensitive to N while N small. When N>45, $\Delta\alpha$ floats around 10^{-8}. So $\Delta\alpha$ is not sensitive to N when N >45.

6.2. Experiment 2

We set δm as 10^{-6} g, analysis the sensitive of $\Delta\alpha$ to N, only considering the uniform mass, neglecting the error of P and d.

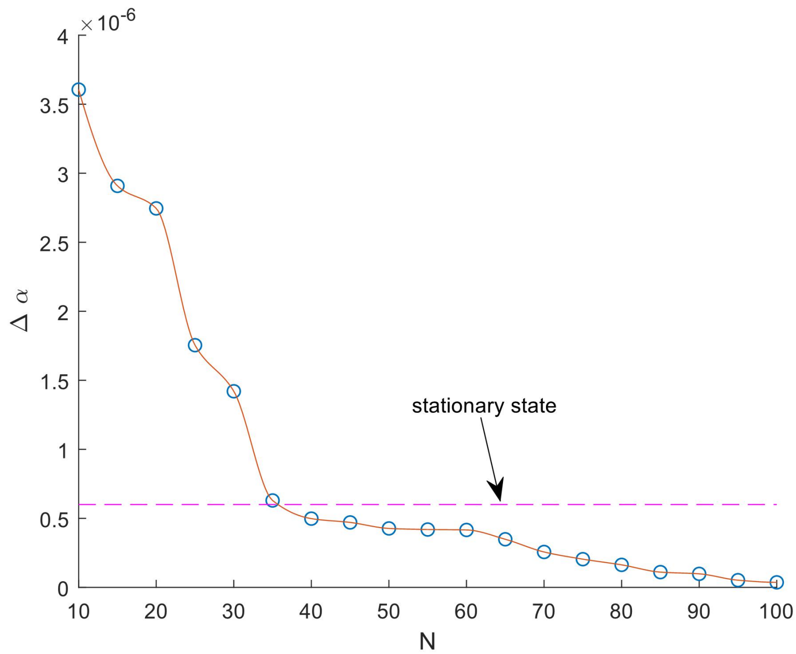

Figure 13 Sensitivity analysis of the number of microelements in mass

The graph shows that when N<35, $\Delta\alpha$ falls sharply as N increases. When N>35, $\Delta\alpha$ is falling slow. We regard it keep at the stationary state.

6.3. Experiment 3

We set δd as 10^{-7} m, analysis the sensitive of $\Delta\alpha$ to N, only considering the uniform distribution of d, neglecting the error of P and m.

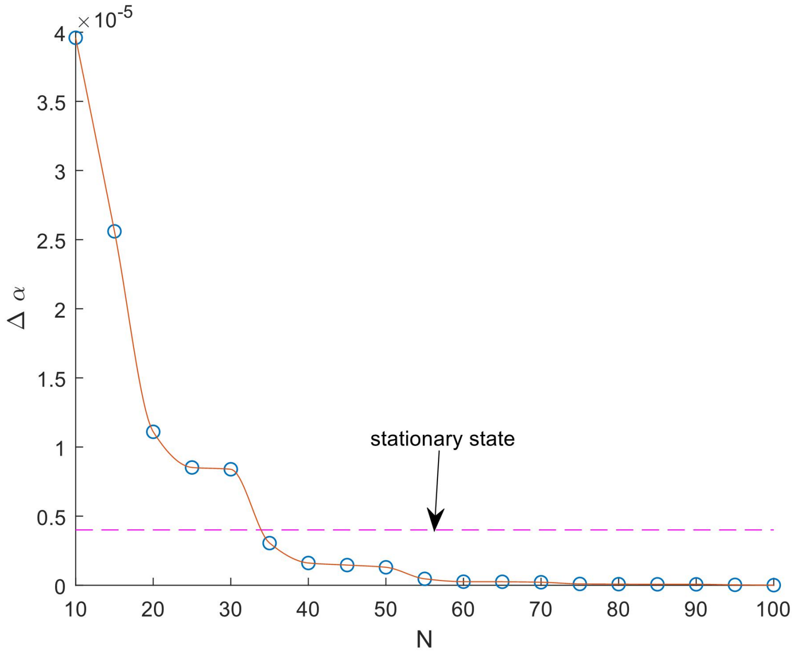

Figure 14 Sensitivity analysis of the number of microelements in position

The graph shows that when N<40, $\Delta\alpha$ falls sharply as N increases. $\Delta\alpha$ is sensitive to N while N small. When N>40, $\Delta\alpha$ changes little.

Above all, $\Delta\alpha$ is sensitive to N when N small according to the results of three experiments. And it also shows that the increase of N cannot help with the accuracy when N is larger than 50. Consequently, it is reasonable to assume N as 100 in the former model.

7. Model Improvement

7.1. Model Defects

In the previous model solving, only one factor is considered in every calculation. However these three factors work together to influence drift angle (α) in the real situation. So the results are the maximum errors of the three factors. At the same time, it is easier to achieve when its error is greater because of the technological limitation.

Now this problem has been transformed into optimization problems which can be calculated by algorithm particle swarm optimization.

7.2. Optimize model by Algorithm Particle Swarm Optimization

We take the precision of the energy, the precision of the mass processing, and the precision of the distance of the laser array as a vector of a particle that is moving in 3 dimensional spaces.

Table 3 Initialization of PSO

Position of Particle	X_i	$(\frac{\delta P}{P},\ \delta m,\ \delta d)$
Velocity of Particle	V_i	(V_{i1}, V_{i2}, V_{i3})
Best position of Particle	Pbesti	(P_{i1}, P_{i2}, P_{i3})
Best position of Particle group	Gbest	(G_{i1}, G_{i2}, G_{i3})

When finding these two optimal values, the particle updates Vi and Xi according to the following formulas:

$$X_i = X_i + V_i * t$$

$$V_i = V_i + (c1 * r1(Gbest - Pbest) + (c2 * r2 * (Pbest - X_i)))$$

Where: c1 and c2 acceleration constants, $r1, r2 \sim U(0,1)$

Fitness Function:

$$Fitness(X_i) = \begin{cases} (\lg(\frac{\delta P}{P}))^2 + (\lg(\delta m))^2 + (\lg(\delta d))^2 & \Delta\alpha < 1.9e-7 \\ 0 & \Delta\alpha > 1.9e-7 \end{cases}$$

solve the optimal solution by PSO

Step 1: Initialize the particle swarm, N=100, X_i and V_i are random numbers

Step 2:calculate $Fitness(X_i)$

Step 3:if $Fitness(X_i) > Pbesti$ $Pbesti = Fitness(X_i)$

Step 4: if $Fitness(X_i) > Gbesti \quad Gbesti = Fitness(X_i)$

Step 5: update V_i and X_i

Step 6: if $\Delta \text{Gbest} < 0.001$ break.

8. Strength and Weakness

8.1. Strength

The model did a successful work to reduce the computation， which only considered the influence of one factor and ignored another two factors.
The model used numerical method and quantized the laser beam precision, uneven sail mass and laser beam uniformity. In this way, we reduced computation and calculated the maximum error.

8.2. Weakness

Computation rose sharply as the increase of stimulation times. So our time step was a little big. However, stimulated performance is maybe different from the real process.
Laser beam transmission in the atmosphere is multi-directional, while only the vertical plane one-way attenuation was considered during the establishment. The complex composition of the atmosphere was greatly simplified in the construction of the attenuation model, which actually required further analysis.

9. Conclusion

We analyzed the spacecraft's dynamics during the acceleration after it was driven by the laser array. What we attached most energy to was the deflection angle of the light sail. According to our assumptions, there were three pivotal affecting factors, the accuracy of the beam, the margin of error in the fabrication of the light sail and the uniformity of the laser beam. The ideal situation was that the spacecraft flyby the Proxima Centauri b alone the line between the launch and the destination. However, with the torque caused by those factors, there existed an angle shift. Then in the process of analyzing the dynamics, we tried to calculated the deviation angle and discuss how those three factors influenced the angle.

In our result, the spacecraft was supposed to arrive in a sphere whose center was the Proxima Centauri b and radius was the Earth-Moon distance as *figure 7* showed. This sphere gave a solid angle. In other words, the angle shift caused by small perturbations should be limited to the solid angle or less. The closer, the better. We chose numerical simulation to get the relations between factors and angle shift. Finally, we calculated out the relative precision of the laser's power named

$\frac{\delta P}{P}$ was $10^{-8.306}$ or less, the margin error in the fabrication of the light sail δm was $10^{-5.391}$ and the uniformity of the laser beam shown as δd was $10^{-4.314}$. Then we were aware of the affect of the number of laser emitters and showed that in sensitivity analysis. As it was shown, different number of laser emitters led to different ability to resist the effects of three factors.

Finally, the model was improved by the way of overall consideration of three factors. Through PSO method, an optimal solution was searched out. And the optimal solution to the relative precision of the laser's power was $10^{-8.769}$. The optimal solution to the margin error in the fabrication of the light sail was $10^{-6.032}$. The optimal solution to the uniformity of the laser beam was $10^{-4.938}$.

10. Reference

[1] Li T Z, Research on energy attenuation of one-way ateospheric transmisson[J] Journal of Applied Optics, 1996(02):28-29
[2] Zhou P, Hou J, Liu Z J, Zhao Y J. Photon propulsion and its feasibility analysis[J]. Infrared and Laser Engineering, 2007(06):805-808
[3] unknow, "Density of air" https://en.wikipedia.org/wiki/Density_of_air Accessed: 2018-11-11

11. Appendix

```
clc;clear all;
N=100;
% m=ones(1,N*N);%
% bias_m=5E-5;
bias_m=0;
% bias_d=0.00000001;
bias_d=0;
m=0.005*ones(1,N*N)/(N*N)+bias_m*randn(1,N*N);
c=3*10^8;
bias=0.0001;
% bias=0;
fa_vector=[0;0;1];
apha=0;beta=0;gama=0;
```

```
theta=0;
R_X=[1,0,0;
    0,cos(apha),-sin(apha);
    0,sin(apha),cos(apha)];
R_Y=[cos(beta),0,sin(beta);
    0,1,0;
    -sin(beta),0,cos(beta)];
R_X_=[1,0,0;
    0,cos(-apha),-sin(-apha);
    0,sin(-apha),cos(-apha)];
R_Y_=[cos(-beta),0,sin(-beta);
    0,1,0;
    -sin(-beta),0,cos(-beta)];
R_Z=[cos(gama),-sin(gama),0;
    sin(gama),cos(gama),0;
    0,0,1];

P=zeros(N*N,3);
for i=1:N
    for j=1:N
        P((i-1)*N+j,:)=[j-0.5*(N+1),i-0.5*(N+1),0]+bias_d*randn(1,1);
    end
end
P=P';
for i=1:N*N
    P(:,i)=R_X_*R_Y*R_Z*R_Y_*R_X*P(:,i);
end
I_X=sum(m.*P(2,:).*P(2,:)+P(3,:).*P(3,:));
I_Y=sum(m.*P(1,:).*P(1,:)+P(3,:).*P(3,:));
I_Z=sum(m.*P(2,:).*P(2,:)+P(1,:).*P(1,:));
I_XY=sum(m.*P(1,:).*P(2,:));
I_YZ=sum(m.*P(2,:).*P(3,:));
I_XZ=sum(m.*P(1,:).*P(3,:));
III=[I_X,I_XY,I_XZ;I_XY,I_Y,I_YZ;I_XZ,I_YZ,I_Z];
I=5E10*ones(3,N*N)/(N*N);
I(1,:)=0;
I(2,:)=0;
% I(3,:)=100001;

M_X=sum(I(3,:).*P(2,:));
M_Y=sum(I(3,:).*P(1,:));
M_Z=sum(I(1,:).*P(3,:));
M=[M_X;M_Y;M_Z];
v_apha=0;
v_beta=0;
```

```
t=0;
v=0;
APHA=[0];
BETA=[0];
v_all=[0;0;0];
all=[0;0;0];
t0=0.5;
while(t<1200)
    t=t+t0;

    FF=(I(3,:)+bias*randn(1,N*N))./(c+v);
    v=v+t0*mean(FF./m);
    m=m/sqrt(1-(v/c)^2);
    M_X=sum(FF.*P(2,:));
    M_Y=sum(FF.*P(1,:));
    M_Z=0;
    M=[M_X;M_Y;M_Z];
    I_X=sum(m.*P(2,:).*P(2,:)+P(3,:).*P(3,:));
    I_Y=sum(m.*P(1,:).*P(1,:)+P(3,:).*P(3,:));
    I_Z=sum(m.*P(2,:).*P(2,:)+P(1,:).*P(1,:));
%       I_XY=sum(m.*P(1,:).*P(2,:));
%       I_YZ=sum(m.*P(2,:).*P(3,:));
%       I_XZ=sum(m.*P(1,:).*P(3,:));
%       III=[I_X,I_XY,I_XZ;I_XY,I_Y,I_YZ;I_XZ,I_YZ,I_Z];
%       all__=III\M;

    apha__=M_X/I_X;
    beta__=M_Y/I_Y;
%       v_all=v_all+0.1*all__;
    v_apha=0.1*v_apha+apha__*t0;
    v_beta=0.1*v_beta+beta__*t0;
%       all=all+v_all*0.1;
    apha=apha+v_apha*t0;
    beta=beta+v_beta*t0;
%       apha=all(1);
%       beta=all(2);
%       apha=apha+0.5*apha__*t0^2;
%       beta=beta+0.5*beta__*t0^2;

    APHA=[APHA,apha];
    BETA=[BETA,beta];
    R_X=[1,0,0;
        0,cos(apha),-sin(apha);
        0,sin(apha),cos(apha)];
```

```
    R_Y=[cos(beta),0,sin(beta);
        0,1,0;
        -sin(beta),0,cos(beta)];
    R_X_=[1,0,0;
        0,cos(-apha),-sin(-apha);
        0,sin(-apha),cos(-apha)];
    R_Y_=[cos(-beta),0,sin(-beta);
        0,1,0;
        -sin(-beta),0,cos(-beta)];
    R_Z=[cos(gama),-sin(gama),0;
        sin(gama),cos(gama),0;
        0,0,1];
    for i=1:N*N
        %           P(:,i)=R_X_*R_Y*R_Z*R_Y_*R_X*P(:,i);
        P(:,i)=R_X*R_Y*R_Z*P(:,i);
    end
    %      fa_vector_=R_X_*R_Y*R_Z*R_Y_*R_X*fa_vector;
    fa_vector_=R_X*R_Y*R_Z*fa_vector;
    theta=[theta,asin(norm(fa_vector_(1:2)))];
end
figure(1)
scatter3(P(1,:),P(2,:),P(3,:))
figure(2)
plot(APHA)
figure(3)
plot(BETA)
figure(4)
plot(theta)
```