

Problem B

Stability Research of Quadcopter UAV Under Unstable Wind

Team # 380

Abstract

In order to find the maximum wind speed for the safe operation of the quadcopter UAV, so that it can stay within 20cm of the target position, this paper establishes a wind field model composed of basic wind, gust, gradual wind, and random wind to simulate the disturbance of wind; the hover dynamics model of the quadcopter UAV is established on the basis of the aircraft coordinate system to analyze its state when the aircraft is hovering; the force-motion model quadcopter UAV is established on the basis of the geodetic coordinate system which is used to analyze the position change of the quadcopter UAV; based on the force model, in order to make the UAV swing at the target position as small as possible, this paper establishes a quantitative self-adjusting UAV The feedback mechanism model enables it to resist wind speed disturbance and stay at the target position.

This paper improves the relevant parameters of the quadcopter UAV, and gives the definition of the maximum wind speed and the wind speed change factor. Through the random simulation of the unstable wind, the relationship between the wind speed of the unstable wind and the time is obtained, as shown in the figure (11). Based on the above parameter settings and wind speed simulation, the maximum wind speed under different wind speed change factors is solved, and the time change of the distance of the UAV from the target position is given. For different wind speed change factors, this paper calculates that the expected value of the maximum wind speed for safe operation of the quadcopter is 5.3678m/s. At the same time, this paper also conducts a numerical simulation experiment of the wind speed, and the average value of the simulated maximum wind speed is 5.3487m/s.

Key Words: quadcopter UAV, disturbance of unstable wind, feedback control, simulation experiment

Content

1. Introduction.....	4
2. Restatement and Analysis of Problems.....	4
2. 1 Restatement.....	4
2. 2 Analysis of Problems.....	4
3. Assumptions and Symbols.....	5
3. 1 Assumptions.....	5
3. 2 Notations.....	5
4. Model.....	8
4. 1 Modeling the wind.....	8
4.1.1 An overview of the wind field.....	8
4.1.2 basic wind model.....	8
4.1.3 gust wind model.....	9
4.1.4 Gradient wind model.....	9
4.1.5 random wind model.....	9
4.1.6 Definition of maximum wind speed.....	9
4.1.7 Calculation of wind force.....	10
4.2 Dynamics model of drone hovering.....	10
4.2.1 The structure of the UAV.....	10
4.2.2 Necessary assumptions.....	10
4.2.3 Definition of transformation matrix.....	11
4.2.4 Analysis of hover state.....	14
4.2.5 Horizontal force analysis.....	15
4.3 UAV control system.....	16
4.3.1 The overview of UAV control system.....	16

4.3.2 The quantitative solution of UAV control system.....	17
5. Model Solving and Analysis.....	19
5.1 Setting of relevant parameters.....	19
5.1.1 Definition of UAV stability.....	19
5.1.2 Air viscosity and air density.....	19
5.1.3 The setting of UAV parameters.....	19
5.2 Definition of change factor ε	20
5.3 Solution of maximum wind speed.....	21
5.4 Analysis of wind speed change factors.....	26
6. Model evaluation and improvements.....	28
6.1 Advantages.....	28
6.2 Disadvantages.....	29
6.3 Improvement.....	29
7. Conclusions.....	30
8. Reference.....	31
Appendix.....	32
Main Code.....	32

1. Introduction

In recent years, UAVs (unmanned aerial vehicles) have been increasingly used in various fields. In particular, the quadcopter UAV is a popular multi-rotor UAV. With the advancement of modern technology, especially the rapid development of artificial intelligence technology, the future UAV industry application model has broad application prospects. However, in the process of performing missions in the air, UAVs are easily affected by wind and other factors, especially when hovering. Therefore, it is of great significance to study how UAVs are affected by wind during hovering flight.

2. Restatement and Analysis of Problems

2.1 Restatement

This paper studies the stability of a quad-rotor aircraft in the wind. The four-rotor UAV that we study has a mass of 1.5 kg and is driven by four rotors, each of which can generate up to 7 Newtons of thrust. Each center of the drone's rotors is 50 cm away from its center of mass, and arranged in a square. Our task is to determine a maximum wind speed where the drone can sustain within 20 cm away from the target location.

2.2 Analysis of Problems

As the drone flies in the air, it will be disturbed by the wind from time to time. When the UAV is exposed to the wind, its flying attitude will change. At this point, the drone's internal control system will sense this change through the drone's own sensors, and the drone will then make adjustments accordingly. There will be a delay between when the wind starts to kick in and when the drone adjusts. Our mission is to control the drone to move no more than 20 centimeters during that time. In other words, if the wind speed does not change, the UAV will remain relatively stable by maintaining one attitude against the wind all the time. But the reality is that the wind speed is changing all the time, and the drone's reaction to the wind speed is also a reaction to the wind speed at the last moment. Therefore, in order to solve the problem, the wind speed and direction should be numerically simulated first. For the numerical signal of wind speed received by the UAV and the distance signal from the target position, the UAV can be controlled to respond to it, so that the UAV only dithers within a circle with a radius of 20cm. In addition, for the stable flight of UAV, the lift force strategies generated by the four wings of UAV against different winds are analyzed through knowledge of gas dynamics, so as to enable the UAV to fly stably under unstable winds.

3. Assumptions and Symbols

3.1 Assumptions

1) In order to simplify the problem and make the model fit the reality better, this paper regards the aircraft as a rigid body, and ignores its elastic deformation;

2) Regarding the ground coordinate system as an inertial coordinate system to ignore the influence of the earth's rotation and revolution on the aircraft;

3) Referring to the reference, the data shows that the direction of the wind changes uniformly and slowly, so this paper assume sudden change barely occurs in a short time;

4) The acceleration of gravity remains unchanged in the flying field of the aircraft;

5) The shape and mass of the drone are X-symmetrical about its center;

6) Ignore the ground effect and the curvature of the earth, and the influence of the rotation of the earth on the aircraft is not considered, and the acceleration of gravity is constant.

3.2 Notations

Variables	Explanations
V_b	the average wind speed
V_g	gust wind speed
$V_{g\max}$	peak gust wind speed
t_1	the time when the gust began
T_g	gust wind cycle
V_r	gradient wind speed

$V_{r\max}$	peak speed of gradual wind
t_{r_1}	beginning time of the gradient wind
t_{r_2}	the ending time of the gradient wind
T_v	the lasting time of the gradient wind
V_n	speed of random wind
$V_{n\max}$	peak velocity of random wind
$R_{am}(-1,1)$	random number uniformly distributed between -1 and 1
ω_n	a random variable in $0.5 \sim 2\pi rad / s$
φ_n	a random variable in $0 \sim 2\pi rad / s$
G	the force of gravity on the drone
F	the force generated by the four propellers of the drone
f	the force of the wind on the drone
C	air viscosity coefficient
ρ	air mass density
S	the area of the top surface of the drone
θ	the angle between the UAV and the horizontal plane
v	wind speed
θ_y	pitch angle

θ_p	roll angle
θ_r	yaw angle
R_b^e	the conversion matrix from the body coordinate system to the ground coordinate system
ρ	the air density
θ	the angle between the drone plane and the horizontal plane
$F(t)$	the force required to adjust the flight state
x_{\max}	the maximum distance of the drone from the target
F_{\max}	maximum horizontal component of lift provided by the rotor of the drone

4. Model

4.1 Modeling the wind

4.1.1 An overview of the wind field

Because the problem studied in this paper is to determine the maximum wind speed that the quad-rotor UAV can maintain stable, it is necessary to model the wind field. By reading related literature, the Von Karman turbulence model and Dryden model are commonly used to simulate atmospheric wind speed, but these two theories are more suitable for high-altitude wind fields. The flying height of quadcopter UAVs is generally in the ultra-low altitude range, and it is not suitable to use the Von Karman turbulence model and Dryden model to establish a wind field.

Therefore, this paper uses a simple natural wind model to establish a wind field. The ultra-low altitude range naturally has a variety of characteristics such as suddenness, persistence, periodicity and uncertainty, so in accordance with these characteristics, the following four models are used to establish a wind field, which includes basic wind, gradient wind, gust wind and random wind respectively.

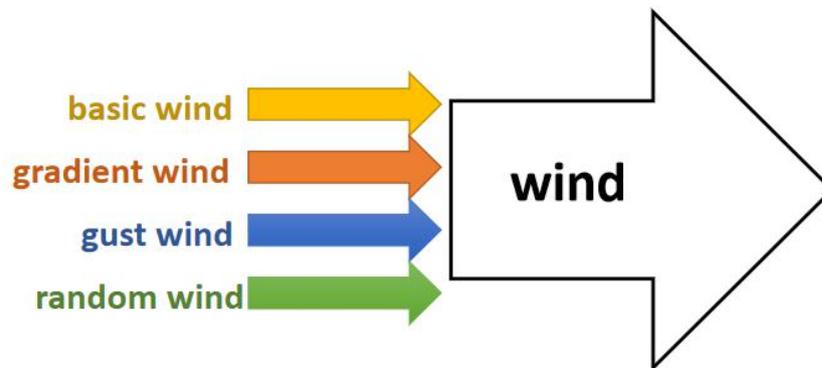


Fig.1 The composition of the wind

4.1.2 basic wind model

The basic wind speed is used to describe the basic value of the wind force in the whole process, and the characteristics of the rest wind speed are superimposed on the basis of the basic wind speed. It represents the basic size of the wind force in the whole process, which is close to the average value of the wind speed. It has stable characteristics and is represented by the constant value k .

$$V_b = k \quad (1)$$

Where, V_b is the basic wind velocity, k is a constant.

4.1.3 gust wind model

Gust wind shows the instantaneity of wind, which is expressed by a trigonometric function. In this way, it can not only reflect the transient characteristics of gust, but also reflect its periodicity in a small period of time:

$$V_g = \begin{cases} 0, (t < t_1 \text{ or } t > t_1 + T_g) \\ \frac{V_{g\max}}{2} \left(1 - \cos \left(2\pi \frac{t-t_1}{T_g} \right) \right), (t_1 \leq t \leq t_1 + T_g) \end{cases} \quad (2)$$

In the above equation, V_g is the velocity of gust wind, and $V_{r\max}$ is the velocity of gradient wind; t_{r_1} is the beginning time, while t_{r_2} is the ending time, and T_v is the lasting time.

4.1.4 Gradient wind model

In the process of increasing or decreasing wind, we can use gradual wind to represent it, using a linear piecewise function:

$$V_r = \begin{cases} 0, (t \leq t_{r_1} \text{ or } t \geq t_{r_2} + T_v) \\ V_{r\max} \frac{t-t_{r_1}}{t_{r_2}-t_{r_1}}, (t_{r_1} \leq t \leq t_{r_2}) \\ V_{r\max}, (t_{r_2} \leq t \leq t_{r_2} + T_v) \end{cases} \quad (3)$$

In this above equation, V_r is the velocity of gradient wind; $V_{r\max}$ is the peak velocity of gradient wind; t_{r_1} is the beginning time; t_{r_2} is the ending time; T_v is the lasting time.

4.1.5 random wind model

The speed of random wind V_n is used to describe the randomness of wind speed at flight altitude:

$$V_n = V_{n\max} R_{am}(-1,1) \cos(\omega_n t + \varphi_n) \quad (4)$$

In the above equation, V_n is the velocity of random wind; $V_{n\max}$ is the peak speed of the random wind; $R_{am}(-1,1)$ is a uniformly distributed random number between -1 and 1.

4.1.6 Definition of maximum wind speed

Based on the wind speed description in the above 4 parts, from equation (1) to equation (4), the simulated wind speed acting on the aircraft is:

$$V = V_b + V_g + V_r + V_n \quad (5)$$

In the above equations, V_b , V_g , V_r and V_n respectively represent basic wind, gust wind, gradual wind, and random wind.

From the restatement of the problem in Section 2.1, the problem to be solved in this article is that the quad-rotor UAV can maintain a stable maximum wind speed, so we define the expression of the maximum wind speed as:

$$V_{\max} = V_b + V_{g\max} + V_{r\max} + V_{n\max} \quad (6)$$

4.1.7 Calculation of wind force

According to the data, the expression of wind force is

$$f = \frac{1}{2} C \rho S \sin \theta v^2 \quad (7)$$

Among them, C represents the viscosity coefficient of the air, ρ represents the air density, S represents the area of the drone plane, θ is the angle between the drone plane and the horizontal plane, and N represents the wind speed.

4.2 Dynamics model of drone hovering

4.2.1 The structure of the UAV

There are two common four-rotor UAV flight structures, as shown in figure (). There is no essential difference in theoretical analysis between the two structures. Considering that there are more X-shaped structures in actual flight, this article uses X structure As the flying structure of a quadcopter UAV. In terms of control algorithms, although the X-type structure has a slightly larger algorithm complexity, it has the advantages of more flexible flight and faster response.

According to the meaning of the question, each center of the rotor of the UAV in this paper is 50 centimeters away from its center of mass, arranged in a square, so this paper chooses X-type quadcopter aircraft for research.

The four rotors of the four-rotor aircraft adopt a combination configuration of forward and reverse rotation, and a pair of adjacent rotors rotate in opposite directions, which mutually cancel the effect of anti-torque on the aircraft. As shown in the figure, the No. 1 and No. 3 rotors rotate counterclockwise, and the No. 2 and No. 4 rotors rotate counterclockwise. This configuration simplifies the structure of the quadcopter and makes it easy to realize its control technology.

4.2.2 Necessary assumptions

The dynamic equation of the four-rotor UAV hovering can be derived by Newton-Euler formula. In order to facilitate the establishment of the model, it is necessary to omit or simplify the physical quantities that have less influence on the aircraft. Therefore, the following assumptions are made :

- a. Ignore the ground effect and the curvature of the earth, do not consider the influence of the earth's rotation on the aircraft, and the acceleration of gravity is constant.
- b. Regard the airframe structure and rotor as rigid bodies, ignoring the elastic deformation and vibration of the airframe.
- c. The four motors and propellers are installed symmetrically, and other parameters are the same except for the positive and negative polarities.
- d. The fuselage is symmetrical about each axis of the fuselage coordinate system, and the mass distribution of the fuselage is uniform and the center of mass coincides with the center of the shape.

4.2.3 Definition of transformation matrix

Establishment of ground coordinate system and airframe coordinate system^[1].

The ground coordinate system is used to study the motion state of the four-rotor aircraft relative to the ground, and to determine the spatial position of the airframe. According to the right-hand rule, this article selects the take-off location of the aircraft (the position before take-off initialized by the aircraft control system) as the origin of the coordinate system. The X_e axis points to the geographic north, the Y_e axis points to the geographic east, and the Z_e axis is perpendicular to the ground.

The aircraft coordinate system is fixedly connected to the aircraft. The center of mass of the quad-rotor aircraft is taken as the origin. The longitudinal axis of symmetry after taking the weight of the aircraft is the X_b axis, and the front of the fuselage is the positive direction of the X axis; The lateral symmetry axis of the center is the Y_b axis, and the right side of the fuselage is the positive direction of the Y_b axis; and the definition of the Z_b axis satisfies the right-hand rule, and the positive direction points to the top of the fuselage.

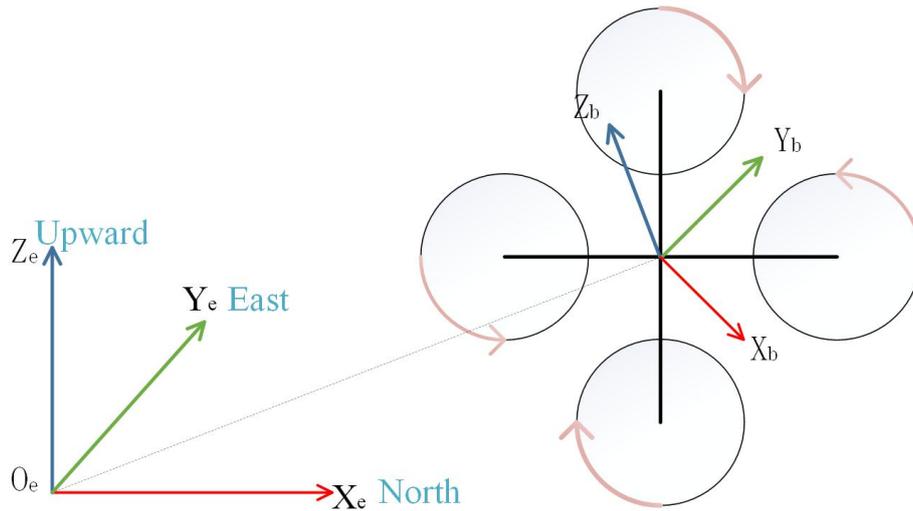


Fig.2 The Change of Coordinate System

The coordinate transformation from the ground coordinate system to the body coordinate system can be solved by the rotation matrix. The rotation matrix is defined below.

$$R = [b_1, b_2, b_3] \tag{8}$$

The rotation matrix has the following properties:

$$RR^T = R^T R = I_3 \tag{9}$$

Where, I_3 is 3rd order identity matrix.

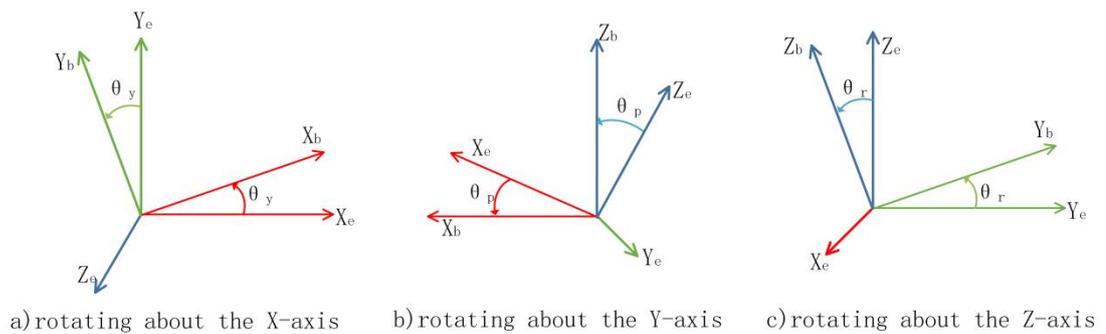


Fig.3 rotating about the axis

Through the attitude angle of the aircraft (pitch angle θ_y , roll angle θ_p , yaw angle θ_r), the coordinate conversion matrix between the aircraft body coordinate system and the earth surface inertial coordinate system can be obtained, and the change from the earth surface inertial coordinate system to the body coordinate system can be obtained by Three-step rotation to complete [3].

First, assuming that the airframe only has yaw motion, the conversion from the ground coordinate system to the airframe coordinates is a rotation change around the Z axis, as shown in Figure (3-a), the rotation matrix R_z can be obtained:

$$R_z = \begin{bmatrix} \cos \theta_y & \sin \theta_y & 0 \\ -\sin \theta_y & \cos \theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

Next, assuming that the body only has pitching motion, the conversion from the ground coordinate system to the body coordinates is a rotation around the Y axis, as shown in Figure (3-b), the rotation matrix R_y can be obtained:

$$R_y = \begin{bmatrix} \cos \theta_p & 0 & -\sin \theta_p \\ 0 & 1 & 0 \\ \sin \theta_p & 0 & \cos \theta_p \end{bmatrix} \quad (11)$$

Then, assuming that the body only has rolling motion, the conversion from the ground coordinate system to the body coordinates is a rotation around the X axis, as shown in Figure (3-c), the rotation matrix can be obtained:

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_r & \sin \theta_r \\ 0 & -\sin \theta_r & \cos \theta_r \end{bmatrix} \quad (12)$$

And finally, the combination of these three hypotheses namely rotating about the Z , Y , and X axis is the way to achieve the transformation of the ground coordinate system to the body coordinate system, which is denoted here as:

$$\begin{aligned} R_e^b &= R_z(\theta_y) \cdot R_y(\theta_p) \cdot R_x(\theta_r) \\ &= \begin{bmatrix} \cos \theta_y \cos \theta_p & \sin \theta_y \cos \theta_r + \cos \theta_y \sin \theta_p \sin \theta_r & \sin \theta_y \sin \theta_r - \cos \theta_y \sin \theta_p \cos \theta_r \\ -\sin \theta_y \cos \theta_p & \cos \theta_y \cos \theta_r - \sin \theta_y \sin \theta_p \sin \theta_r & \cos \theta_y \sin \theta_r + \sin \theta_y \sin \theta_p \cos \theta_r \\ \sin \theta_p & -\cos \theta_p \sin \theta_r & \cos \theta_p \cos \theta_r \end{bmatrix} \end{aligned} \quad (13)$$

Using the properties of the rotation matrix, the conversion matrix R_b^e from the body coordinate system to the ground coordinate system can be obtained:

$$\begin{aligned} R_b^e &= (R_e^b)^{-1} = (R_e^b)^T = R_z(-\theta_y) \cdot R_y(-\theta_p) \cdot R_x(-\theta_r) \\ &= \begin{bmatrix} \cos \theta_y \cos \theta_p & -\sin \theta_y \cos \theta_r + \cos \theta_y \sin \theta_p \sin \theta_r & \sin \theta_y \sin \theta_r - \cos \theta_y \sin \theta_p \cos \theta_r \\ \sin \theta_y \cos \theta_p & \cos \theta_y \cos \theta_r + \sin \theta_y \sin \theta_p \sin \theta_r & \cos \theta_y \sin \theta_r + \sin \theta_y \sin \theta_p \cos \theta_r \\ -\sin \theta_p & \cos \theta_p \sin \theta_r & \cos \theta_p \cos \theta_r \end{bmatrix} \end{aligned} \quad (14)$$

4.2.4 Analysis of hover state

With regard to the research^[4], the dynamics model of the quadcopter UAV has been well established. These studies are carried out under the assumption that the wind speed is zero. This article refers to their model and converts it into a dynamic model of a quadcopter drone hovering in the wind. The analysis is as follows.

The force analysis of the quad-rotor UAV shows that there are three main sources of force acting on the aircraft: the gravity on the body, the combined external force generated by the rotation of each rotor of the quad-rotor, and the force of the wind, which are introduced below.

The total lift force generated by the i th rotor is denoted as F_i , and

$$F_i \in [0,7] \quad (15)$$

Next, calculate the total lift F generated by the rotor in the ground coordinate system and distribute it to each axis in the inertial coordinate system, and the component force of F on each axis:

$$\begin{aligned} F_t &= \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = R_b^e \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \sum_{i=1}^4 (F_i - f_i) \\ &= \begin{bmatrix} \sin \theta_y \sin \theta_r + \cos \theta_y \sin \theta_p \cos \theta_r \\ -\cos \theta_y \sin \theta_r + \sin \theta_y \sin \theta_p \cos \theta_r \\ \cos \theta_p \cos \theta_r \end{bmatrix} \sum_{i=1}^4 (F_i - f_i) \end{aligned} \quad (16)$$

Among them, R_b^e is the conversion matrix from the body coordinate system to the ground coordinate system in the formula (13).

The gravity experienced by the drone is:

$$F_g = [0,0,mg]^T \quad (17)$$

Calculated from the wind force in section 4.1.7, the air resistance of the drone is:

$$f = \frac{1}{2} C \rho S \sin \theta v^2 = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} C \rho S \sin \theta v_x^2 \\ \frac{1}{2} C \rho S \sin \theta v_y^2 \\ \frac{1}{2} C \rho S \sin \theta v_z^2 \end{bmatrix} \quad (18)$$

Combining formulas (16) to (18), the resultant force of the quadcopter UAV in linear translation can be obtained. Combining with Newton's second law, the linear motion equation of the body can be obtained as:

$$mV'_e = F_i - f - F_g \quad (19)$$

In the above formula, m is the total take-off mass of the four-rotor UAV, expand this formula to get:

$$m \begin{bmatrix} x''_e \\ y''_e \\ z''_e \end{bmatrix} = \begin{bmatrix} \sin \theta_y \sin \theta_r + \cos \theta_y \sin \theta_p \cos \theta_r \\ -\cos \theta_y \sin \theta_r + \sin \theta_y \sin \theta_p \cos \theta_r \\ \cos \theta_p \cos \theta_r \end{bmatrix} \sum_{i=1}^4 (F_i - f_i) - \begin{bmatrix} \frac{1}{2} C \rho S \sin \theta v_x^2 \\ \frac{1}{2} C \rho S \sin \theta v_y^2 \\ \frac{1}{2} C \rho S \sin \theta v_z^2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \quad (20)$$

After rearranging the above equation, we can get:

$$\begin{cases} x''_e = \frac{1}{m} \left[(\sin \theta_y \sin \theta_r + \cos \theta_y \sin \theta_p \cos \theta_r) \sum_{i=1}^4 (F_i - f_i) - \frac{1}{2} C \rho S \sin \theta v^2 \right] \\ y''_e = \frac{1}{m} \left[(-\cos \theta_y \sin \theta_r + \sin \theta_y \sin \theta_p \cos \theta_r) \sum_{i=1}^4 (F_i - f_i) - \frac{1}{2} C \rho S \sin \theta v_y^2 \right] \\ z''_e = \frac{1}{m} \left[(\cos \theta_p \cos \theta_r) \sum_{i=1}^4 (F_i - f_i) - \frac{1}{2} C \rho S \sin \theta v^2 \right] - g \end{cases} \quad (21)$$

4.2.5 Horizontal force analysis

The quad-rotor drone is simplified into a mass point, as shown in the figure below, it is subjected to gravity G , the combined force generated by the four propellers F , and the wind f on the drone.^[5]

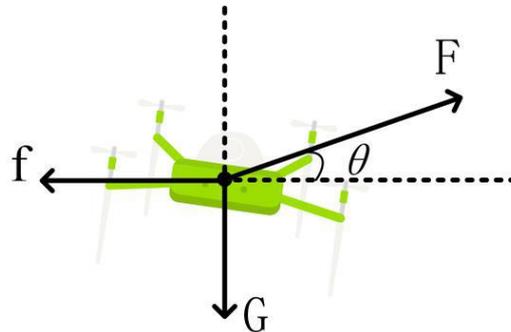


Fig.4 Horizontal force analysis of UAV

By consulting the reference, the expression of wind force is:

$$f = \frac{1}{2} C \rho S \sin \theta v^2 \quad (22)$$

Among them, C represents the viscosity coefficient of the air, ρ represents the air density, S represents the area of the drone plane, θ is the angle between the drone plane and the horizontal plane, and v represents the wind speed.

Using Newtonian equations, we have:

$$F \cos \theta = G \quad (23)$$

$$F \sin \theta - f = ma \quad (24)$$

Among them, m represents the mass of the drone, and A represents the acceleration generated by the combined force received by the drone.

With regard to formula(21),we can know that

$$a = \begin{bmatrix} x_e'' \\ y_e'' \\ z_e'' \end{bmatrix} = \begin{bmatrix} \frac{1}{m} \left[(\sin \theta_y \sin \theta_r + \cos \theta_y \sin \theta_p \cos \theta_r) \sum_{i=1}^4 (F_i - f_i) - \frac{1}{2} C \rho S \sin \theta v^2 \right] \\ \frac{1}{m} \left[(-\cos \theta_y \sin \theta_r + \sin \theta_y \sin \theta_p \cos \theta_r) \sum_{i=1}^4 (F_i - f_i) - \frac{1}{2} C \rho S \sin \theta v_y^2 \right] \\ \frac{1}{m} \left[(\cos \theta_p \cos \theta_r) \sum_{i=1}^4 (F_i - f_i) - \frac{1}{2} C \rho S \sin \theta v^2 \right] - g \end{bmatrix} \quad (25)$$

4.3 UAV control system

4.3.1 The overview of UAV control system

During the flight of the UAV, it needs to constantly adjust its flight attitude according to the changes in the wind to ensure that it can maintain balance in the air. Wind changes can be divided into wind direction changes and wind speed changes. We assume that in a very short period of time, only one of the wind speed and wind direction changes. In other words, this article assumes that when one of the speed or direction of the wind changes, the wind will not change the next time before the drone successfully adjusts its own state.

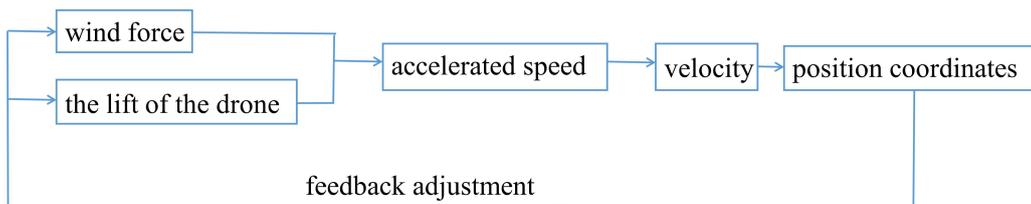


Fig.5 Schematic diagram of drone feedback adjustment

As shown in the figure above, the combined force of the force of the wind and the UAV's own lift makes the UAV produce an acceleration^[6]. Under the action of the combined force, the speed and position coordinates of the drone are constantly changing. When the distance of the drone from the target position is close to the critical value, the drone's sensor device will detect the change of the position coordinate. Then, the UAV's control system will control the UAV's lift and adjust the UAV's motion state.

Wind changes can be divided into wind direction changes and wind speed changes. For wind changes, if the wind speed does not reach the maximum wind speed, the wind changes can be divided into larger winds and smaller winds; changes in wind direction can be divided into changes in the horizontal plane and changes in the vertical plane. Study each situation separately to get the adjustment strategy of the UAV.

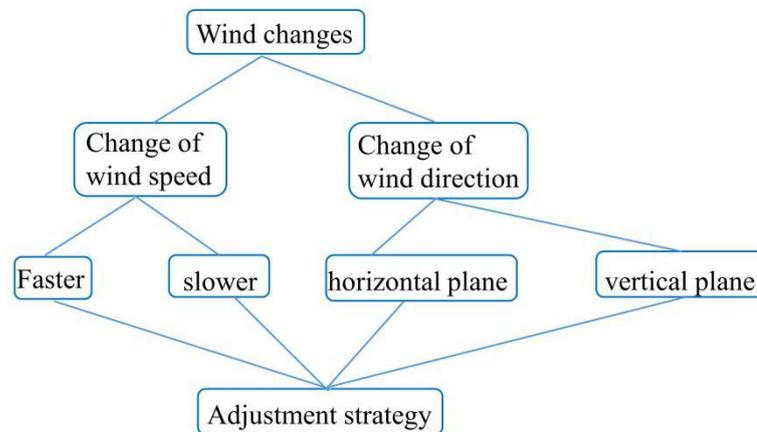


Fig.6 Schematic diagram of UAV adjusting its flight status according to wind characteristics

For changes in wind speed, when the wind speed increases, the force of the wind on the UAV becomes greater. The UAV needs to increase the rotation speed of the rotor to provide greater lift, so as to maintain balance; on the contrary, when the wind speed decreases, The force of wind on the UAV is reduced, and the UAV can reduce the rotation speed of the rotor to reduce the lift, so as to maintain balance.

For the change of wind direction, when the wind direction changes in the horizontal plane, the drone can adjust the speed difference between the rotors. According to the working principle of the quadcopter drone, it can be known that there is a speed difference between the rotors of the drone. When the drone will roll or pitch. In this way, the angle of the drone can be adjusted to achieve a new balance while maintaining dynamic balance. When the direction of the drone changes in the vertical plane, the lift of the drone can be adjusted to ensure that the drone maintains a dynamic balance. In addition, when the wind direction changes in other angles, vector decomposition can be used to decompose the wind speed changes into the horizontal and vertical planes, and the corresponding adjustments can be made according to the above rules.

4.3.2 The quantitative solution of UAV control system

In the former section, the adjustment process of UAV is described qualitatively. In the actual process, the adjustment process of UAV needs to be solved quantitatively. After the UAV is subjected to a certain wind force, the UAV needs to adjust its flight state according to its own state and the size of the wind force through the lift provided by the rotor.

The force analysis of the UAV can be obtained, the UAV receives the lift provided by the gravity, the rotor and the horizontal wind force. The maximum lift provided by the rotor is 28 Newtons. The maximum component of lift in the horizontal direction is shown in the following figure.

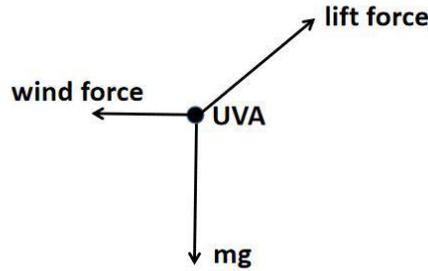


Fig.7 The force analysis

The maximum horizontal component of lift provided by the rotor of the drone can be obtained as:

$$F_{\max} = \sqrt{F_{\text{wind}}^2 + F_{\text{lift}}^2} \quad (26)$$

The UAV needs to continuously determine the force $F(t)$ required to adjust the flight state at this moment according to the wind speed and the magnitude of the wind force at the previous moment. Suppose that the position coordinate of the drone at this time is x , and the maximum distance of the drone from the target area is x_{\max} .

If $2f(t-1) < F_{\max}$, then the adjustment force required by the drone at time t is:

$$F(t) = f(t-1) + f(t-1) * \frac{x}{x_{\max}} \quad (27)$$

Otherwise, if $2f(t-1) > F_{\max}$, then the adjustment force required by the drone at time t is:

$$F(t) = f(t-1) + [F_{\max} - f(t-1)] * \frac{x}{x_{\max}} \quad (28)$$

5. Model Solving and Analysis

5.1 Setting of relevant parameters

5.1.1 Definition of UAV stability

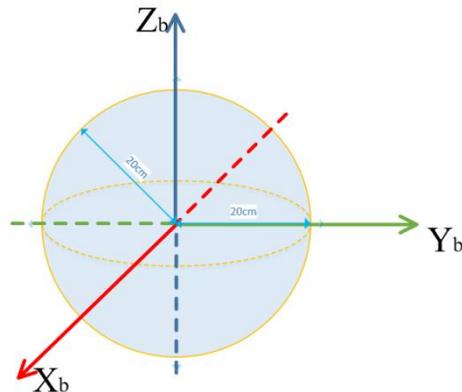


Fig.8 The stability space of UAV

In order to make the UAV hover in the air and work normally, this article believes that the conditions for the UAV to stabilize are as follows:

(1) Any action made by the UAV in the hovering state will not cause its displacement offset relative to the target center to exceed 20cm.

(2) If the UAV can maintain conditions (1) within 60s, the UAV is considered stable

5.1.2 Air viscosity and air density

according to the reference, this paper sets the parameters in the formula (7) as follows:

$$C = 0.3 \quad (29)$$

$$\rho = 1.225 \text{ kg} / \text{m}^3 \quad (30)$$

5.1.3 The setting of UAV parameters

First of all, by consulting the information of the quadcopter UAV, this article sets the basic parameters of the UAV as shown in the table below.

Table 1 The basic parameters of the UAV

Type	length (m)	width (m)	height (m)

flight control board + battery	0.30	0.30	0.10
rack arm	0.50	0.05	0.04
propeller	0.40	0.04	0.005

Then, According to the figure below, define the windward area S_1 of the top surface and the windward area S_2 of the side

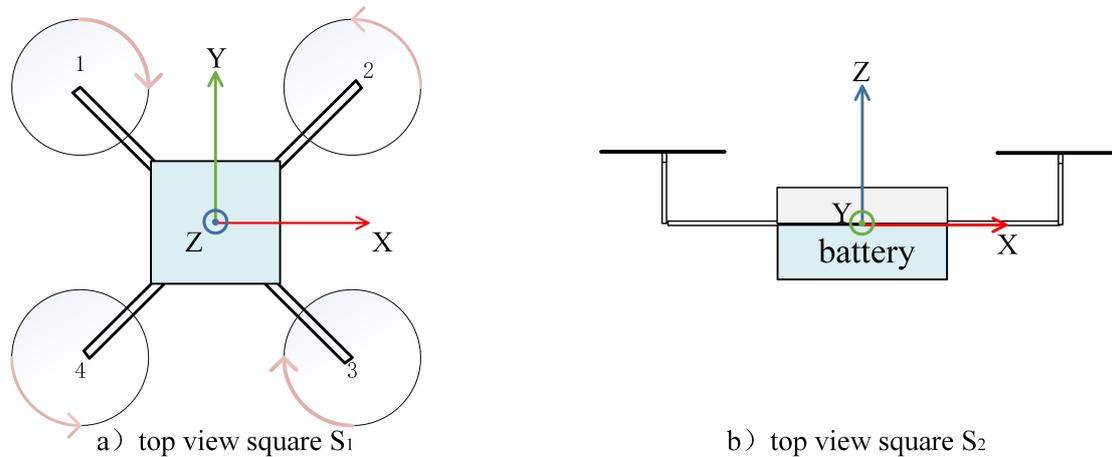


Fig.9 Windward area of top view and test chart

Through simple mathematical knowledge of plane geometry, the following data is obtained:

$$S_1 = 0.0583m \quad (31)$$

$$S_2 = 0.6102m \quad (32)$$

Finally, we set the wind speed sampling frequency of the quad-rotor UAV to 5Hz, that is, the UAV's wind speed sensing module samples the outside wind speed every 0.2s. In addition, we set the control and adjustment time of the quad-rotor UAV to 0.2s, that is, after the UAV receives the input value of the wind speed sensor module, it takes 0.2s to adjust to the attitude of the wind speed value.

5.2 Definition of change factor ε

ε is a value between 0 and 1, we call it the wind speed change factor, which is the amount that indicates how fast the wind speed changes in the natural world.

The wind speed components in formulas (1), (2), (3), and (4) are shown as follows:

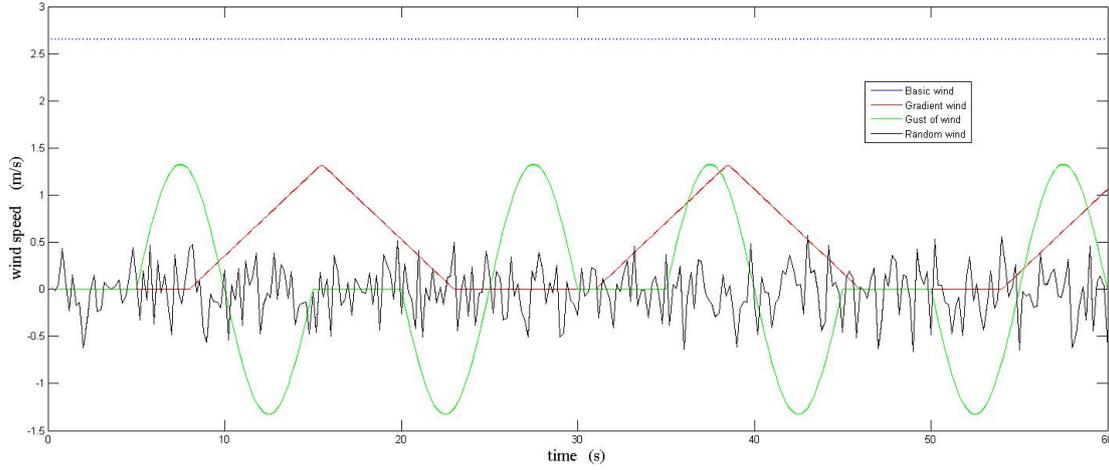


Fig.10 The wind speed components change graph

With regard to the research by E. Kuantama^[2], the dynamics model of the quadcopter UAV has been well established. These studies are carried out under the assumption that the wind speed is zero. This article refers to their model and converts it into a dynamic model of a quadcopter drone hovering in the wind. The analysis is as follows.

It can be seen from the figure that there is a definite functional relationship between gust and gradual wind over time, while random wind has uncertainty, and the mean value of random wind is zero, so the expression that defines the maximum wind speed is

$$V_{\max} = V_b + V_{g\max} + V_{r\max} \quad (33)$$

However, there is a maximum value for random wind, which is

Since gust and gradual wind are both components of the wind and have a definite functional relationship, this article believes that they have the same importance and make both equal to εV_b , i.e.

$$V_{g\max} = V_{r\max} = \varepsilon V_b \quad (34)$$

From the formulas (33) and (34), the maximum wind speed is

$$V_{\max} = (1 + 2\varepsilon)V_b \quad (35)$$

Next we will discuss the maximum wind speed factor and solve the maximum wind speed.

5.3 Solution of maximum wind speed

The wind in nature is random and constantly changing. In order to facilitate the study of the influence of wind speed change factors, the two random numbers of random wind in the formula (4) are set to be the same, and the start time of gust and

gradual wind is set to be the same. Therefore, the trend of wind speed changes with time in the diagrams (11), (13) and (15) below are basically the same.

The period of gust is $T_g = 10s$, the cycle of the gradient wind $T_r = 15s$. the analysis is as follows

(1) If wind speed change factor $\varepsilon = 0.5$

Let the basic wind increase from 0, look for the maximum wind speed, increase by $0.01m/s$ in each test, and get the basic wind speed as $V_b = 2.65m/s$, the wind speed reaches the maximum. If the wind speed continues to increase, the UAV does not meet the stability constraints in Section 5.1. The wind speed changes with time are shown in the figure (11) below.

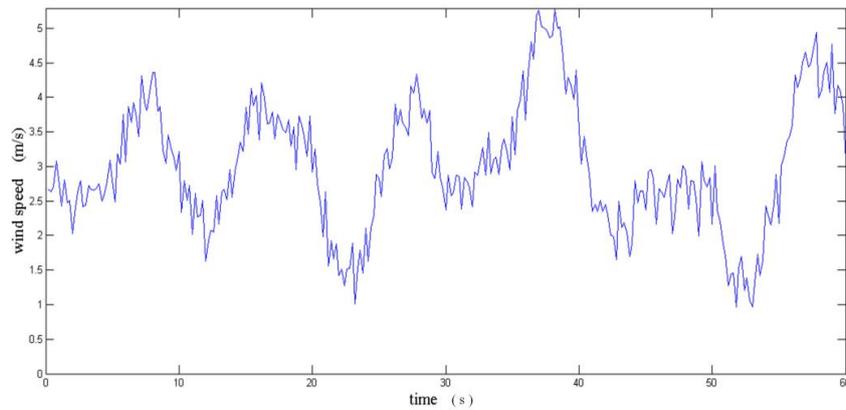


Fig.11 Wind speed graph over time when $V_b=2.65m/s$

The figure above represents the simulated wind speed. The peak value of the image is defined as V_{\max}^{Sim} , which represents the maximum value of the simulated wind speed.

The figure above shows a real-time simulation of wind speed, its peak is $5.2994m/s$, i.e. $V_{\max}^{Sim} = 5.2994$.

Using the formula (35), the theoretical maximum wind speed is calculated as

$$V_{\max} = (1 + 2\varepsilon)V_b = (1 + 2 \times 0.5) \times 2.65 = 5.30m/s \quad (36)$$

From the results, we can see that $V_{\max}^{Sim} \approx V_{\max}$, which indicates that the maximum wind speed model established in this paper accords with the reality.

At this time, the offset distance from the center of the target when the quadcopter drone is hovering is shown in the figure below

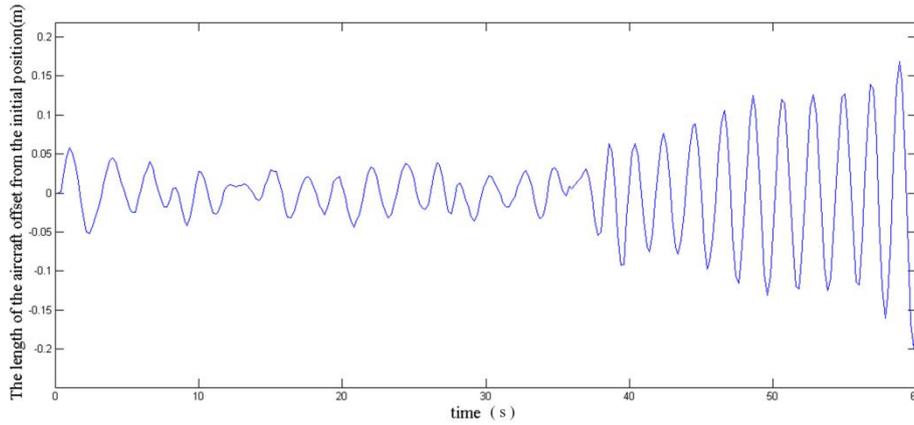


Fig.12 When $V_b=2.65\text{m/s}$, Variation diagram of UAV displacement offset

As can be seen from the figure above, the quad-rotor UAV can stay within 20cm of the target center within 0~60s.

The following shows the wind speed variation diagram and the displacement offset variation diagram of UAV when $V_b = 2.66\text{m/s}$.

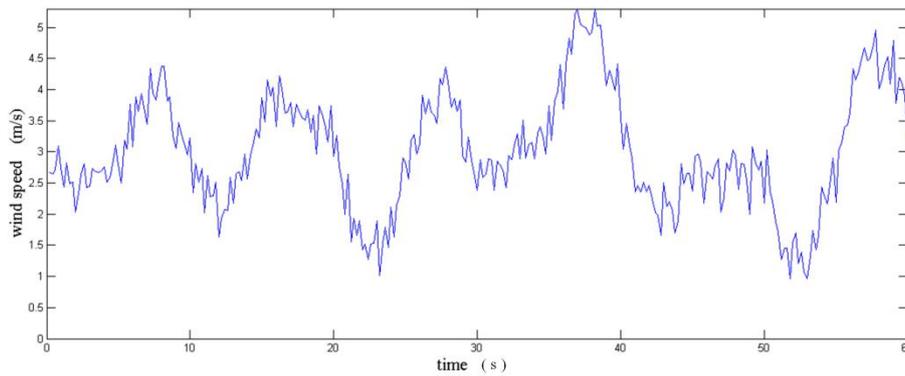


Fig.13 Wind speed change graph when $V_b=2.66\text{m/s}$

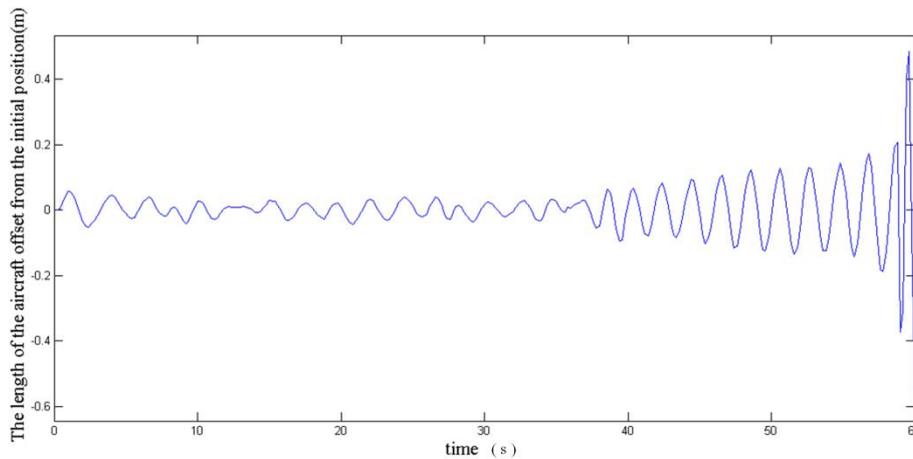


Fig.14 UAV displacement offset change diagram when $V_b=2.66\text{m/s}$

It can be seen from the above figure that when the time increases to about 59s, the displacement offset of the UAV exceeds 20cm. We can draw a conclusion that We can draw a conclusion that if $V_b = 2.65m/s$, the corresponding maximum velocity V_{max} is correct.

(2) If wind speed change factor $\varepsilon = 0.8$, same as the method in (1), finding the basic wind corresponding to the maximum speed as $V_b = 2.14m/s$, The variation of wind speed over time is shown in the figure below.

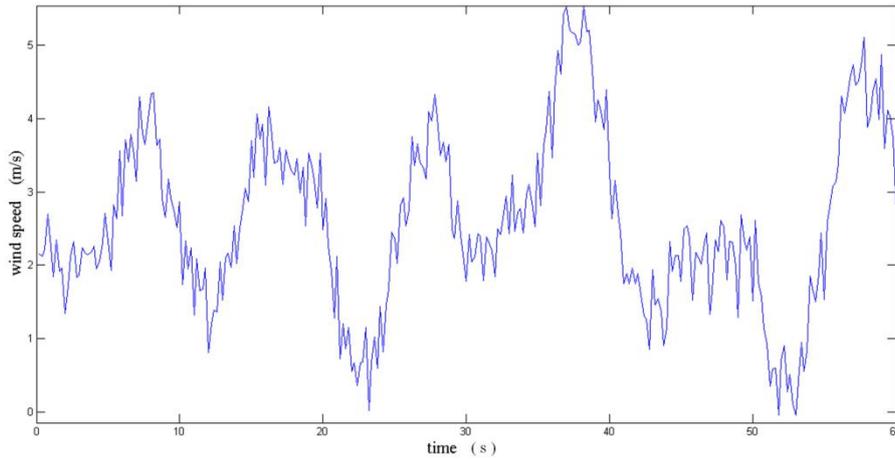


Fig.15 Wind speed change diagram when the maximum wind speed is reached($\varepsilon=0.8$)

The peak in the above figure is 5.5375m/s i.e. $V_{max}^{Sim} = 5.5375m/s$. Using the formula (35), the theoretical maximum wind speed is calculated as

$$V_{max} = (1 + 2\varepsilon)V_b = (1 + 2 \times 0.8) \times 2.14 = 5.564m/s \quad (37)$$

From the results, we can draw a conclusion that $V_{max}^{Sim} \approx V_{max}$.

At this point, the distance from the target center offset in the hover state of the four-rotor UAV is shown in the figure below.

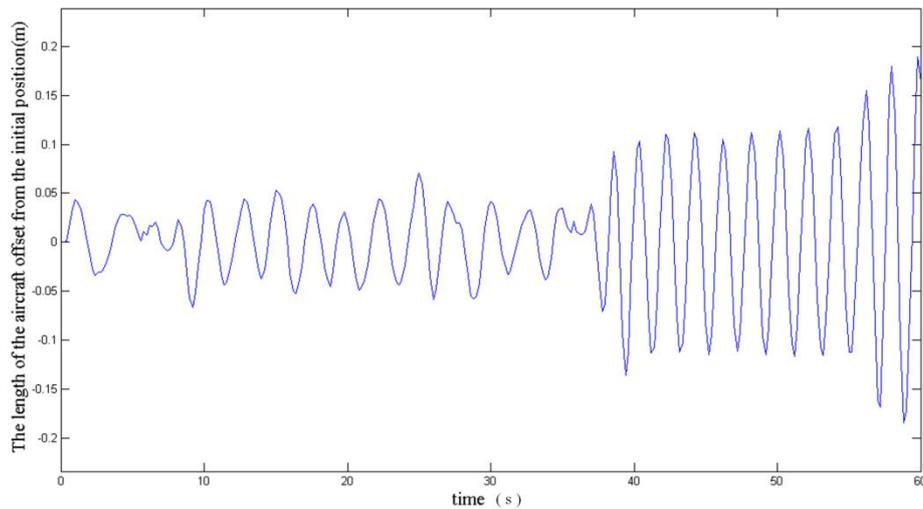


Fig.16 variation diagram of displacement offset when the maximum wind speed critical is reached ($\varepsilon=0.8$)

As can be seen from the above picture, the UAV can maintain a stable state within 60s.

(3) If wind speed change factor $\varepsilon = 0.2$, same as the method in (1), finding the basic wind corresponding to the maximum speed as $V_b = 3.91\text{m/s}$, The variation of wind speed over time is shown in the figure below.

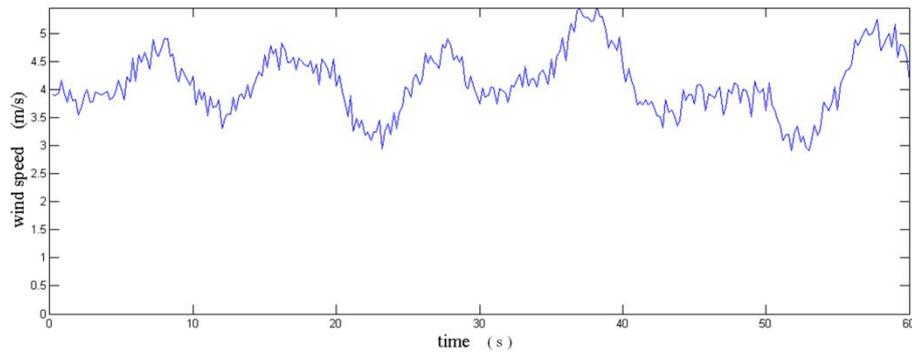


Fig.17 Wind speed change diagram when the maximum wind speed is reached ($\varepsilon=0.2$)

The peak in the above figure is 5.4619m/s , i.e. $V_{\max}^{\text{Sim}} = 5.4619\text{m/s}$. Under the circumstances, using formula (35), calculate the theoretical maximum wind speed is

$$V_{\max} = (1 + 2\varepsilon)V_b = (1 + 2 \times 0.2) \times 3.91 = 5.474\text{m/s} \quad (38)$$

From the results, we can draw a conclusion that $V_{\max}^{\text{Sim}} \approx V_{\max}$.

At this time, the offset distance from the center of the target when the quadcopter drone is hovering is shown in the figure below.

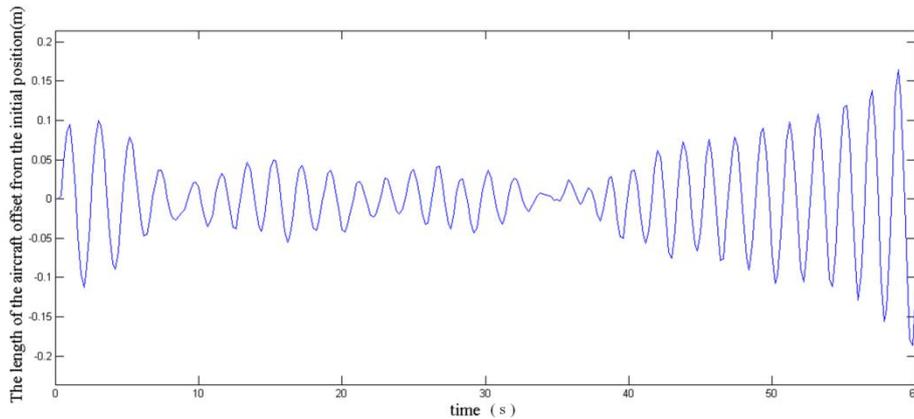


Fig.18 variation diagram of displacement offset when the maximum wind speed critical is reached ($\epsilon=0.2$)

As can be seen from the above figure, the drone can maintain a stable state within 60s.

5.4 Analysis of wind speed change factors

The variation factor of wind speed is taken as the independent variable, and the theoretical maximum wind speed V_{max} simulation maximum wind speed V_{max}^{Sim} basic wind speed V_b is taken as the dependent variable. Its variation relationship is shown in the figure below

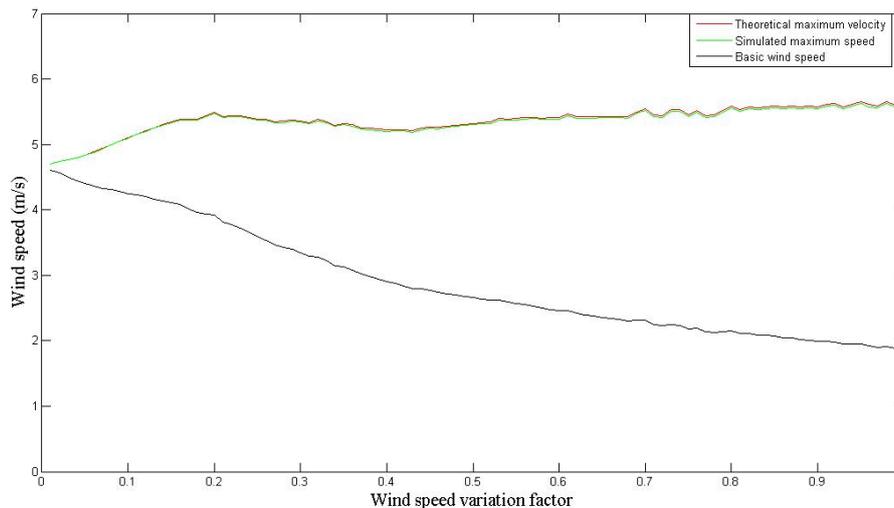


Fig.19 The variation relationship between factor and wind speed

The analysis of the above figure is as follows: the abscissa represents the wind speed change factor, and the ordinate represents the basic wind speed of the maximum wind speed obtained by traversal under the condition of the wind speed change factor, which is represented by a black line. According to the formula (35), the theoretical maximum wind speed value is calculated and indicated by the red line. The green line represents the real-time change graph of the simulated wind speed.

Since the greater the change factor, the greater the volatility of the wind speed, combined with the formula (35), the value of the fluctuating part $2\varepsilon V_b$ will become larger, and the maximum wind speed V_{\max} is constant, so V_b gradually decreases with the increase of the change factor small.

The theoretical maximum wind speed V_{\max} and the simulated maximum wind speed V_{\max}^{Sim} in the figure (20) are very close. Enlarge the two as shown in the figure below

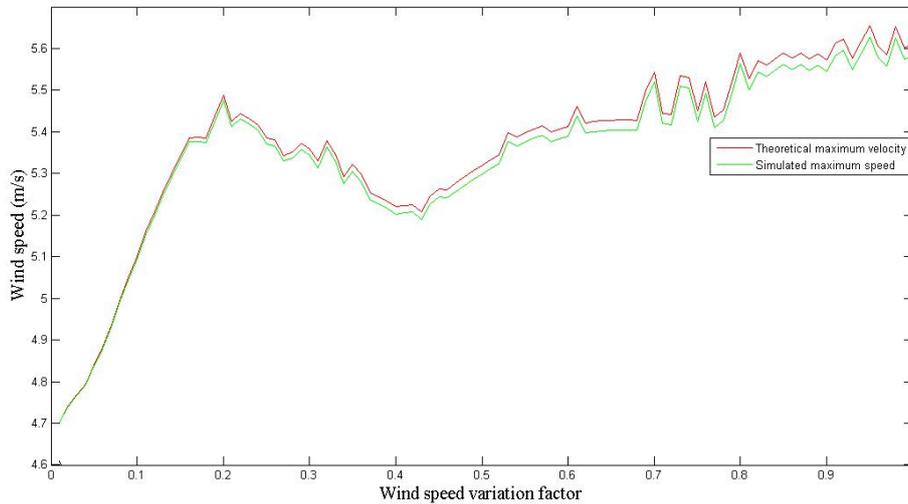


Fig.20 The enlarged view of Fig.19

The analysis of the above figure is as follows: With the increase of the wind speed change factor, the theoretical value and the simulated value of the maximum wind speed tend to be stable. Calculate the average value of the two under different change factors, and get $V_{\max} = 5.3678m/s$, $V_{\max}^{Sim} = 5.3487m/s$

Combining this result with the formulas (36), (37), (38) in Section 5.3, we know

$$V_{\max}^{Sim} \approx V_{\max} \quad (39)$$

The above conclusions show that the theoretical model of maximum wind speed established in this paper can fit the reality well and show the correctness of the model.

5.5 interpretation of result

Through the simulation of different wind speeds, this paper concludes that the maximum wind speed that the four-rotor UAV can maintain within 60s under the limit of not exceeding 20cm of the target center is: 5.2795m/s. At this time, when the basic wind speed of the wind is 2.65m/s, the maximum gust wind speed is 1.33m/s, the maximum gradual wind speed is 1.33m/s, the maximum random wind speed is 0.66m/s, and the wind speed variation factor $\varepsilon = 0.5$.

6. Model evaluation and improvements

6.1 Advantages

1. A wind model is established in this paper. The wind speed is decomposed into basic wind speed and disordered wind speed, which includes gust wind, gradual wind and random wind.

In addition, the wind model is refined into 4 small parts, which can better reflect the authenticity of the model. At the same time, for the convenience of subsequent calculation, we designed the definition of maximum wind speed, and synthesized the 4 small parts into maximum wind speed.

1.1 The average value of wind speed is used to represent the basic wind, which reflects the stability of the basic wind.

1.2 Use trigonometric function to represent the gust, which not only reflects the characteristics of the gust sometimes and sometimes, but also shows the periodicity of the time of the gust.

1.3 The gradual wind is represented by the linear piecewise function, which can better reflect the gradual increase or decrease of wind force.

1.4 Two random variables are introduced into the expression of random wind, which can better reflect the randomness of wind force.

2. The numerical simulation model of unstable wind is established, and the variation of wind speed with time is given, which makes the wind speed close to the real wind and facilitates the calculation. In addition, the shaking condition of the UAV under unstable wind is given, and the variation of the distance of the UAV deviating from the target position with time is given. Finally, the expected value of the theoretical maximum wind speed and the average value of the simulated wind speed are calculated.

3. In this paper, the ground coordinate system and the airborne coordinate system are established, and the rotation transformation matrix in the transformation process of the two coordinate systems is deduced to quantify the rotation process. Meanwhile, the form of the matrix can be easily understood and calculated by the computer, so as to improve the reaction speed of UAV.

4. Based on the UAV sensing the wind speed at the last moment and simulating the relatively stable structure of the spring, the UAV's response mechanism is established. At the same time, the UAV's reaction time against wind speed changes is taken into account, which is more in line with the real situation. For example, when the drone is at a location close to the target, the adjustment space for the drone is relatively large. At this time, it is not necessary to use the maximum lift to adjust the attitude of the drone. On the basis of the wind speed, adjust the drone with a smaller lift and a smaller angle; when the drone is close to the critical distance from the target position, it is not only necessary to ensure that the drone will not leave the target beyond the specified distance Range, but also to ensure that the UAV can move to the target position to reduce the distance between the UAV and the target position. At this time, the UAV adjusts the angle to provide a larger lift.

6.2 Disadvantages

1. This model only studies the movement of the UAV under the horizontal action, but fails to study how the UAV moves when the wind direction changes. But in fact, the direction of the wind in nature changes at any time. When the drone is windy in the air, it is likely to receive the effect of wind from above or below. Except for the horizontal direction, this article does not make a quantitative study of wind in other directions. This model just qualitatively explains the wind in other directions.

2. After the drone is affected by the wind, it should continuously adjust its posture according to the current wind direction as soon as possible, so as to achieve balance in the shortest possible time. Otherwise, the drone will be too far away from the target. The research process of the adjustment strategy of UAV in this paper is not sufficient. There is no quantitative description of how the drone adjusts itself under the action of wind. How to coordinate the four rotors to adjust the attitude of the drone;

3. The research on the adjustment strategy of UAV in this article is not enough. In fact, when the wind changes, the UAV has different adjustment strategies for the same change in different positions. For example, when the drone is at a place close to the target, the adjustment space for the drone is relatively large. At this time, it is not necessary to use the maximum lift to adjust the attitude of the drone; When approaching the critical distance, it is necessary not only to ensure that the drone will not leave the target distance beyond the specified range, but also to ensure that the drone can move to the target position to reduce the distance between the drone and the target position.

6.3 Improvement

In view of the shortcomings of the model analyzed above, this article can improve the model from the following aspects:

In the process of modeling wind, the wind in different directions should be studied, not only the impact of horizontal wind on the drone, but also the effect of wind in the numerical plane on the drone. This paper has obtained a relatively sufficient horizontal wind model. We can continue to adjust the direction of the wind

on the basis of the research in this article. For wind with an oblique direction, orthogonal decomposition can be used to decompose the wind speed into horizontal and vertical directions. Then we can study separately and make adjustment strategies.

Quantitatively describe the adjustment strategy of the UAV. After the drone is exposed to the wind, it needs to be adjusted according to the direction and size of the wind. This process should be quantitative. In other words, after receiving a certain direction and a certain magnitude of wind, how the drone makes adjustments should be described quantitatively. The four rotors of the quadcopter drone must be fully coordinated to make adjustments. The maximum lift that any rotor can provide is 7 Newtons. No rotor can exceed this upper limit.

When making adjustment strategies, this paper can be improved by considering the impact of the location of the drone on the specific adjustment strategy. UAVs have different adjustment strategies for the same changes in different positions. For example, when the drone is far away from the target position and is already close to the critical distance, not only must it be ensured that the drone does not leave the target distance beyond the specified range, but also that the drone can move to the target position to reduce the distance between the human and the machine to the target location. When the drone is at a place close to the target, the adjustment space of the drone is relatively large. At this time, it is not necessary to use the maximum lift to adjust the attitude of the drone. The flying attitude of the drone can be adjusted according to the characteristics of the spring. That is, the farther away from the target position, the greater the adjustment force of the drone.

7. Conclusions

In this paper, the stability of quadcopter UAV in wind is studied. In order to study the force of UAV in the air, the wind field model is first established in this paper. By analyzing the characteristics of wind in nature, the wind is divided into basic wind, gradual wind, gust wind and random wind.

Each wind has different characteristics. These winds add up to the real wind in nature. Then, according to the characteristics of different winds, equations are established to describe them. The definition of maximum wind speed is given in this paper. By making a precise definition of maximum wind speed, it lays a foundation for the calculation in the following part.

The dynamic model of the quadcopter UAV is also studied. For the convenience of research, the ground coordinate system and UAV body coordinate system are established in this paper, and the conversion formulas between different coordinate systems are given. By analyzing the hovering state of the UAV, the stress of the UAV is analyzed.

Drones need to constantly adjust their posture to the wind. In this process, the maximum lift that the UAV's rotor can provide is limited, so this paper studies the unmanned flight control system. From a qualitative perspective, the flight control strategy of UAV can be divided into two situations: wind speed change and wind direction change. The two situations can be studied respectively to obtain the flight control strategy of UAV. By adjusting the lift force of UAV, the dynamic balance of

UAV can be ensured. In addition, when the wind direction changes within other angles, the change of wind speed can be decomposed into horizontal and vertical planes through vector decomposition, and corresponding adjustments can be made. From a quantitative point of view, the UAV continuously adjusts its flight posture according to the wind force and various factors in previous moments. Therefore, this paper studies the quantitative flight adjustment of UAV by imitating the principle of spring.

Finally, a series of simulation experiments are carried out in this paper. By attaching different values to the parameters, the maximum wind force that the UAV can withstand under different circumstances is calculated.

8. Reference

- [1] Qi Yuexin, Design and implementation of control system for four-rotor UAV [D]. Harbin Institute of Technology, 2018.
- [2] Allison, S., Bai, H., & Jayaraman, B. (2020). Wind estimation using quadcopter motion: A machine learning approach. *Aerospace Science and Technology*, 98.
- [3] ZHENG Jiajing, LI Ping. Fault Tolerant Control of Actuator Additive Fault for quadcopter Using Sliding Mode Observer. *Journal of Huaqiao University(Natural Science)*. 2019;40(4):437-443. doi:10.11830/ISSN.1000-5013.201810019
- [4] S Bouabdallah, R Siegwart. Backstepping and sliding - mode techniques applied to an indoor micro quadrotor [C]. *Proceedings of the 2005 IEEE International Conference on IEEE*, 2005: 2247 -2252.
- [5] Derafa L , Benallegue A , Fridman L . Super twisting control algorithm for the attitude tracking of a four rotors UAV[J]. *Journal of the Franklin Institute*, 2012, 349(2):685-699.
- [6] S. Salazar-Cruz, Escareo J , Lara D , et al. Embedded control system for a four-rotor UAV[J]. *International Journal of Adaptive Control and Signal Processing*, 2007, 21(2-3).

Appendix

Main Code

```
fre=0.2;
kk=fre/0.1;
ss=zeros(1,100);
Vmaxlilun=zeros(1,100);
Vbs=zeros(1,100);
Vmaxmoni=zeros(1,100);
for s=1:100
    ss(s)=s/100;
for V0=0:0.01:10
    dVmax=s/100*V0;
    Vb=V0;
    Vgmax=dVmax;
    t1=5;
    Tg=10;
    Vmax=dVmax;
    tr1=8;
    Tv=15;
    Vnmax=dVmax/2;
    wn=0.2*pi;
    V=zeros(1,600/kk);
    rand('seed',1);
    r=rand(1,600/kk);
    r=(r-0.5)*2;
```

```
tt=zeros(1,600/kk);
```

```
Vg=zeros(1,600/kk);
```

```
Vr=zeros(1,600/kk);
```

```
Vn=zeros(1,600/kk);
```

```
V=zeros(1,600/kk);
```

```
for t=1:1:600/kk
```

```
    if (t/(10/kk)<t1 || (t/(10/kk)<(2*t1+Tg) && t/(10/kk)>t1+Tg) ||
        (t/(10/kk)<(3*t1+2*Tg) && t/(10/kk)>2*t1+2*Tg) ...
```

```
            || (t/(10/kk)<(4*t1+3*Tg) && t/(10/kk)>3*t1+3*Tg))
```

```
        Vg(t)=0;
```

```
    else
```

```
        Vg(t)=Vgmax*sin(2*pi*(t/(10/kk)-t1)/Tg);
```

```
    end
```

```
    if (t/(10/kk)<tr1 || (t/(10/kk)<2*tr1+Tv && t/(10/kk)>tr1+Tv) ||
        (t/(10/kk)<3*tr1+2*Tv && t/(10/kk)>2*tr1+2*Tv))
```

```
        Vr(t)=0;
```

```
    elseif(tr1<=t/(10/kk) && t/(10/kk)<=tr1+0.5*Tv)
```

```
        Vr(t)=Vrmax*(t/(10/kk)-tr1)/(0.5*Tv);
```

```
    elseif(2*tr1+Tv<t/(10/kk) && t/(10/kk)<=2*tr1+1.5*Tv)
```

```
        Vr(t)=Vrmax*(t/(10/kk)-2*tr1-Tv)/(0.5*Tv);;
```

```
    elseif(3*tr1+2*Tv<t/(10/kk) && t/(10/kk)<=3*tr1+2.5*Tv)
```

```
        Vr(t)=Vrmax*(t/(10/kk)-3*tr1-2*Tv)/(0.5*Tv);;
```

```
    elseif(tr1+0.5*Tv<=t/(10/kk) && t/(10/kk)<=tr1+Tv)
```

```
        Vr(t)=Vrmax*(-t/(10/kk)+tr1+Tv)/(0.5*Tv);
```

```
    elseif(2*tr1+1.5*Tv<t/(10/kk) && t/(10/kk)<=2*tr1+2*Tv)
```

```

    Vr(t)=Vrmax*(-t/(10/kk)+2*tr1+2*Tv)/(0.5*Tv);;
elseif(3*tr1+2.5*Tv<t/(10/kk) && t/(10/kk)<=3*tr1+3*Tv)
    Vr(t)=Vrmax*(-t/(10/kk)+3*tr1+2*Tv)/(0.5*Tv);;
end

Vn(t)=Vnmax*r(t)*cos(wn*t);

V(t)=Vb+Vg(t)+Vr(t)+Vn(t);

tt(t)=t/(10/kk);

end

%{
plot(tt,V)

xlabel('time    (m/s)','FontName','Times New Roman','FontSize',15)
ylabel('wind speed    (m/s)','FontName','Times New Roman','FontSize',15)

set(gca,'YLim',[0 max(V)]);

%}

S0=0.0583;

S=0.6102;

xpl=zeros(1,600/kk);

vpl=zeros(1,600/kk);

apl=zeros(1,600/kk);

C=0.3;

p=1.293;

g=9.8;

Fxmax=sqrt(28^2-(1.5*9.8)^2);

sinamax=Fxmax/28;

k=0;

```

```
Fp1=0;

sina=0;

Ff=zeros(1,600/kk);

for t=2:1:600/kk

    Ff(t)=1/2*C*p*(S0*sqrt(1-sina^2)+S*sina)*(V(t)-vpl(t))^2;

    sina=Ff(t)/sqrt(Ff(t)^2+Fp1^2)*sinamax;

    if (0<(xpl(t-1)) && (xpl(t-1))<0.2)

        Fp1=-Ff(t-1)-max(Ff)*xpl(t-1)/0.2;

    end

    if ((xpl(t-1)<=0) && -0.2<(xpl(t-1)))

        Fp1=-Ff(t-1)-max(Ff)*xpl(t-1)/0.2;

    end

    if (xpl(t-1)>=0.2)

        Fp1=-Fxmax;

    end

    if (xpl(t-1)<=-0.2)

        Fp1=Fxmax;

    end

    apl(t)=(Ff(t)+Fp1)/1.5;

    vpl(t)=vpl(t-1)+apl(t)*0.2;

    xpl(t)=xpl(t-1)+vpl(t)*0.2;

    if (abs(xpl(t))>0.2)

        k=1;
```

```
    end

end

if(k==1)

    Vmaxlilun(s)=(1+2*s/100)*V0;

    Vmaxmoni(s)=max(V);

    Vbs(s)=V0;

    break

end

end

end

plot(ss,Vmaxlilun,'r');

hold on;

plot(ss,Vmaxmoni,'g');

%hold on;

%plot(ss,Vbs,'k');

legend('Theoretical maximum velocity','Simulated maximum speed');

xlabel('Wind speed variation factor','FontName','Times New Roman','FontSize',15)

ylabel('Wind speed (m/s)','FontName','Times New Roman','FontSize',15)

set(gca,'YLim',[4.6 5.7]);

%{

plot(tt,xpl);

xlabel('time (m/s)','FontName','Times New Roman','FontSize',15)

ylabel('The length of the aircraft offset from the initial position(m)','FontName','Times New Roman','FontSize',15)

set(gca,'YLim',[min(xpl)-0.05 max(xpl)+0.05]);

%}
```

Secondary code

```
fre=0.2;
kk=fre/0.1;
s=0.5;
for V0=2.65:0.01:2.66
dVmax=s*V0;
Vb=V0;
Vgmax=dVmax;
t1=5;
Tg=10;
Vrmax=dVmax;
tr1=8;
Tv=15;
Vnmax=dVmax/2;
wn=0.2*pi;
V=zeros(1,600/kk);
rand('seed',1);
r=rand(1,600/kk);
r=(r-0.5)*2;
tt=zeros(1,600/kk);
Vg=zeros(1,600/kk);
Vr=zeros(1,600/kk);
Vn=zeros(1,600/kk);
V=zeros(1,600/kk);
for t=1:1:600/kk
```

```

if (t/(10/kk)<t1 || (t/(10/kk)<(2*t1+Tg) && t/(10/kk)>t1+Tg) ||
(t/(10/kk)<(3*t1+2*Tg) && t/(10/kk)>2*t1+2*Tg) ...

```

```

|| (t/(10/kk)<(4*t1+3*Tg) && t/(10/kk)>3*t1+3*Tg))

```

```

Vg(t)=0;

```

```

else

```

```

Vg(t)=Vgmax*sin(2*pi*(t/(10/kk)-t1)/Tg);

```

```

end

```

```

if (t/(10/kk)<tr1 || (t/(10/kk)<2*tr1+Tv && t/(10/kk)>tr1+Tv) ||
(t/(10/kk)<3*tr1+2*Tv && t/(10/kk)>2*tr1+2*Tv))

```

```

Vr(t)=0;

```

```

elseif(tr1<=t/(10/kk) && t/(10/kk)<=tr1+0.5*Tv)

```

```

Vr(t)=Vrmax*(t/(10/kk)-tr1)/(0.5*Tv);

```

```

elseif(2*tr1+Tv<t/(10/kk) && t/(10/kk)<=2*tr1+1.5*Tv)

```

```

Vr(t)=Vrmax*(t/(10/kk)-2*tr1-Tv)/(0.5*Tv);;

```

```

elseif(3*tr1+2*Tv<t/(10/kk) && t/(10/kk)<=3*tr1+2.5*Tv)

```

```

Vr(t)=Vrmax*(t/(10/kk)-3*tr1-2*Tv)/(0.5*Tv);;

```

```

elseif(tr1+0.5*Tv<=t/(10/kk) && t/(10/kk)<=tr1+Tv)

```

```

Vr(t)=Vrmax*(-t/(10/kk)+tr1+Tv)/(0.5*Tv);

```

```

elseif(2*tr1+1.5*Tv<t/(10/kk) && t/(10/kk)<=2*tr1+2*Tv)

```

```

Vr(t)=Vrmax*(-t/(10/kk)+2*tr1+2*Tv)/(0.5*Tv);;

```

```

elseif(3*tr1+2.5*Tv<t/(10/kk) && t/(10/kk)<=3*tr1+3*Tv)

```

```

Vr(t)=Vrmax*(-t/(10/kk)+3*tr1+2*Tv)/(0.5*Tv);;

```

```

end

```

```

Vn(t)=Vnmax*r(t)*cos(wn*t);

```

```

V(t)=Vb+Vg(t)+Vr(t)+Vn(t);

```

```
tt(t)=t/(10/kk);  
  
end  
  
p1=plot(tt,Vb,'b');  
  
hold on;  
  
p2=plot(tt,Vr,'r');  
  
hold on;  
  
p3=plot(tt,Vg,'g');  
  
hold on;  
  
p4=plot(tt,Vn,'k');  
  
legend([p1(1),p2(1),p3(1),p4(1)],'Basic wind','Gradient wind','Gust of wind','Random  
wind')  
  
xlabel('time (s)','FontName','Times New Roman','FontSize',15)  
  
ylabel('wind speed (m/s)','FontName','Times New Roman','FontSize',15)
```