

Wind Control Strategy for Quadcopter

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Abstract

Recently, quadcopter has become one of the most popular machines world-widely. It's smaller and lighter than other UAV but still good in maneuverability. Now it acts an indispensable role in many fields like emergency treatment, dispatch, etc. So how to control a quadcopter and make it as stable as possible is what people want to know. Because of the lighter weight, wind becomes a kind of severe influence which may damage the stability of a quadcopter. Real nature wind never shows symmetry and changeless, which makes a big trouble in quadcopter control method. This thesis describes a model based on Newton-Euler function and PID controller, to find a better control strategy under turbulence wind based on Dryden model. Finally, by simulation based on our model, the distance between the quadcopter and the target will reach 20cm when the velocity of wind is 10.63 m/s.

1. Model

To find a better model for quadcopter controller, first we use an appropriate method to local the quadcopter, which can enable us describe the location and movement accurately and make the equations of motion as easy as enough. Then we can analyze the quadcopter by Newton-Euler function, and get the equations of motion for programming. To generate a kind of wind which is similar to the real, we build three transfer functions to create turbulence wind based on Dryden model, and combine the turbulence wind created and the equations of motions together to simulate the real quadcopter in the air.

1.1. Coordinate system

To describe the movement of quadcopter clearly, we need to set two coordinate systems^[1]. One is the ground coordinate system whose observer is on the earth. The ground coordinate system enables us to get precise location of the quadcopter,

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which means that we can get (X_g, Y_g, Z_g) at a certain moment. The original point O_g is set at the starting point of the UAV, and $O_g Z_g$ is perpendicular to the ground. $O_g X_g$ shows the direction of the movement of the UAV. $O_g Y_g$ is same with the direction of $O_g Z_g \times O_g X_g$.

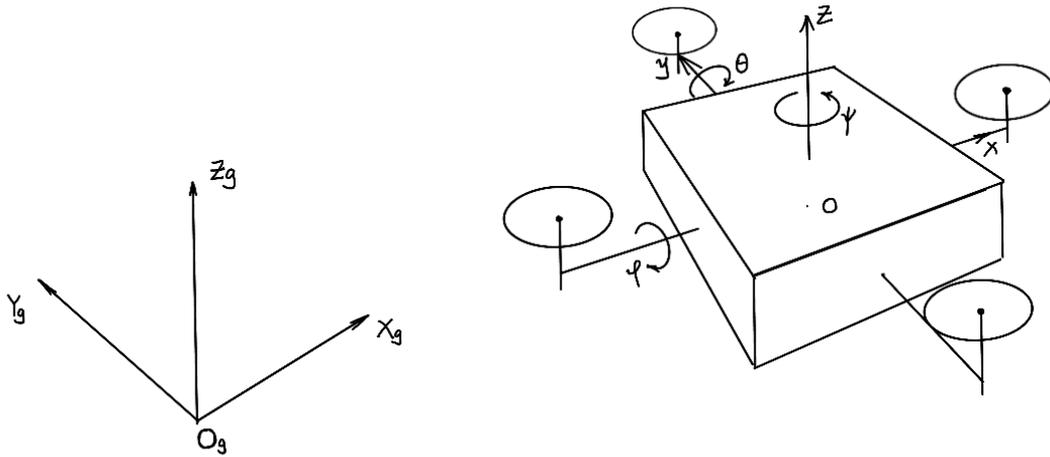


Fig.1 Coordinate system

Another coordinate system is the rigid body coordinate system $O - XYZ$ whose observer is on the quadcopter and follows every moment of it. In our model, we assume that the UAV is a rigid body, which means that its mass and form never change. The barycenter is also the geometry center and the original point of the rigid body coordinate system.

According to the two coordinates above, we can use Euler angles to describe the rotation of the UAV in the air. Euler angles contains three different angle: roll angle θ , pitch angle φ and yaw angle ψ . The meaning of these angles is shown on the figure above. We also need transformation matrix of the two coordinates which shows below:

$$R(\varphi, \theta, \psi) = \begin{bmatrix} \cos \theta \cos \psi & \cos \phi \sin \varphi \sin \theta - \sin \psi \cos \theta & \cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi \\ \sin \psi \cos \theta & \sin \varphi \sin \theta \sin \psi + \cos \theta \cos \psi & \sin \theta \sin \psi \cos \varphi - \sin \varphi \cos \phi \\ -\sin \theta & \cos \theta \sin \varphi & \cos \varphi \cos \theta \end{bmatrix}$$

1.2. Derivation of kinematics model

Before the derivation, we need to make some hypothesis to simplify the problem:

- Neglect the influence of movements of the earth, which means the ground coordinate system is an inertial system. Also neglect the variance of the acceleration of gravity since the variance of height is small.
- The shape and construction of the quadcopter is geometrically symmetric. The mass is even, and the barycenter is also the geometry center.
- The quadcopter we analyze is a rigid body. Neglect all form and mass changes in the whole process.
- Neglect the influence of wind on the main body of the UAV since the main body has smaller interface with the wind comparing to fans^[2].

1.2.1 Aerodynamic analysis

As we assumed before, the quadcopter is mainly made up of four total same rotors, and the main body is negligible in the aerodynamic analysis. So we only analyze one rotor, and the conclusion can be used in other three rotors. Finally combine them together, we can get the aerodynamic model of the quadcopter.

The basic analysis diagram shows in Fig.2. V_w is the speed of wind. V_d is induction speed of the rotor which defined as follow^[3, 4]:

$$||V_d|| = \sqrt{\frac{F_T}{2\rho A}}$$

F_T is the thrust of the rotor. ρ is the density of air. A is the area of the fan. Ω is the rotate speed of the rotor. V is total induction speed.

F_T and M_T are proportional to Ω^2 . M_T is torque of the rotor^[5].

$$F_T = k_F \Omega^2, M_T = k_M \Omega^2$$

k_F is the pull force coefficient, and k_M is torque coefficient.

Under wind disturbance, the total aerodynamic thrust is the sum of F_T and F_w ^[3, 4]:

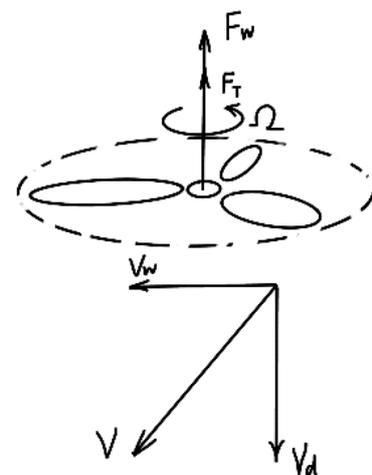


Fig.2 Aerodynamic analysis of one rotor

$$F_R = F_T + F_w = 2\rho AV_d V$$

It's obvious that if there has no wind disturbance, $V_d = V$. So that $F_R = F_T = 2\rho AV_d^2$, which is the same as the definition of V_d . The total torque of the rotor is also the sum of M_w and M_T :

$$M_R = M_w + M_T = k_{drag} V^2$$

k_{drag} is a coefficient related to A , ρ and shape of the fan^[4].

1.2.2 Overall motion analysis for quadcopter

From the analysis above, we now consider movement of quadcopter is composed of translation and rotation. We analyze them separately in different coordinate system.

For translation, in the ground coordinate system $O_g - X_g Y_g Z_g$, we can get equations of translation from Newton-Euler equation:

$$m\ddot{\mathbf{X}} = R \left(\sum_{i=1}^4 \mathbf{F}_{T_i} + \sum_{i=1}^4 \mathbf{F}_{W_i} \right) + m\mathbf{G}$$

In the equation, $\mathbf{X} = [x \quad y \quad z]^T$ is the position of quadcopter. m is mass, and R is translation matrix mentioned before. $\mathbf{G} = [0 \quad 0 \quad -g]$ is the gravity matrix.

For rotation, we can get equation of rotation in the rigid body coordinate system $O - XYZ$:

$$\sum \tau = J\dot{\omega} + \omega \times J\omega$$

$\sum \tau$ is the sum of external torque and I is rotational inertial of quadcopter. $\omega = [p \quad q \quad r]^T$ is angular velocity. Combine every torque the UAV takes, we get the equation:

$$J\dot{\omega} = -\omega \times J\omega + \omega \times J_{rr} + M_B + M_w$$

$\omega \times J_{rr}$ is gyro torque generated by rotation of rotor, and $J_{rr} = [0 \quad 0 \quad J_r \Omega_r]$ where J_r is rotational inertial of rotor. M_B is torque made by thrust of rotor^[6]:

$$M_B = \begin{bmatrix} l(-F_{T_2}^2 + F_{T_4}^2) \\ l(-F_{T_3}^2 + F_{T_1}^2) \\ M_{T_1} - M_{T_2} + M_{T_3} - M_{T_4} \end{bmatrix}$$

l is the distance from rotor center to geometry center. M_w is torque made by wind^[6]:

$$M_w = \begin{bmatrix} l(-F_{w_2}^2 + F_{w_4}^2) \\ l(-F_{w_3}^2 + F_{w_1}^2) \\ M_{w_1} - M_{w_2} + M_{w_3} - M_{w_4} \end{bmatrix}$$

From all the deduction above, we can get the equations of motion as follow:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} U_1(C_\varphi S_\theta C_\psi + S_\varphi + S_\psi)/m \\ U_1(C_\varphi S_\theta C_\psi - S_\varphi + C_\psi)/m \\ U_1(C_\varphi C_\theta)/m - g \\ qr(J_{yy} - J_{zz})/J_{xx} + q\Omega_r J_r/J_{xx} + U_2 l/J_{xx} \\ pr(J_{zz} - J_{xx})/J_{yy} - p\Omega_r J_r/J_{yy} + U_3 l/J_{yy} \\ pr(J_{xx} - J_{yy})/J_{zz} + U_4 l/J_{zz} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_1 \\ W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix}$$

Define $U = [U_1 \quad U_2 \quad U_3 \quad U_4]^T$ as control vector:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ b(-\Omega_2^2 + \Omega_4^2) \\ b(\Omega_1^2 - \Omega_3^2) \\ d(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{bmatrix}$$

Define $W = [W_1 \quad W_2 \quad W_3 \quad W_4]^T$ as wind disturbance vector:

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^4 F_{w_i}/m \\ l(-F_{w_2}^2 + F_{w_4}^2)/J_{xx} \\ l(-F_{w_3}^2 + F_{w_1}^2)/J_{yy} \\ l(M_{R_1} - M_{R_2} + M_{R_3} - M_{R_4})/J_{zz} \end{bmatrix}$$

Now we get all information of motion. What we need to do next is design controller for control vector U and construct wind model to get wind disturbance vector W .

1.3 Turbulence wind model

As we said before, there is no pure wind in nature. Most wind we feel is actually turbulence wind. However, turbulence is one of the most difficult question in physics. Scientists try so hard to solve it but until now people do not get a perfect model of turbulence. We can calculate turbulence by N-S equation at a horribly high cost. But we won't do that, we generate turbulence wind data by Dryden model which filtrates white noise by special transfer functions and simulates real turbulence wind.

Assume that we have three different white noise who obeys Gaussian distribution. Also assume that $u(r)$, $v(r)$ and $w(r)$ represents three different direction components of the turbulence.

According to Dryden model, the frequency of turbulence velocity is^[7]:

$$\left\{ \begin{array}{l} \Phi_u(\omega) = \sigma_u^2 \frac{L_u}{\pi U_x} \frac{1}{1 + (L_u \frac{\omega}{U_x})^2} \\ \Phi_v(\omega) = \sigma_v^2 \frac{L_v}{\pi U_y} \frac{1 + 12(L_v \frac{\omega}{U_y})^2}{[1 + 4(L_v \frac{\omega}{U_y})^2]^2} \\ \Phi_w(\omega) = \sigma_w^2 \frac{L_w}{\pi U_z} \frac{1 + 12(L_w \frac{\omega}{U_z})^2}{[1 + 4(L_w \frac{\omega}{U_z})^2]^2} \end{array} \right.$$

ω is frequency, and L , σ are turbulence coefficients. They can be determined by the following equations^[8]:

$$\left\{ \begin{array}{l} 2L_w = h \\ L_u = 2L_v = \frac{h}{(0.177 + 0.000823h)^{1.2}} \\ \sigma_w = 0.1u_{20} \\ \frac{\sigma_u}{\sigma_w} = \frac{\sigma_v}{\sigma_u} = \frac{1}{(0.177 + 0.000823h)^{0.4}} \end{array} \right.$$

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u_{20} is the wind velocity in the height of $6.096m$. h is height.

White noise $n(t)$ is filtrated by a filter whose transfer function is $G(s)$, and get the output $x(t)$:

$$\Phi(\omega) = |G(i\omega)|^2 = G^*(i\omega)G(i\omega)$$

Decompose frequency of turbulence velocity by the formula above, and get transfer functions:

$$G_u(s) = \frac{K_u}{T_u s + 1}, \quad G_v(s) = \frac{K_v}{T_v s + 1}, \quad G_w(s) = \frac{K_w}{T_w s + 1}$$

K and T can be determined by L , σ :

$$K_u = \sigma_u \sqrt{\frac{L_u}{\pi V}}, \quad K_w = \sigma_w \sqrt{\frac{L_w}{\pi V}}, \quad K_v = \sigma_v \sqrt{\frac{L_v}{\pi V}}$$
$$T_v = \frac{2L_v}{\sqrt{3}V}, \quad T_u = \frac{L_u}{V}, \quad T_w = \frac{2L_w}{\sqrt{3}V}$$

So once we set certain u_{20} and h , we can calculate all the other parameters needed in Dryden model. Here we set $u_{20} = 20m/s$, $h = 1m$, and get the turbulence wind like Fig.3.

In our model, turbulence does not always exist. What we want to simulate is quadcopter meets turbulence unexpectedly. So we only allow turbulence to perform in a special region [$startTime$, $startTime + duration$], and set turbulence to zero in other regions. The diagram is like Fig.4.

2. Simulation

Now the complete model shows on the table. All we need to do is design a wonderful simulation program. First we need to determine the construction of the program basically. Then the control vector U should be designed. Some parameters of the quadcopter should also be determined. The structure of control system is shown in Fig.5.

Here X is the status of next stage of quadcopter, and it contains all the information that we need.

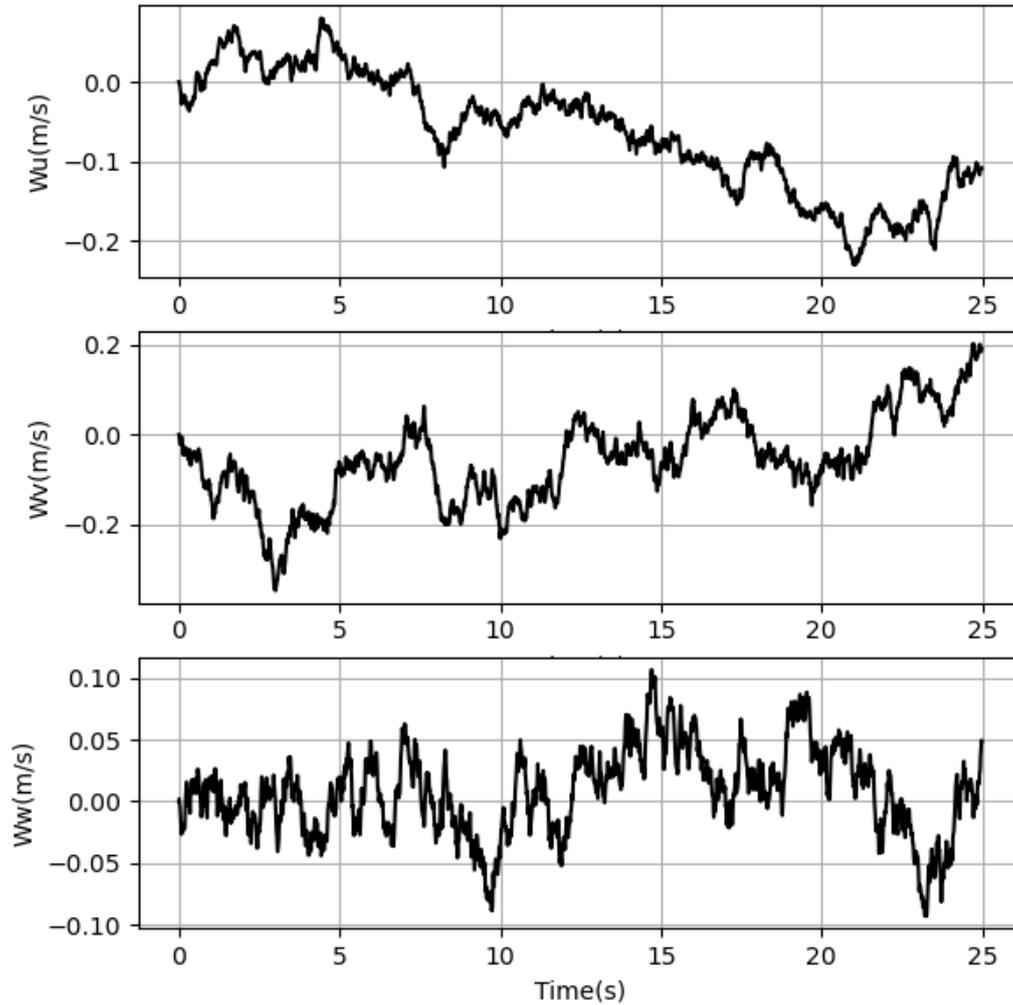


Fig.3 Turbulence wind generated by Dryden model ($u_{20} = 20m/s, h = 1m$)

$$X = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad \varphi \quad \theta \quad \psi \quad p \quad q \quad r]^T$$

In the system, controller and turbulence provide U and W to the quadcopter. Quadcopter accepts the parameters and calculates new status X_{n+1} , and sends the new status to the controller. Then the Controller calculates new U_{n+1} and turbulence provides next wind data generated before.

2.1 Parameters

Before we move to the next part, we need to calculate some important parameters based on the question:

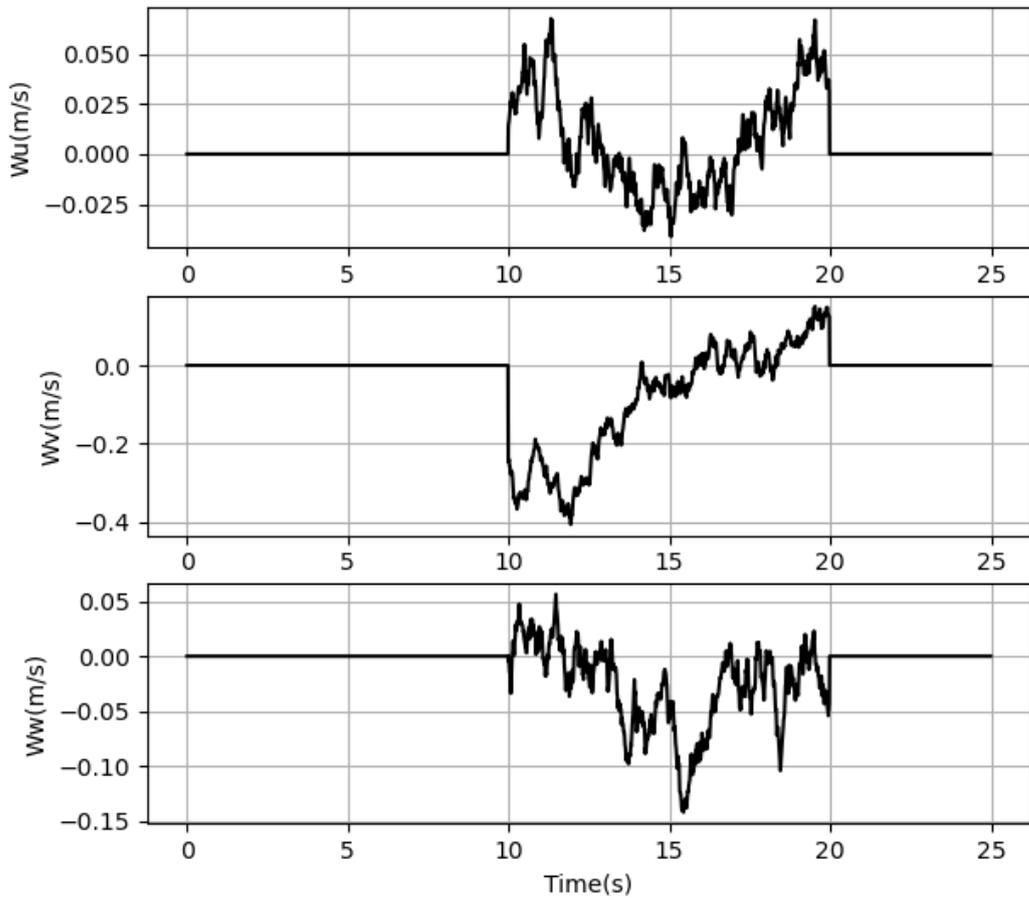


Fig.4 Turbulence wind in certain region ($u_{20} = 20m/s, h = 1m$)

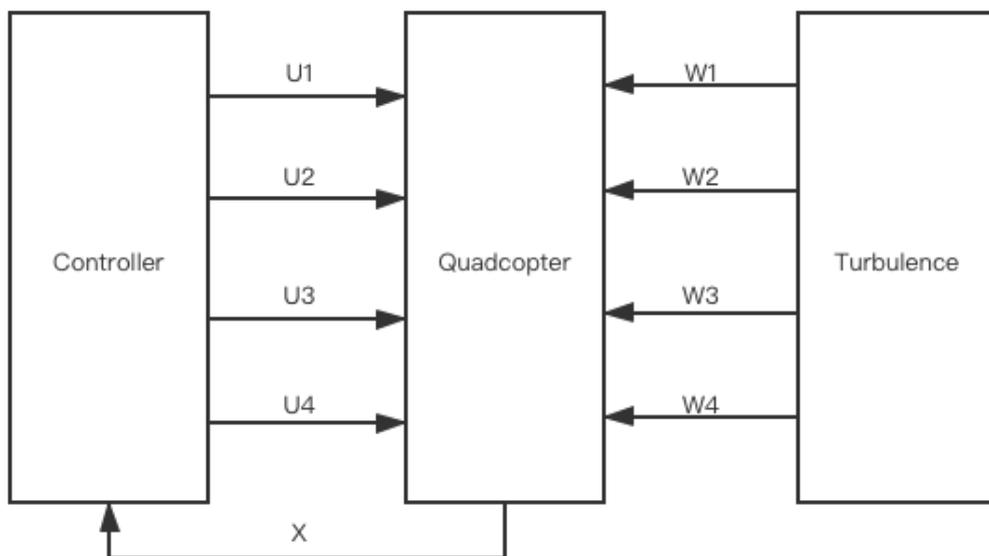


Fig.5 Structure of control system

$$\begin{cases} F_{Tmax} = 7N \\ m = 1.5kg \\ l = 0.5m \end{cases}$$

In these conditions, if we have a certain k_F of the rotor, than from the equation $F_{Tmax} = k_F \Omega_{max}^2$ we can easily get Ω_{max}^2 which limits the max value of the control vector U who is determined by Ω_i^2 . To simplify the model, we assume that k_F is the same value of b . The value of b in a kind of quadcopter is $5.324 \cdot 10^{-5} N \cdot s^2$ [6], so,

$$\Omega_{max} = 362s^{-1}$$

From m and l we can calculate rotational inertia of the quadcopter. As we assumed, the mass of the quadcopter is evenly distributed. So we idealize the quadcopter as an even square board whose side length is $a = \sqrt{2}l$ and show it in Fig.6. According to the symmetry of the rigid body coordinate system we set, it's obvious that $J_{xx} = J_{yy}$.

$$\begin{aligned} J_{xx} = J_{yy} &= \int dJ \\ &= \int_{-a/2}^{a/2} a \frac{m}{a^2} x^2 dx \\ &= \frac{1}{12} m a^2 \\ &= \frac{1}{6} m l^2 \end{aligned}$$

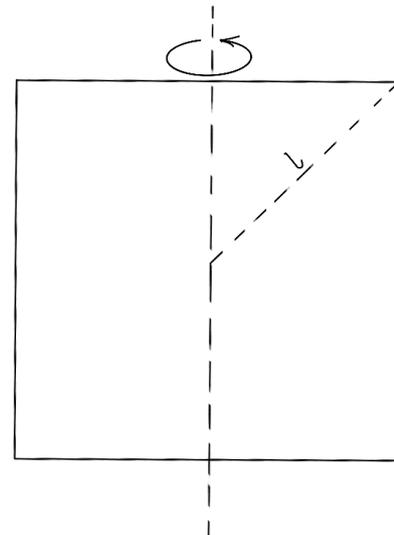


Fig.6 rotational inertia deduction

To get J_{zz} , we can simply use perpendicular axis theorem:

$$J_{zz} = J_{xx} + J_{yy} = \frac{1}{3} m l^2$$

The rotational inertia of the quadcopter in the question:

$$\begin{cases} J_{xx} = 0.0625kg \cdot m^2 \\ J_{yy} = 0.0625kg \cdot m^2 \\ J_{zz} = 0.125kg \cdot m^2 \end{cases}$$

Other parameters determined by rotors refer to a kind of quadcopter^[6]. All the parameters shows int Tab.1.

Tab.1 Quadcopter parameters

| Para | Name | Value | Unit |
|----------|----------------------------------|-----------|----------------|
| m | Mass | 1.5 | kg |
| l | Arm length | 0.5 | m |
| J_r | The rotational inertia of rotors | 7.321E-05 | $kg \cdot m^2$ |
| J_{xx} | The rotational inertia in x-axis | 0.0625 | $kg \cdot m^2$ |
| J_{yy} | The rotational inertia in y-axis | 0.0625 | $kg \cdot m^2$ |
| J_{zz} | The rotational inertia in z-axis | 0.125 | $kg \cdot m^2$ |
| b | Lift coefficient | 5.324E-05 | $N \cdot s^2$ |
| d | Resistance coefficient | 8.721E-07 | $N \cdot s^2$ |

2.2 PID controller

Traditional PID controller has a pretty long history in control science. Simple theory and easy to calibrate make it the most widely used control method in industrial engineering. It is a kind of closed loop control method which dominates the variable by its errors and no need for the exact model of the controlled variable. In the real flying procession of quadcopter, we have many variable which can't be determined precisely. So PID controller is the best method for quadcopter.

2.2.1 Basic introduction to PID controller

Structure of PID controller shows in Fig.7.

$u(t)$ is the controlled variable, and $r(t)$ is the target value of the controlled variable. $y(t)$ is output value. $e(t)$ is error between output value and target value:

$$e(t) = r(t) - y(t)$$

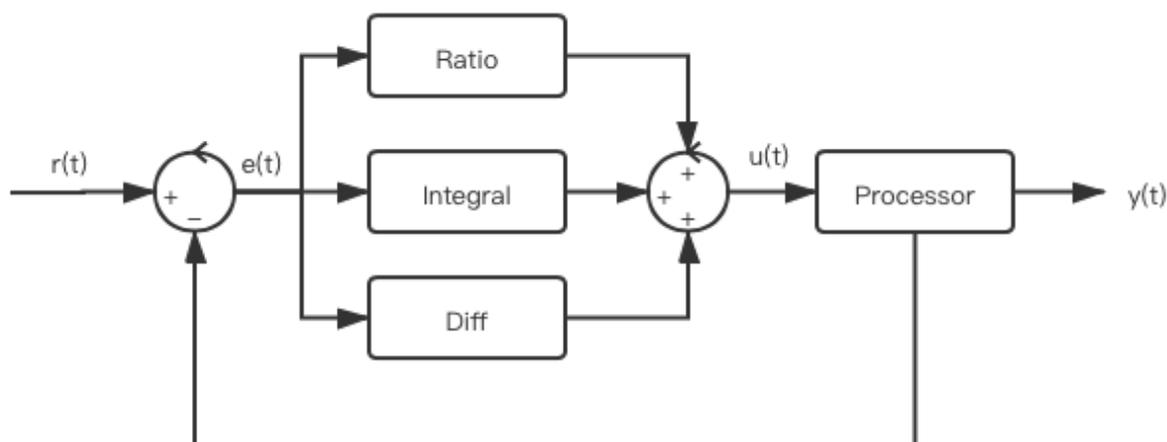


Fig.7 Structure of PID controller

The biggest character of PID controller is that it combines ratio part, integral part and differential part. We control proportion of each part by changing the PID coefficients: K_p , K_i and K_D .

Ratio part can diminish disturbance quickly but may generate new error after the control process. Disturbance is diminished quicker if K_p is bigger. However, bigger K_p also means that the system may vibrate violently. So an appropriate K_p is very important. The ratio part can be expressed as follow:

$$U_{k_p} = K_p e(t)$$

Integral part can diminish the steady-state error to improve the quality of control method. In the same way, K_i needs an appropriate value. The integral part can be expressed as follow:

$$U_{k_i} = K_i \int_0^{\tau} e(t) dt$$

Differential part can diminish errors proactively and also stabilize the control system. In the same manner, we can express the differential part:

$$U_{k_d} = K_d \frac{de(t)}{dt}$$

Above all, we get the PID controller:

$$U = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{de(t)}{dt}$$

2.2.2 Design control vector

We use four PID controllers to control $U = [U_1 \ U_2 \ U_3 \ U_4]^T$. While the quadcopter flies, the only disturbance in our model is turbulence wind which shows great randomness. In this situation, Integral part can't perform well. So we set $K_i = 0$ and mainly use the ratio part and differential part. Design the controller^[6]:

$$\begin{cases} u_1 = K_{pz} e_z + K_{dz} \frac{de_z}{dt} \\ u_2 = K_{p\varphi} e_\varphi + K_{d\varphi} \frac{de_\varphi}{dt} \\ u_3 = K_{p\theta} e_\theta + K_{d\theta} \frac{de_\theta}{dt} \\ u_4 = K_{p\psi} e_\psi + K_{d\psi} \frac{de_\psi}{dt} \end{cases}$$

From the equations of motion we deduced before, it's easy to know that locations variables are conjugated and anger variables are conjugated. So in the following analysis, we mainly discuss z and ψ . PID coefficients are very important values which needed to be determined carefully since they can influence performance of PID controller greatly.

Now we set target height is $1m$ and don't consider turbulence, and change PID coefficients to find the best values.

From Fig.8 to Fig.10, K_p changes in $[20,60,120]$. As we can see, when $K_p = 20$, the whole system will be stable near $t = 4s$. As long as K_p goes bigger, the system gets the stable state quicker. When $K_p = 60$, it only takes $1.83s$ to reach the stable state. But when K_p becomes too large, convergence time doesn't change too much. What's more, it seems the quadcopter "jumps" before the stable state, which means the rotor needs to provide huge force to make the quadcopter move back. However, our motor doesn't have this ability. So a very large K_p is not what we want.

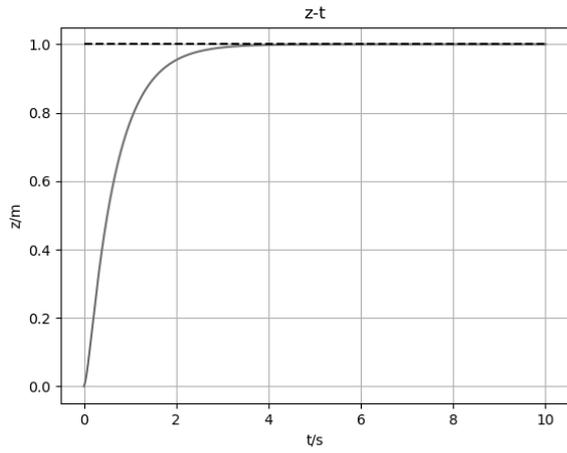


Fig.8 20, 0, 14

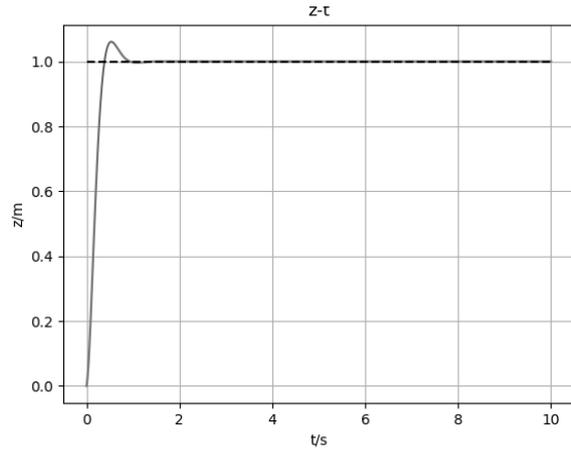


Fig.11 60, 0, 10

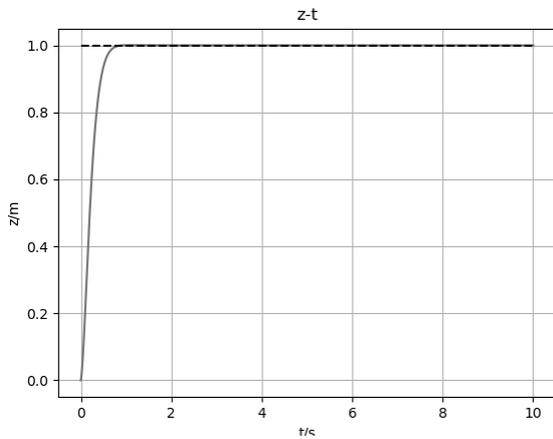


Fig.9 60, 0, 14

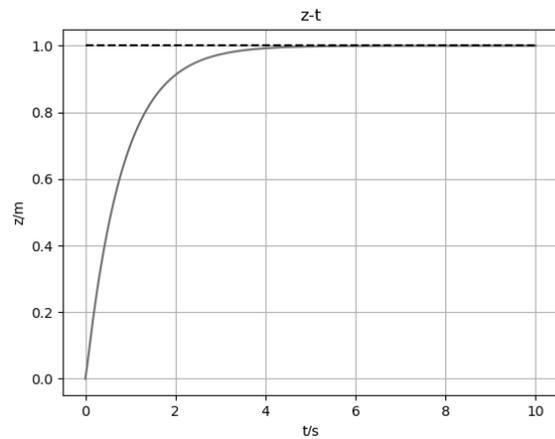


Fig.12 60, 0, 50

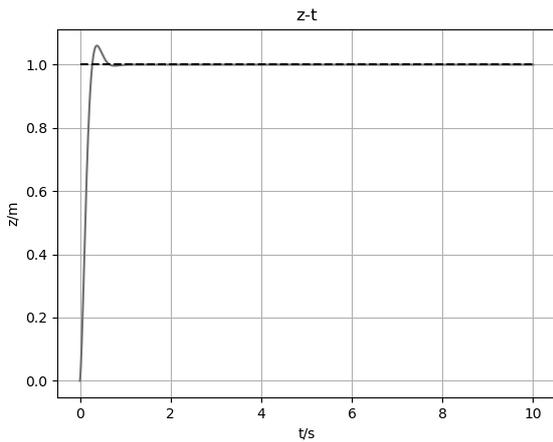


Fig.10 120, 0, 14

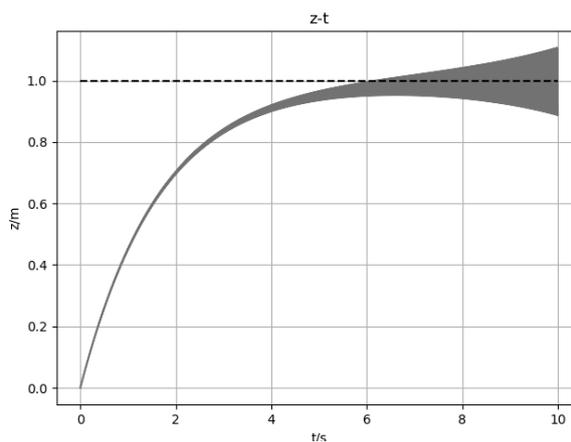


Fig.13 60, 0, 100

From Fig.11 to Fig.13, K_d changes in [10,50,100]. The conclusion is that we need to set K_d a small value since the system becomes slower as K_d goes bigger. And

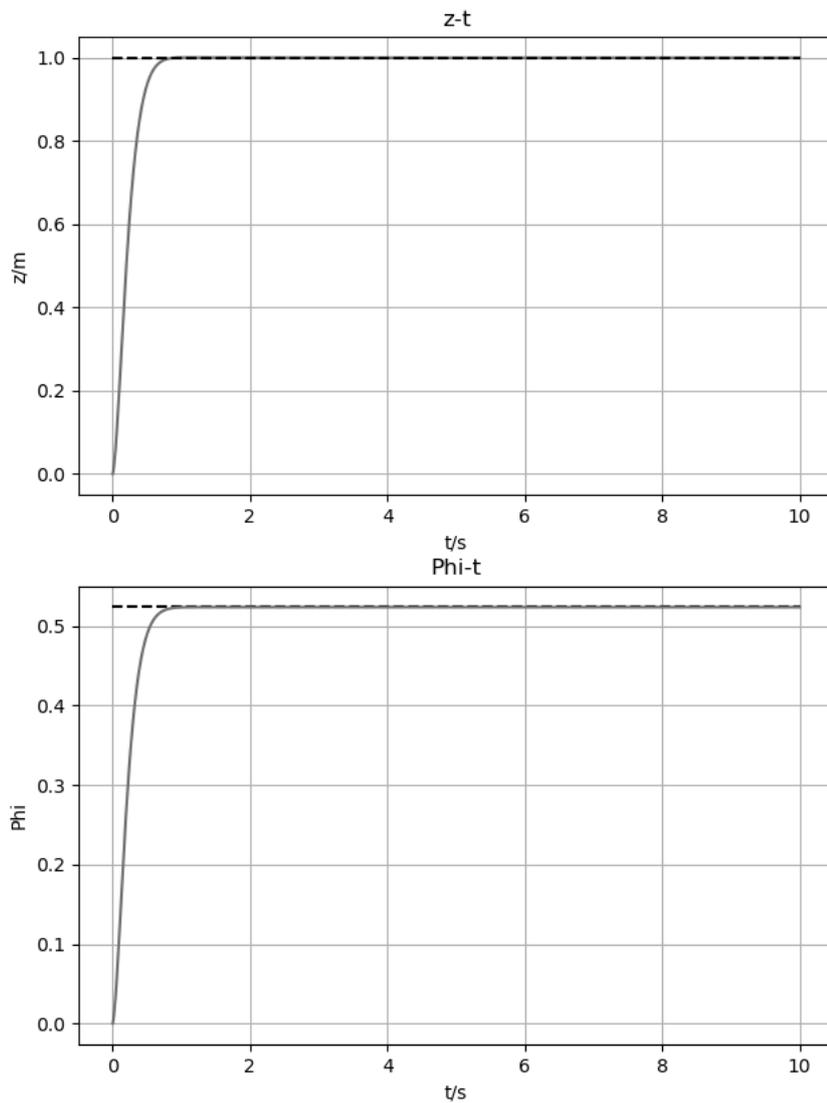


Fig.14 Selected PID controller

when K_d is too large, the quadcopter vibrates in a very high frequency. Of course such kind of motion is not what we want.

So we finally set $K_{pz} = 60$ and $K_{dz} = 14$. The process to determine $K_{p\psi}$ and $K_{d\psi}$ goes the same way, and $K_{p\psi} = 65$, $K_{d\psi} = 15$. Fig.14 shows the process of the PID controller we selected.

2.3 Quadcopter in turbulence

We have seen that we can determine the turbulence wind at height h by setting u_{20} . It is easy to know that turbulence wind goes severer if u_{20} is getting larger. We can calculate the total turbulence wind that the quadcopter takes by using vector addition:

$$V_{wind} = V_u + V_v + V_w$$

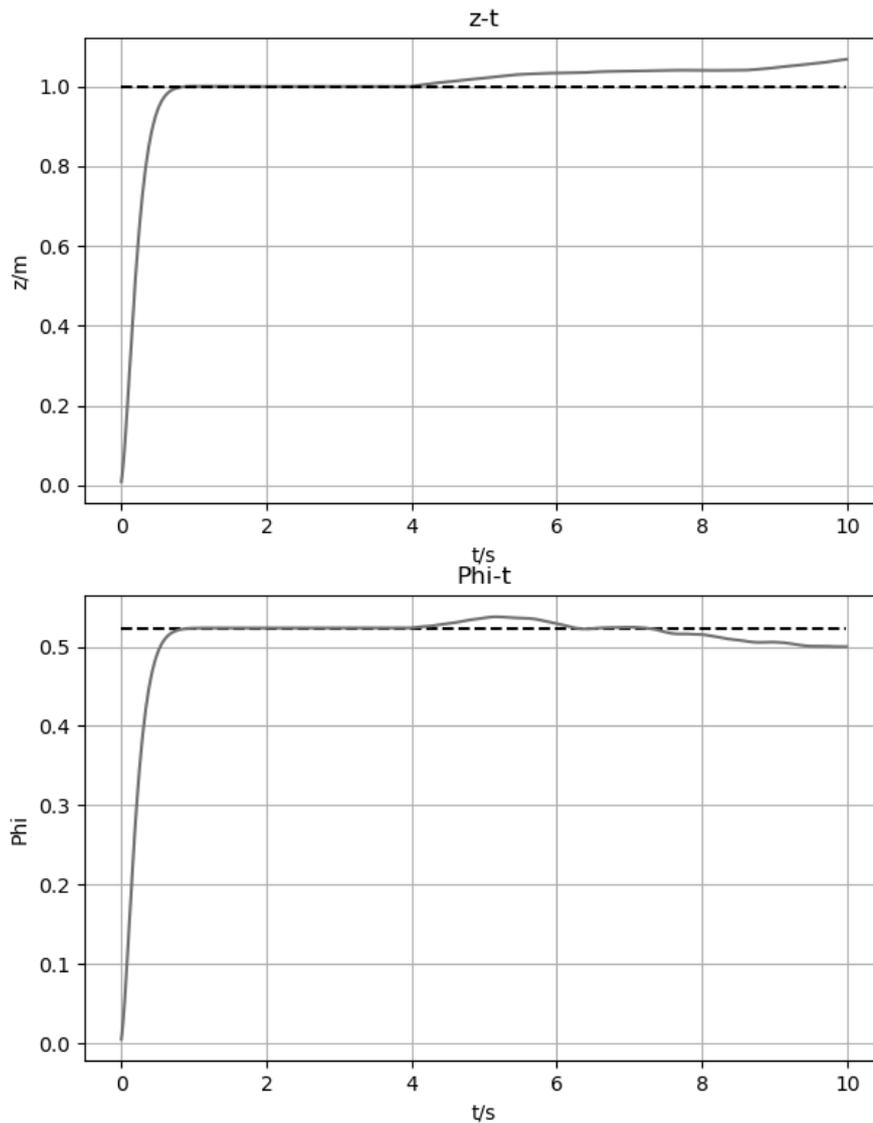


Fig.15 Quadcopter in turbulence ($u_{20} = 10m/s$)

So our strategy is, in order to find when the quadcopter leaves its target, magnifying u_{20} until the quadcopter leaves more than $20cm$ far away. Then calculate the wind speed now, and that's the value we want. The turbulence wind performs in $[4, 10]$.

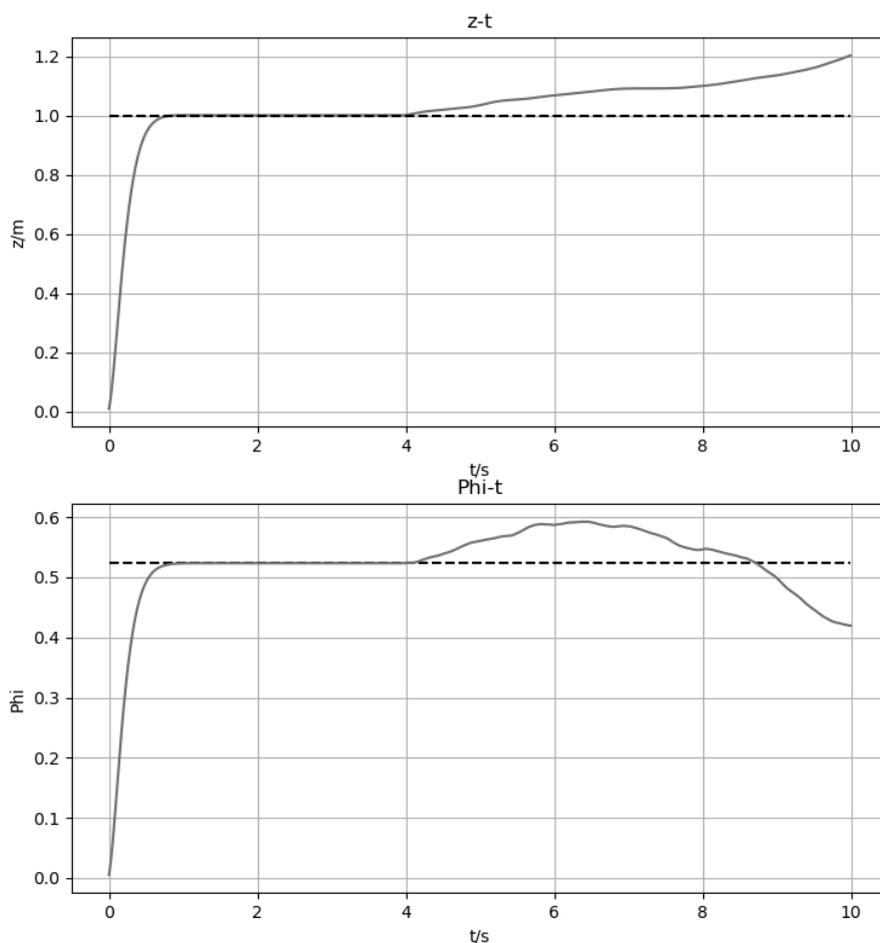


Fig.16 Quadcopter in turbulence ($u_{20} = 23m/s$)

When the u_{20} is slow, we can get Fig.15. Both z and ψ are disturbed by the turbulence wind. But both have small errors because the PID controller controls well. Until now, the quadcopter doesn't leave the target point too far away. As the wind is getting stronger, the influence is also getting stronger. PID controller can't control the UAV well since the wind is too strong. When $u_{20} = 23m/s$, Fig.16 shows that the distance between the quadcopter and the target reaches 20cm. Calculate the velocity of wind:

$$\|V_{wind}\|^2 = \sqrt{V_u^2 + V_v^2 + V_w^2} = \sqrt{7.51^2 + 6.24^2 + 4.20^2} = 10.63m/s$$

4 Conclusion

We design control strategy for wind control for quadcopter, which enables the quadcopter given in the question stay stable in the turbulence wind whose max

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velocity is 10.63 m/s. Equations of motion based on Newton-Euler equations in the ground coordinate system and the rigid body coordinate system are the core of the model, which serve as guidance for simulation programming. PID controller is used to control the control vector, and after selection of coefficients it can make the whole system more stable.

4.1 Limitations and Possible Improvement

We don't have enough time to complete the whole model. So the current model have some limitations and possible improvement:

- The main body takes pressure from wind. In order to simplify the question, we neglect the influence of wind on the main body. To improve, new model must consider this influence.
- The rotational inertia should be calculated precisely. In the current model, we idealize the quadcopter as an even square board. But the mass of the quadcopter distributes mainly around its center. Rotors' mass is small.
- Turbulence wind is the only kind of wind we consider. Real nature wind is made up by several different types of wind.
- PIL coefficients selection is rough. Because of the tight time, we choose PIL coefficients by change values manually. But coefficients are really important. It's better to apply some search algorithms, GA(Genetic Algorithm) and Q-Learning algorithm here to find the best coefficients.
- Dryden model for turbulence is not very precise. Real turbulence wind data are better in simulation.

4.2 Advantages

- The model uses PIL controller, which means that the model have wonderful adaptability. It can work for different quadcopters by changing PIL coefficients.
- The model stabilizes the quadcopter very fast, which means it responds quickly.
- The model can control the quadcopter well when wind is not too strong. So it can satisfy consumers who use quadcopter in daily life.
- The dynamic equations deduced by this model can describe motions of quadcopter precisely.

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