Compost Pile Sizes

Team # 240
Problem B

Abstract

In this paper, we present a model for a compost pile made of kitchen waste. Our main objectives are finding the volume that maximizes the composting rate and analyzing its dependence on the ambient temperature and the input organic materials. A set of initials conditions and metabolic parameters are taken into account, which determine the evolution of the system during a period of 100 days. The system is described using differential

equations from Semenov's formulation and mass and energy conservation. In order to analyze the composting rate, the mass of reacted oxygen was used as a reference. The results were obtained using a numerical integration method and optimized by variation of the input volume and surface of the pile. Once the optimal value was determined, its dependence with the ambient temperature and input organic composition was analyzed. The simulations showed an optimal volume of $6.33m^3$ and $9.20m^3$ for a semi-sphere and

a four-sided truncated pyramid pile, respectively. These results indicate that the oxygen concentration is a priority over insulation in composting. Also, the optimal volume increases with the ambient temperature and decreases with the input biomass concentration. Finally,

we have compared the evolution of our system with results from other models and they shared a similar pattern. Thus, our model and optimal volumes are acceptable.

1 Background

It is hard to say for how long humans have used composting. Nowadays, this process is a solution to many problems. Generating composting from food waste is an easy, low-cost method to reduce residues and green-house effect. It also improves the quality of the land while it is transformed into fertilizer. In European countries, food waste is 45% of the municipal solid waste, and this percentage increases in developing countries. For this reasons, it is fundamental to know what affects the composting process to control it and improve its quality.

The composting process occurs when organic waste is decomposed by microorganisms under aerobic conditions. This combustion reaction principally generates carbon dioxide, steam and heat, and increases the internal temperature of the pile. In composting there are three main phases: mesophilic, termophilic and stationary.

Studying composting is a complex research, not only in biology but also in physics and chemistry, as the pile is heterogeneous and many living things are involved; like bacteria, fungi, protozoa, actinomycetes and rotifers. Moreover, food waste also contains inert materials such as plastic or glass due to the recollection method. The factors which affect composing are the C/N ratio, porosity, aeration, temperature, moisture and the metabolism of the microorganisms. The quality of the product depends on maturity and stability.

2 Introduction

The main objectives of this paper are: the optimization of the size and shape of a compost pile and the analysis of the dependence of optimal volume with the ambient temperature and the input organic materials. In order to determine them, we have developed a model for a compost pile made of kitchen vegetable based on Semenov's formulation.

The method of study consists of an analysis of the four main internal characteristics of the compost pile: temperature, oxygen content, moisture content and input organic materials, in dependence on the pile size. The pile has been assumed to have uniform internal temperature with an ambient temperature that ranges between 5 and 20 $°C$ in a 24-hour period. Due to the fact that all the four variables are related, we have initially focused on the temperature and oxygen content to optimize the pile volume as they are the primary sources of bacteria and thus key factors in the composting rate.

In order to characterize the composting rate for its optimization, we have taken the mass of oxygen transformed to carbon dioxide as a reference due to its relation with the consumed biomass and metabolic rate. As the composting reference is related to the internal temperature and oxygen composition, the model has been used to determine the evolution of these two variables for a period of 100 days. The simulations of the model with different sizes and shapes were performed to optimize the composting rate. Finally, we analyzed the effect of the ambient temperature and input organic materials in the composting rate using the most effective pile size.

3 Theory

3.1 Chemical Kinetics

In composting, the main chemical reaction is an exothermic combustion carried out by the microorganisms of the compost.

$$B + O_2 \longrightarrow CO_2 + H_2O + \Delta \tag{1}$$

(Biomass) (Heat)
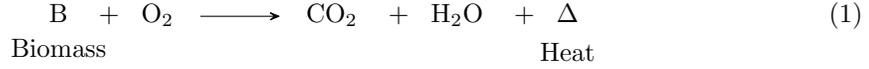
According to Chemical Kinetics, the reaction rate of this combustion can be experimentally expressed using a second order approximation as the product of the kinetic constant $k(T)$ and internal concentrations of both reagents,

$$v = k(T)[B][O_2] \tag{2}$$

as the reaction takes place inside the pile. The kinetic constant has been observed to follow a pseudo experimental Arrhenius equation given by

$$k(T) = \frac{A_1 e^{-\frac{E_1}{RT}}}{1 + A_2 e^{-\frac{E_2}{RT}}} \tag{3}$$

where T is the internal temperature, R the ideal gas constant, A_1 the pre-exponential factor for oxidation of biomass growth, A_2 the pre-exponential factor for oxidation of cellulosic material, E_1 the activation energy for biomass growth and E_2 the activation energy for inhibition of biomass growth.

The reaction rate describes the variation of the internal concentration of the reactants over time. Thus, the consumption of oxygen and the generation of steam by the reaction, $[O_2]_r$ and $[H_2O]_r$ respectively, results in

$$-\frac{d[O_2]_r}{dt}, \frac{d[H_2O]_r}{dt} \propto \frac{A_1 e^{-\frac{E_1}{RT}}}{1 + A_2 e^{-\frac{E_2}{RT}}}[B][O_2] \tag{4}$$

It is important to remark that the internal concentration of oxygen $[O_2]$ is not the initial concentration of oxygen less the consumed by the reaction $[O_2]_r$, because there is an exchange of oxygen between the pile and the atmosphere.

3.2 Composting

The composting C is the result of the metabolism of the microorganisms in the pile: the higher the metabolism, the better the compost. Also, the metabolism can be described as the consumption of biomass or oxygen. In consequence, the composting is proportional to the concentration of reacted oxygen inside the pile, as

$$C \propto m_{O2,r} = V_T[O_2]_r \tag{5}$$

where V_T is the total volume of the pile which is approximately constant.

Using equation (4), the composting will be determined with the expression

$$C \propto \int_0^{\Delta t} dt\, k(T)[O_2][B] \,. \tag{6}$$

3.3   Balances of Matter and Energy

The pile of compost is an open system so there are exchanges of matter and energy with the environment. The balances of matter and energy are used to determine the concentration of each component and the temperature, in order to determine the composting rate.
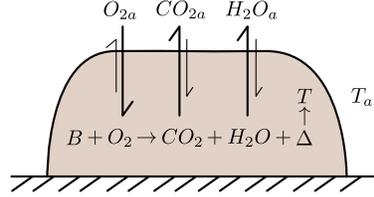
Figure 1: Scheme of the dynamic exchange of matter of the pile

3.3.1   Matter

The balance of matter for O_2 is

$$[O_2] = [O_2]_0 - [O_2]_r + [O_2]_e - [O_2]_s \tag{7}$$

where $[O_2]_0$ is the initial concentration, $[O_2]_r$ is the concentration that has already reacted, $[O_2]_e$ is the concentration that enters in the pile and $[O_2]_s$ the concentration that goes out of the pile.

Using the equation (4) deduced in subsection 3.1, the expression (7) derives to

$$\frac{d[O_2]}{dt} = -F_b k(T)[B][O_2] + \frac{d([O_2]_e - [O_2]_s)}{dt} \tag{8}$$

where the adimensional coefficient F_b appears to take into account the relationship between the amount of biomass consumed and the oxygen consumed, acting as a stoichiometric ratio.

To study the term of entrance and exit of oxygen, it is necessary to know how the air circulates through the pile. It is assumed that the air circulates through the pores in the pile. The volume of each of the pores V_P decreases if the total volume V increases as the pile becomes more compact. However, the number of pores n_P also increases, so the relation (9) can be approximated as a constant.

$$K_P = V_P/n_P \tag{9}$$

The mass transfer coefficient X_m can be descrived as the total air that enters the pile per unit of volume,

$$X_m = \frac{K_P v_{air}}{V} \tag{10}$$

where v_{air} is the speed of the air that passes through the pile. This definition is coherent as the mass transfer coefficient will increase with v_{air} and will decrease with V because if a pile is bigger it will have a smaller volume of pores.

From equation (10) we can express the mass transfer based on the difference of oxygen concentrations between the air and the pile, the X_m and the V/S ratio of the pile's shape.

$$\frac{V}{S} \frac{d([O_2]_e - [O_2]_s)}{dt} = X_m([O_2]_a - [O_2]) \tag{11}$$

Finally, the balance of matter can be expressed as the differential equation,

$$\frac{d[O_2]}{dt} = -F_b k(T)[B][O_2] + \frac{X_m S}{V}([O_2]_a - [O_2]) \tag{12}$$

The differential equation (12) is the first of the two differential equations that Semenov's mathematical model contemplates.

Regarding the other components, the balance of matter is equivalent, but in the water and the carbon dioxide the term of reaction is positive and in the biomass the terms of entry and exit are zero.

3.3.2 Energy

The energy balance of the system leads to the second Semenov's equation [3]. Firstly, the pile is considered to have a uniform internal temperature because the cold air going through the pores mantain the pile in the same temperature. Therefore, the heat diffusion is equal in any point of the surface that is in contact with the air.

The thermal energy exchange (13) can be understood in the form of heat $Q\frac{dT}{dt}$. The temperature variation in the interior will depend on the enthalpy ΔH of the reaction and its speed, equation (2), since the enthalpy does not include the temporary factor. It also depends on the loss of temperature due to the contact with the environment, which is proportional with the coefficient of heat transfer X_t, the surface-volume ratio of the system V/S and the ambient temperature T_a.

$$Q\frac{dT}{dt} = \Delta H k(T)[B][O_2] - X_t \frac{S}{V}(T - T_a) \tag{13}$$

Reinterpreting Q as the product of the total density ρ for the heat capacity C_V and the enthalpy redefined as Q_b for the stoichiometry ratio F_b, the Semenov's second equation ends being

$$\rho C_V \frac{dT}{dt} = Q_b F_b k(T)[B][O_2] - X_t \frac{S}{V}(T - T_a) \ . \tag{14}$$

4 Method

4.1 General Assumptions

Due to the characteristics of our system, the following assumptions can be made:

- The compost pile has a uniform internal temperature.

- The heat transfer to the surroundings is convective and linked with the temperature difference only by the heat transfer coefficient, X_t.

- The concentration of oxygen inside the pile is uniform.

- Oxygen transfer from the surroundings is convective and linked to the concentration difference only by the mass transfer coefficient, X_m.

- The climate follows a range of temperatures between 5 and 20 degrees C in a 24-hour period, which can be described as a sinusoidal function.

$$T_a[^\circ C] = 12.5 + 7.5 \sin\left(2\pi \ t[days]\right) \tag{15}$$

4.2 Model

Our model is based on the differential equations of the component's concentrations and temperature inside the pile. The components we have analyzed are oxygen, biomass and steam as they are key factors in the metabolism of the microorganisms. Unlike the Semenov's formulation, we have taken into account that the biomass diminishes as we are considering periods of time of 100 days.

The procedure to determine the composting rate of the pile starts setting up the initial conditions: pile's shape S, pile's size V, concentration of biomass $[B]_0$, internal concentration of water $[H_2O]_0$, internal concentration of oxygen $[O_2]_0$ and internal temperature T_0. Two different forms of pile have been tested: semisphere and four-sided truncated pyramid.

The system is described with five differential equations,

$$\rho C_V \frac{dT}{dt} = Q_b F_b k(T)[B][O_2] - X_t \frac{S}{V}(T - T_a) \,. \tag{16}$$

$$\frac{d[O_2]}{dt} = -F_b k(T)[B][O_2] + \frac{X_m S}{V}([O_2]_a - [O_2]) \tag{17}$$

$$\frac{d[H_2O]}{dt} = +F_b k(T)[B][O_2] + \frac{X_m S}{V}([H_2O]_a - [H_2O]) \tag{18}$$

$$\frac{d[B]}{dt} = -F_b' k(T)[B][O_2] \tag{19}$$

$$C = \alpha \int_0^{\Delta t} dt \, k(T)[O_2][B] \,. \tag{20}$$

where α is only a constant of proportionality between the composting rate and mass of reacted oxygen.

4.3 Simulations

The differential equations of the model were solved by numerical integration using $Python$ 3.5 obtaining the evolution of the temperature and concentrations of the components, as well as the composting rate versus the pile's volume.

$T(t)$ and $[H_2O](t)$ are the control for the model as they should be between 40-60 degrees C and 50-60% respectively. These variables are the first validation of the model.

$C(V)$ has been normalized as its value depends on the proportional parameter α that it is not known. The optimization has been done by a semisphere and by four-sided truncated pyramid as pile's form.

Once this volume is obtained, the T_a parameter is changed to observe its relation with the optimal volume. The same procedure is repeated for the density ρ and initial concentration of biomass $[B]_0$.

5 Results

5.1 k(T) vs Temperature
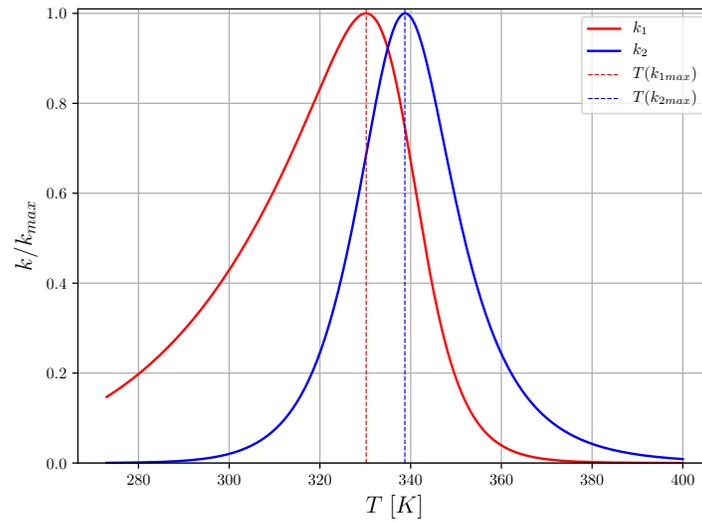
Figure 2: Checking the model coherence

5.2 Temperature vs Time
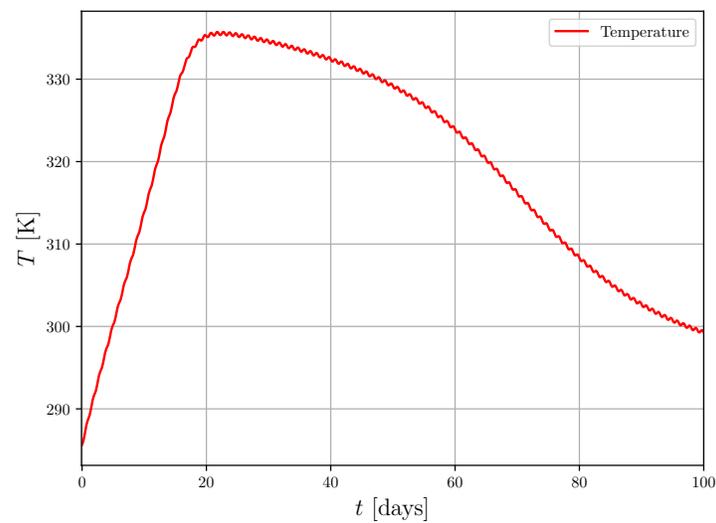
Figure 3: Evolution of internal temperature through time

5.3 $[H_2O]$ vs Time
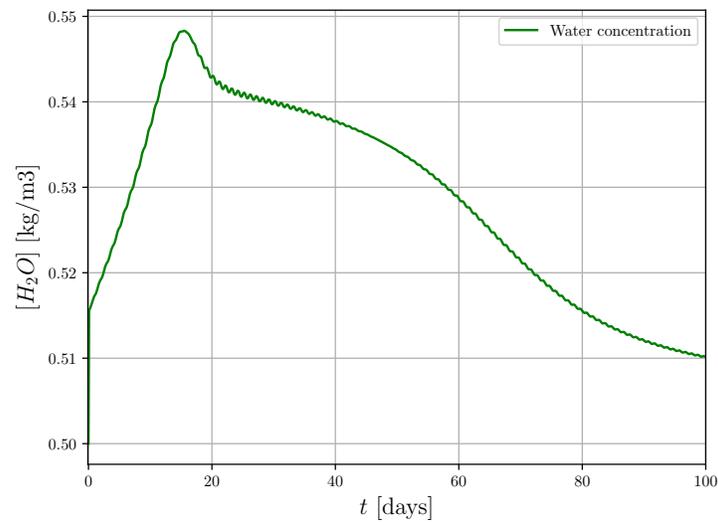
Figure 4: Evolution of internal water concentration through time

5.4 $[B]$ vs Time
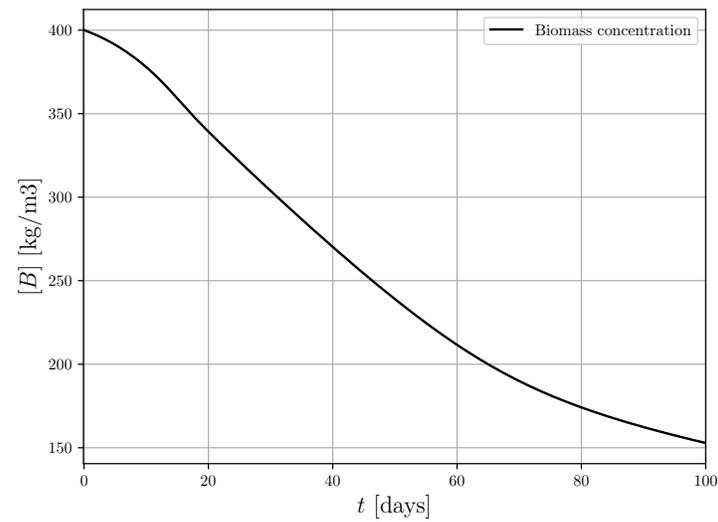
Figure 5: Evolution of organic matter concentration through time

5.5 $[O_2]$ vs Time
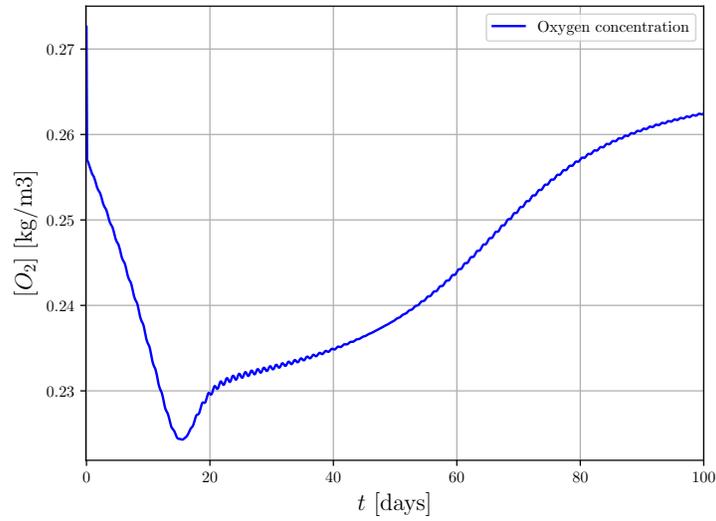
Figure 6: Evolution of internal oxygen concentration through time

5.6 Optimization
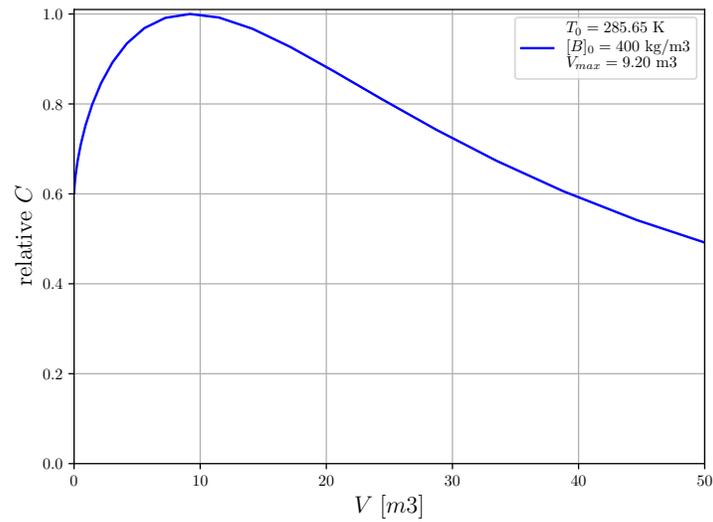
Figure 7: Optimization of the volume of a semispherical pile to maximize composting
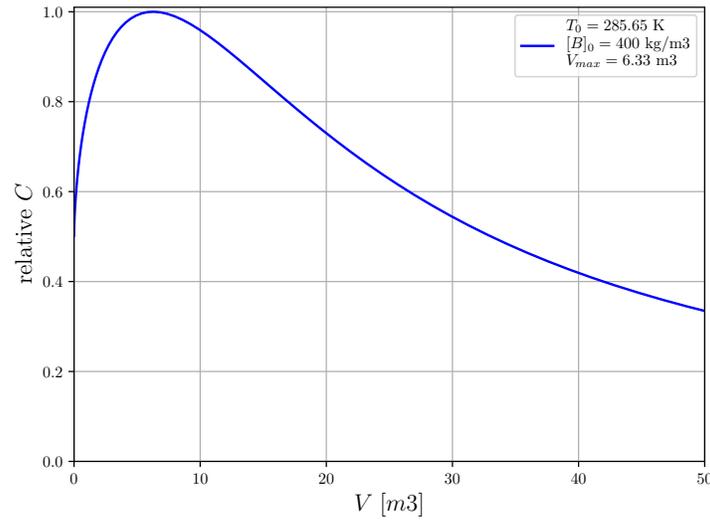
Figure 8: Optimization of the volume of a four-sided truncated pyramid pile to maximize composting

5.7 Relation between volume, composting, ambient temperature and input organic materials

Table 1: Relation of composting (of the optimal volume) between [B]

C	$[B](kg/m^3)$
0.9	50
3.2	100
2.9	200
2.7	400
2.5	600
2.4	800
2.3	1000

Table 2: Relation between composting (of the optimal volume) and ambient temperature

C	$T_a(K)$
2.5	273
2.6	281
2.7	287
2.8	294
2.9	299
3.2	310
3.5	325
3.7	333

6   Discussion

Value of the constants

Before optimizing the volume of pile, the values of the constants were calculated to check the coherence with the data from the problem. It has been found that the factors regarding the kinetic constant where not suited for this problem as the efficient temperature given from these factors was between $80 - 100°C$. The kinetic was reescalated to the $40 - 60°C$ provided by the problem. This solution was also done in [4].

Model

The results from the model are coherent with the predicted concentrations and temperatures. Moreover, it has been compared with results from other articles and they showed the same tendency and shape [2].

The graph T vs t showed three different phases: (1) 0 - 20 days, the temperature increases due to the activity of the microorganisms to the optimal range, (2) 20 - 60 days, the temperature slowly decreases but is still in the optimal range, (3) 60 - 100 days, there not enough biomass the keep the temperature in the optimal range and it decreases until it reaches the ambient temperature.

This phases are also observed in the oxygen and water concentration graph but not in the biomass graph as it is only consumed and it is not replaced by new one.

Optimal volume

Two shapes have been considered in this model: semi-sphere and four-sided truncated pyramid. Its optimal volumes are 9.2 and $6.32m^3$ respectively. These values are relatively small and thus the approximation of the uniformity of internal temperature has a small error in the results.

The maximum composting rate of the pyramid is a little bit greater than the semi-sphere, so for this order of volumes it is more important the oxygen flux through the pile than the isolation from the outside temperature.

Depence of the T_a and $[B]$ in optimal volume

The variation of both factors in the optimal volume has been analyzed separately.

An increase of the outside temperature showed a decrease of the optimal volume. This relation is due to the difference of the temperature from inside and outside is smaller and the isolation factor is less important than the oxygen income. For this reason, smaller volumes have better oxygen income that greater volumes but the isolation factor remains the same.

Moreover, from the internal temperature graph it is shown that the internal temperature has small sinusoidal oscillations that do not affect the overall variation. The model has been also calculated using a constant outside temperature as input and it showed the same pattern. The dependence of the biomass concentration in the optimal value has an unexpected relation as the composting rate is higher with smaller concentrations of biomass. This relation could be explained by the fact that the rest of the pile is better insulating than the biomass and it keeps higher temperatures inside, optimizing the composting rate. This leads to a higher mass of consumed oxygen per biomass mass, increasing our reference for the composting rate.

7   Future Work

For a more reliable prediction it would be necessary to build a 3-D composting model.

In [4], a 3-D compost model of a truncated cone geometry was built to find a prediction of the temperature in composting processes. The energy balance was applied to the system solid-air with an equation that includes volumetric heat capacity (C_v), chemical oxidation and biological growing (A_1,E_1) and inhibition (A_2,E_2).

The equations used are similar to those used in 2-D. The 3-D model equation (21) used is obtained from a Fourier heat transfer equation. It is similar to the equations presented above (24) if we add a third derivative direction for T (in the equations we used the variation of temperature is shown only in 2 directions).

$$(\rho C)_{ef}\frac{\delta T}{\delta t} = k_{ef}\,\bigtriangledown^2 T + (1-\epsilon)Q_C\rho_C A_C exp[\frac{-E_c}{RT}] + (1-\epsilon Q_b\rho_b)\frac{A_1 exp[\frac{-E_1}{RT}]}{1+A_2 exp[\frac{-E_2}{RT}]} \qquad (21)$$

With:

$$k_{ef} = \epsilon k_a + (1-\epsilon)k_c \qquad (22)$$

$$(\rho C)_{ef} = \epsilon\rho_a C_a + (1-\epsilon)\rho_C C_C \qquad (23)$$

$$\rho c_v V\frac{dT}{dt} = Q_b V F_b \frac{A_1 e^{\frac{-E_1}{RT}}}{1+A_2 e^{\frac{-E_2}{RT}}}BO_2 + X_m S(O_{2,a} - O_2) \qquad (24)$$

The comparison between theoretical parameters and the experimental results conduces to a correction of the theoretical values. This is due to not considering some conditions such as pH or a radiation term for heat absorption in high sunlight conditions.

8 Conclusions

This study shows that, in a pile of kitchen waste, the oxygenation of the pile is a priority over its insulation in the maximization of the composting rate. For this reason the optimal volumes that boost the composting are relatively small.

The evolution of component's concentrations and internal temperature have been determined in order to calculate the metabolism rate in dependence of the pile's volume. The results of these variables have been compared with the results from other models and they all had a similar pattern. In the end, the simulations showed that the optimal volume for a semi-sphere and four-sided truncated pyramid are 9.2 and 6.32 m^3 respectively.

The volume determines both oxygenation of the pile and insulation. An increase of volume leads to a more concentrated pile and the air cannot flow so easily through the pores of the heap, resulting in a decrease of the inside concentration of oxygen as it is consumed by the microorganisms. A decrease of the volume diminishes the insulation as the inside temperature is more susceptible of outside changes due to its lower heat capacity.

On the other hand, the volume depends on the ambient temperature and biomass concentration. Increasing the ambient temperature decreases the optimal pile size, due to the smaller difference of the temperature from inside and outside and so the isolation factor is less important than the oxygen income. On the contrary, the optimal pile size has a minimal value in function of the density of the biomass, which is an unexpected non-intuitive result. It could be explained by the fact that the rest of the pile is better insulating than the biomass and it keeps higher temperatures inside, optimizing the composting rate.

To sum up, the model described in this paper is coherent with other models and the obtained results are acceptable and predictable with the used equations. Nevertheless, this model can be improved assuming that the internal temperature is not uniform and further work is needed.

9   References

[1]Luangwilai, T.; Sidhu, H. S.; Nelson, M. I. Food Bioprod. Process. 2018, 108, 18–26.

[2] Moraga, N. O.; Corvalán, F.; Escudey, M.; Arias, A.; Zambra, C. E. Int. J. Heat Mass Transf. 2009, 52 (25–26), 5841–5848.

[3] Luangwilai, T.; Nelson, M.; Chen, X. D. 2010, No. December 2014.

[4]Teutli; Roque; Gonzalez; Jiménez; Lozano; Peláez. Heat Flux Predictions for a 3-D Compost Model.

10   Appendix

10.1   DATA USED

[4]

Datum	Meaning	Magnitude	Units
ρ	density of the composting pile	547	$\frac{Kg}{m^3}$
C_v	Heat capacity of the reaction mixture	$3.32 \cdot 10^3$	$\frac{J}{Kg \cdot K}$
V	Volume of the composting pile	Variable	m^3
S	Surface of the composting pile in contact with air	Variable	m^2
Q_b	Exothermicity of the biomass grown reaction	$1.32 \cdot 10^4$	$\frac{J}{Kg}$
F_b	Stoichiometry ratio	0.02	dimensionless
$[B]$	Organic matter concentration in the composting pile	Variable	$\frac{Kg}{m^3}$
R	Ideal gas constant	8.31441	$\frac{J}{mol \cdot K}$
A_1	pre-exponential factor for biomass growth	$2 \cdot 10^2$	$\frac{m^3}{Kg \cdot s}$
A_2	pre-exponential factor for the inhibition of the biomass growth	$6.86 \cdot 10^{30}$	dimensionless
E_1	activation energy for biomass growth	$2.71 \cdot 10^4$	$\frac{J}{mol}$
E_2	activation energy for the inhibition of the biomass growth	$2 \cdot 10^5$	$\frac{J}{mol}$
X_m	mass transfer coefficient between the compost heap and its surroundings	0.1	$\frac{m}{s}$
X_t	heat transfer coefficient between the compost heap and its surroundings	50	$\frac{J}{s \cdot m^2 \cdot K}$
T_a	ambient temperature	Variable	K
$[O_2]_a$	ambient oxygen concentration	0.2726	$\frac{Kg}{m^3}$

```python
import numpy as np
from scipy.integrate import odeint

#INPUT DATA
Rini = 1E-1 #m
Rfin = 1E1 #m
NR = 100
VSR = 1./3
Tai = 12.5 + 273.15 #K
Bi = 400 #kg/m3
H2Oa = 0.6

#PARAMETERS
p = 547. #kg/m3
Cv = 3E3 #J/kg K
Qb = 1.32E4 #J/kg
R = 8.31441 #J/mol K
Xt = 50. #J/s m2 K
Xm = 0.1 #m/s
O2a = 0.2726 #kg/m3
ti = 0. #s
tf = 24*60*60*100. #s
N = 1001
F1 = 0.02
F2 = 0.02
F3 = 1./15000
F4 = 0.02
t = np.linspace(ti, tf, N)

def kT(T):
    A1 = 2E8 #m3/kg s
    A2 = 6.86E30
    E2 = 2E5 #J/mol
    return 1E-6*A1*np.exp(-E2*0.13529/(R*T))/(1 + A2*np.exp(-E2/(R*T)))

#DIFERENTIAL EQUATIONS
def pend(y, t):
    O2, H2O, B, T = y
    Ta = Tai - 7.5*np.cos(t*2*np.pi/(3600*24))
    kt = kT(T)
    dydt = [ -F1*kt*O2*B + A*(O2a - O2),
          +F2*kt*O2*B + A*(H2Oa - H2O),
           -F3*kt*O2*B,
           BB*F4*kt*O2*B - C*(T-Ta)]
    return dydt

#OPTIMIZING
dataO2rK = []
dataO2rD = []
dataR = np.linspace(Rini, Rfin, NR)
dataO2 = []
```

```python
#SEMI-SPHERICAL PILE
for Ri in dataR:
    V = 2./3*np.pi*Ri**3
    VS = VSR * Ri
    S = V/VS #m2
    Xm = 20./V #m/s
    A = Xm*S/V
    BB = Qb/(Cv*p)
    C = Xt*S/(V*Cv*p)

    #INITIAL CONDITIONS
    y0 = [O2a, H2Oa, Bi, Tai]
    t = np.linspace(ti, tf, N)
    #DIF EQ SOLVER
    sol = odeint(pend, y0, t)

    O2 = sol[:,0]
    H2O = sol[:,1]
    B = sol[:,2]
    T = sol[:,3]

    #O2reactive
    dataO2 += [O2[-1]]
    dO2r = kT(T)*O2*B
    dataO2rK += [((t[1]-t[0])*dO2r)[-1]]
```