Problem B

High-efficiency Cylindrical Compost Pile Size Design

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Abstract

This paper mainly considers the effect of physical effects on composting efficiency.

First, we need to strip out the issues that need to be considered. Since a series of chemical reactions and biological changes in the composting process are independent of the physical model, only the effects of temperature and oxygen content can be considered. Further abstraction corresponds to the **active heat transfer process** and the **oxygen flow process**. For the active heat conduction process, the law of conservation of heat and Fourier's law can be used to establish the differential equation; for the oxygen circulation process, since it follows the law of heat transfer, it can be similarly solved.

In order to facilitate the analysis, we determined the shape of the compost pile as a common and regular cylinder, which was cut into a fan slice by symmetrical cutting, and then the area micro-element was taken on the sheet for analysis.

Another problem that needs to be considered is the fact that compaction is caused by its own gravity. This is reflected in the physical quantity of density. The **density distribution function under self-weight** is an essential element in the heat conduction and oxygen flow equations. Therefore, research is needed. The density distribution of the compost under its own weight. We use the spring self-weight compression model to simplify the compost material into a spring that cannot be ignored by gravity. Through the Hooke's law and the calculus idea, the amount of compression is obtained, and then the density distribution is determined by the density definition.

Contents

1. Problem Analysis

Our main task is to get the most effective compost by changing the size of the compost. The main indicators to measure the effectiveness of compost are temperature, moisture content, and oxygen content.

Since the moisture content is mainly determined by chemical changes, it is not considered in the physical model; for the temperature distribution, the conditions to be satisfied are: to maintain the temperature within the required range for as long as possible within 24 hours (40) Celsius to 60 degrees Celsius); for oxygen content, it is only necessary to make the internal average oxygen content as much as possible.

At the same time, we found that the oxygen content distribution and heat transfer process follow very similar physical laws. The amount of heat generated by the same amount of microorganisms is the same as the amount of oxygen consumed. The amount of heat diffusion and oxygen absorption is more in the vicinity of the outside. The difference is that heat is transmitted from the inside to the outside, and the oxygen content is diffused from the outside to the inside.

In order to simplify the model as much as possible, first we consider the simplest shape, the cylinder, which only needs to consider the two variables of height and radius, and assumes that the microbial content is fixed at a fixed amount.

2 Model

2.1 Active Heat Conduction Equation

2.1.1 Model Establishment

In order to simplify the model, the first consideration is the compost of a cylindrical shape. First, center the center of the circle, like cutting a cake, and cut the cylinder into a number of longitudinal sections. In this way, the temperature and oxygen distribution of each piece are the same, so only one fan-shaped sheet needs to be considered.

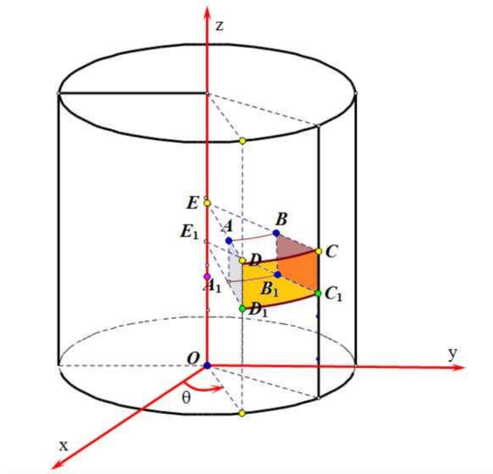

Figure 1 fan-shaped sheet and area micro-element

For the temperature distribution at different positions on the same horizontal line from the center of the circle, first, use **the law of conservation of heat** to list the heat transfer conservation equation at one point:

$$Q = Q_{gen} + Q_{left} + Q_{right} + Q_{up} + Q_{down}$$

Formula description:

- Q indicates the heat remaining at this point;
- Q_{gen} is the heat generated by the point itself;
- The left four part is the heat absorbed or released from other directions, such as Q_{left}—— the heat released to the left （low temperature） point, Q_{right}——the heat absorbed from the right （high temperature） point, Q_{up} ——the heat released to the above（low temperature） point, Q_{down}——the heat absorbed from the point below（high temperature）.

2.1.2 Formula Derivation

Next, the above symbols are expressed by physical formulas, and we can simplify the formula.

- Q and Q_{gen}

Firstly, according to **specific heat capacity calculation formula**:

$$\mathrm{Q} = \mathrm{c} \cdot \mathrm{m} \cdot \Delta \mathrm{t}$$

For a tiny volume $\Delta V = x\,\Delta x\,\Delta h\,\Delta\theta$, for a time interval $t - \Delta t \longrightarrow t$, Its final heat preservation is:

$$Q = c\,\rho(h) \cdot x\,\Delta x\,\Delta h\,\Delta\theta\,[u(x,h,t) - u(x,h,t-\Delta t)]$$

Formula description:

$u(x,h,t)$ indicates the temperature of the volume micro-element at a radius of x , the height from the bottom surface at h, the time t , c is the specific heat capacity of the material, and $\rho(h)$ represents the material density at height h (due to compaction, the height is higher high, the smaller the density, it can be obtained by consulting the data), x is the distance from the center of the circle, and θ is the angle of the sheet.

The heat generated by the point itself is:

$$Q_{gen} = K(h) \cdot x\Delta x\,\Delta h\,\Delta\theta$$

$$K(h) = K_0\,\rho(h)$$

Formula description:

$K(h)$ indicates microbial heat production per unit volume per unit time. Since the amount of microorganisms is proportional to the density of compost $\rho(h)$, the heat production $K(h)$ is also proportional to the density $\rho(h)$, and the proportional coefficient is K_0.

- Q_{left} , Q_{right} , Q_{up} and Q_{down}

For the conduction of heat, **Fourier's law** is required. That is to say, in the heat conduction process, the heat conduction amount passing through a given section per unit time is proportional to the temperature change rate and the cross-sectional area perpendicular to the cross-section direction.

$$Q = -\lambda \frac{dT}{dn} A$$

Among them, the proportionality constant λ is a transport characteristic called thermal conductivity (also called thermal conductivity). In actual tests, the thermal conductivity generally has a linear relationship with temperature, that is:

$$\lambda = \lambda_0 (1 + bT)$$

Change dT to Δ to get:

$$Q_{right} = \frac{\lambda_0\, b\, [1 + u(x,h,t)]\, x\, \Delta h\, \Delta\theta\, [u(x - \Delta x, h, t) - u(x,h,t)]}{\Delta x}$$

$$Q_{left} = -\frac{\lambda_0\, b\, [1 + u(x,h,t)]\, x\, \Delta h\, \Delta\theta\, [u(x,h,t) - u(x + \Delta x, h, t)]}{\Delta x}$$

$$Q_{down} = \frac{\lambda_0\, b\, [1 + u(x,h,t)]\, x\, \Delta x\, \Delta\theta\, [u(x, h - \Delta h, t) - u(x,h,t)]}{\Delta h}$$

$$Q_{up} = -\frac{\lambda_0\, b\, [1 + u(x,h,t)]\, x\, \Delta x\, \Delta\theta\, [u(x,h,t) - u(x, h + \Delta h, t)]}{\Delta h}$$

- Active Heat Conduction Equation

The above formulas are linked and simplified, and finally the following results are obtained.

$$c\, \rho(h) \cdot x\, \Delta x\, \Delta h\, \Delta\theta\, [u(x,h,t) - u(x,h,t - \Delta t)] = K(h) \cdot x \Delta x\, \Delta h\, \Delta\theta$$

$$+ \frac{\lambda_0\, b\, [1 + u(x,h,t)]\, x\, \Delta h\, \Delta\theta \{[u(x - \Delta x, h, t) - u(x,h,t)] - [u(x,h,t) - u(x + \Delta x, h, t)]\}}{\Delta x}$$

$$+ \frac{\lambda_0\, b\, [1 + u(x,h,t)]\, x\, \Delta x\, \Delta\theta\, \{[u(x, h - \Delta h, t) - u(x,h,t)] - [u(x,h,t) - u(x, h + \Delta h, t)]\}}{\Delta h}$$

$$\Longrightarrow\; c\rho(h)\, \frac{\partial u}{\partial t}\, dxdh = K_0\, \rho(h)\, dxdh + \frac{\lambda_0\, b(1 + u)}{dt} (2dh \frac{\partial u}{\partial x} + 2dx \frac{\partial \mathrm{u}}{\partial \mathrm{h}})$$

2.2 Oxygen Flow Equation

According to previous analysis, the models and laws of oxygen flow and heat transfer processes are very similar. The same amount of microorganisms produce the same amount of heat and consume the same amount of oxygen; the amount of heat diffusion and oxygen absorption is more in the vicinity of the outside, and the differential equations of oxygen circulation and heat conduction are also the same. The difference is that heat is conducted from the inside to the outside. The smaller the distance, the higher the temperature; the oxygen content is diffused from the outside to the inside. The smaller the distance, the lower the oxygen content. .

The operation process is similar to the above process, and the following equation is obtained:

$$\frac{\partial n(x,h,t)}{\partial t}\,dxdhdt = -L_0\,\rho(h)\,dxdh + D_0[\rho(0)-\rho(h)](2dh\frac{\partial u}{\partial x} + 2dx\frac{\partial u}{\partial h})$$

Formula description:

- $n(x,h,t)$ means the oxygen content of the volume micro-element at a time radius t at a radius of x and a height h at the bottom;
- $L(h)$ is similar to $K(h)$, indicates the oxygen consumption of microorganisms per unit volume, L_0 is the proportionality factor:

$$L(h) = L_0\,\rho(h)$$

- D_0 is oxygen transmission coefficient (oxygen conduction), which characterizes the passage rate of oxygen after being blocked by solids;

2.3 Vertical Distribution of Elastic Material Density When Considering Self-weight

In the cylinder compost, due to the influence of its own gravity, the density distribution will be “upper and smaller”—that is, the compaction occurs. According to experience, the oxygen passing rate of the compacted object becomes lower. In other words, the higher the density, the harder it is to pass oxygen. In order to study the oxygen content, it is particularly important to study the density distribution of compost under self-weight.

In order to facilitate the study, the compost material is simplified into a **spring model** (non-ideal spring, considering its own weight, compression is within the elastic limit), the amount of compression under the condition of self-weight compression is studied, and the density distribution is determined by the density definition.

2.3.1 Spring Self-weight Compression Model

Let the spring be l_1 under self-weight compression, the original spring length be l_0 , the spring self-weight M, and the spring constant k . Since the compression of the spring is caused by the self-weight that is not concentrated, the Hooke's law cannot be applied as a whole. Now consider that the spring self-weight is concentrated on the free end of the spring top. The spring is simplified as an ideal spring, and the compression length is recorded to l_2 at this time, according to Hooke's law:

$$Mg = k(l_2 - l_0)$$

In the reference[1], it is assumed that the compression of the spring under the self-weight is proportional to the amount of spring compression when the gravity is concentrated, and the proportional coefficient is δ:

$$\delta = \frac{l_1 - l_0}{l_2 - l_0}$$

The result of taking the limit based on the differential addition is : $\delta = \frac{1}{2}$ [1]

The above results show that the spring degree compression is equal to half the amount of compression after the spring mass is concentrated under self-weight. That is: $\delta Mg = k(l_1 - l_0)$

Now divide the spring into n parts, each of which is equal in mass, that is:

$$M = nm_0$$

At this time, the spring coefficient of each part of the spring is nk. Numbered from the free end is 1, 2, …, i, …, n. List the Hooke's Law equation for each part:

$$\begin{cases} \delta m_0 g = nk\Delta l_1 \\ 2\delta m_0 g = \frac{nk}{2}(\Delta l_1 + \Delta l_2) \Rightarrow 3\delta m_0 g = \Delta l_2 \\ 3\delta m_0 g = \frac{nk}{3}(\Delta l_1 + \Delta l_2 + \Delta l_3) \Rightarrow 5\delta m_0 g = \Delta l_3 \\ \cdots \\ (2\mathrm{n} - 1)\delta m_0 g = \Delta l_n \end{cases}$$

$$\Rightarrow \quad \Delta l_1 : \Delta l_2 : \Delta l_3 : \cdots : \Delta l_n = 1 : 3 : 5 : \cdots : 2\mathrm{n} - 1$$

Also due to: $\begin{cases} \sum \Delta l_i = l_1 - l_0 \\ \frac{1}{2} Mg = k(l_1 - l_0) \end{cases}$

Solve the above equations : $\Delta l_1 = \frac{Mg}{2n^2 k}$

2.3.2 Application of the Above Model and Solution of Compost Density

Consider the compost pile as a spring that cannot be ignored by its own weight. Its total mass is M , the reluctance coefficient is k, and the height after compression is *H* (corresponding to l_1 of the spring self-weight compression model). According to the above model, use the density definition: $\rho = \frac{m}{V}$,

the density expression for the i-th part can be obtained as:

$$\rho(i) = \frac{m_0}{\pi R^2 (\frac{l_0}{n} - \frac{(2i-1)Mg}{2n^2 k})} \cdots\cdots\cdots\cdots\cdots (*)$$

$$l_0 = \mathrm{H} + \frac{Mg}{2k}$$

Let h be the height of the i-th part. Now start from the free end of the compost pile and find out the relationship between h and i:

$$\frac{il_0}{n} - \sum_{j=1}^{i} \Delta l_i = H - h$$

Simplify it and make $\mathrm{n} \to \infty$:

$$\frac{(i-\frac{1}{2})l_0}{n} + \lim_{\mathrm{n}\to\infty}\left(\frac{l_0}{2n} + \frac{i^2 Mg}{2n^2 k}\right) = H - h$$

$$\Rightarrow \quad \frac{(i-\frac{1}{2})}{n} = \frac{H-h}{\mathrm{H} + \frac{Mg}{2k}}$$

By replacing i in (*) with h, you can get the spatial distribution of the internal density of the compost:

$$\rho(\mathrm{h}) = \frac{M}{\pi R^2} \cdot \frac{kH + \frac{Mg}{2}}{kH^2 + \frac{(Mg)^2}{4k} + Mgh}$$

Where $R(\text{the radius of the cylinder})$, M , H , g , k are all known or constant.

2.4 The Optimal Size for Changes in the External Environment and Organic Materials

The previous research is based on the changes in the external temperature and the related properties of the organic materials, and they are solved as known quantities or initial conditions. Now we should treat them as independent variables. Our idea is to **decompose the temperature distribution function into two items**, one is the set of items related to the two initial values of external temperature and material (**initial value determinant**), another The term is a set of irrelevant terms, so that the pile size can be solved by setting the external temperature and material property values in the initial value decision to an unknown amount and then adding the constraint of efficient composting.

For example, for the temperature distribution function, split it into two parts:

$$u(x,h,t) \Rightarrow U(x,h,t) + u_0(x,h,t)$$

Formula description:

$u_0(x,h,t)$ is just the initial value determinant.

It should be noted that as an independent variable, the organic material needs to be converted into a variable existing in the specific equation. The specific heat capacity and density of the organic material itself affect the specific heat capacity c , part of the parameters in the density distribution function $\rho(h)$, the proportional coefficient K_0 of the microbial heat production $K(h)$ as a function of the density distribution function $\rho(h)$, and the proportional coefficient L_0 of the microbial oxygen consumption $L(h)$ as a function of the density distribution function $\rho(h)$.

3 Results

3.1 Temperature Distribution with Horizontal Distance and Height

In order to solve the result, a series of initial conditions and proportional coefficients need to be substituted into the equation.

Table 1 Parameter Value

Name or Symbol	Numerical Value
Specific heat capacity c	$1000\ \mathrm{J/(Kg\cdot K)}$
Weight m	$400\ \mathrm{Kg}$
Density $\rho(0)$	$0.3\ \mathrm{g}/cm^3$
Scale Factor $K_0 = K(h)/\rho(h)$	$0.013\ \mathrm{J/(Kg\cdot s)}$
Thermal Conductivity $\lambda_0\ b$	$0.5\ \mathrm{W/(m\cdot K)}$
Ambient temperature T_0	$\begin{cases} 5℃, 0-6h; \\ 10℃, 6-12h; \\ 15℃, 12-18h; \\ 20℃, 18-24h. \end{cases}$

The indicator for measuring efficient composting is temperature - 40 to 60 degrees Celsius and enough oxygen, so restrictions need to be set:

$$\begin{cases} 40 \le u(x,h,t) \le 60\ ; \\ max\ \{n(x,h,t)\}\ . \end{cases}$$

The final result of the solution is:

$$\begin{cases} R = 0.8m\ ; \\ H = 0.8m\ . \end{cases}$$

Draw the distribution of temperature in this dimension with horizontal and vertical directions at stable state:

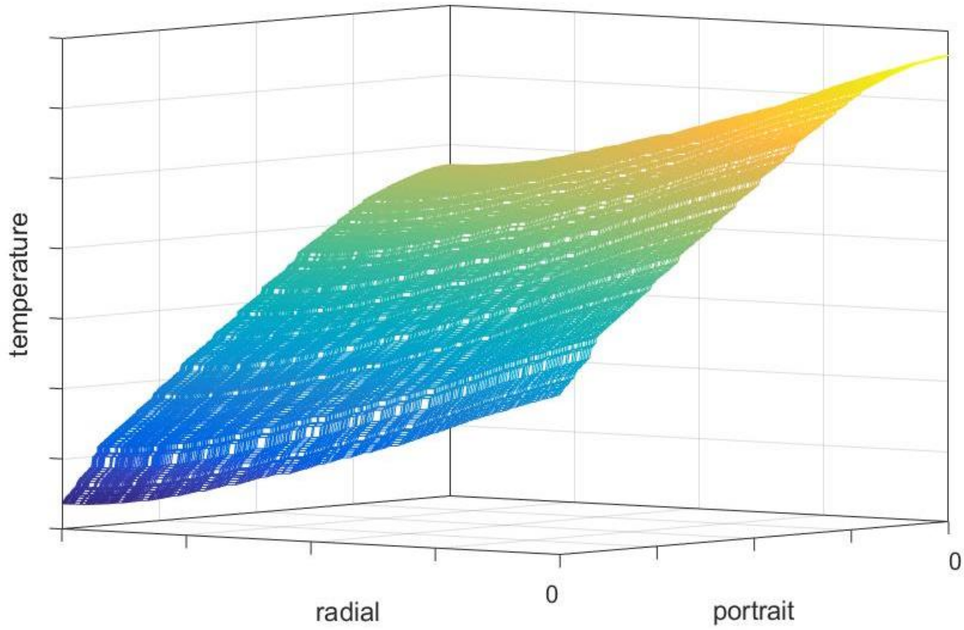

Figure 2 Temperature distribution in the radical and portrait direction

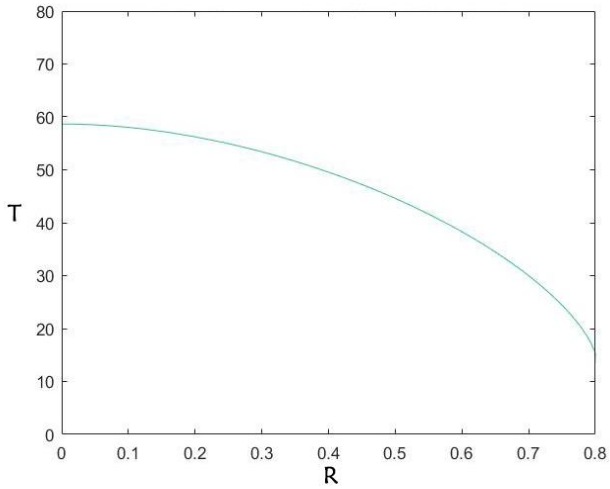

Figure 3　Temperature distribution in the horizontal direction（m）

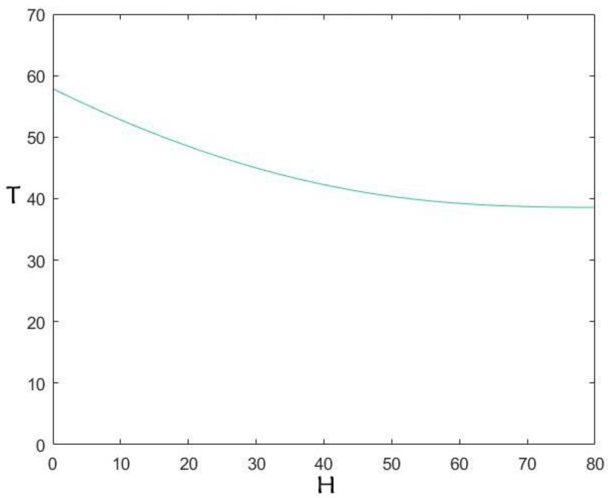

Figure 4　Temperature distribution with height（cm）

The size we choose is reasonable.

First of all, in the horizontal direction, the main criterion we chose is to ensure that the temperature at the center of the circle (the highest temperature) is as close as possible but not exceeding 60 degrees Celsius, and the temperature near the edge is inevitably close to the ambient temperature, which is difficult to reach 40. Above Celsius.

In the vertical direction, the same reason, to ensure that the bottom (the highest temperature) is as close as possible but not more than 60 degrees Celsius.

3.2 Size Selection in Different Environments

In the model building part, we use the idea of constant variable parameters and separate variables. Based on the previous work, we obtain the equations with external temperature and material specific heat capacity and density as independent variables.

Therefore, when solving the model, it is necessary to substitute the constant coefficient value into the equation for solving and simplification.

Unfortunately, although we have established a reasonable and effective physical model, we have not been able to obtain a general formula of the size change with the environment because the equation is too complicated to be solved.

References

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