Asteroid Ocean Impact

Problem B

Team 442

Abstract

Asteroid and space debris impacts have been considered one of the greatest threats to human life and land structures for centuries. Though many are too small to cause any significant damage, there have been incidents which inspired researchers to create more accurate numerical and simulation models describing the effects of such an impact on nearby land. In our work, we present a simplified model of an oceanic impact 1000 km from a coastal city. Using data derived from numerical simulations and wave mechanics theory, we estimate the minimal height of a tsunami wave causing substantial damage to equal 7 m. To cause such a wave, the asteroid would need to weigh $1.7 \cdot 10^{11}$ kg. We are also assuming the most favorable conditions, that is: the maximal angle of approach ($\theta = 90^{\circ}$) and the asteroid to be made of iron ($\rho_a = 8000$ kg/m^3). The damage may also be inflicted through thermal radiation, air burst or seismic effects, though the mass needed to cause them greatly exceeds the previously mentioned minimum.

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Introduction

To estimate the minimum mass of an asteroid sufficient to cause damage to a coastal city, we have to assume a simplified model of an oceanic meteoroid impact. Such models can be of importance in evaluating the damage caused in real-life terrestrial events.

The speed of the meteoroid can realistically be between 11 km/s and 72 km/s [BNGK94] and have a trajectory from 0° (flat approach) and 90° (normal incidence). The asteroid is spherical in shape, and dense and big enough so that it does not break up during the flight in the atmosphere. Our main focus for the distance of 1000 km is the creation and propagation of tsunami waves, mainly their height at the shore. A satisfactory model should also take into account the seismic events correlated with an asteroid impact, the emitted thermal energy and the shock wave created during collision, as those are the most important factors for water impacts (proposed in [RLA17]).

After quantifying the impact of each of those factors we can take a look at the effect they have on the coastal city. We can estimate the level of damage caused as a parameter of the asteroid's size, which along with density dictates its mass.

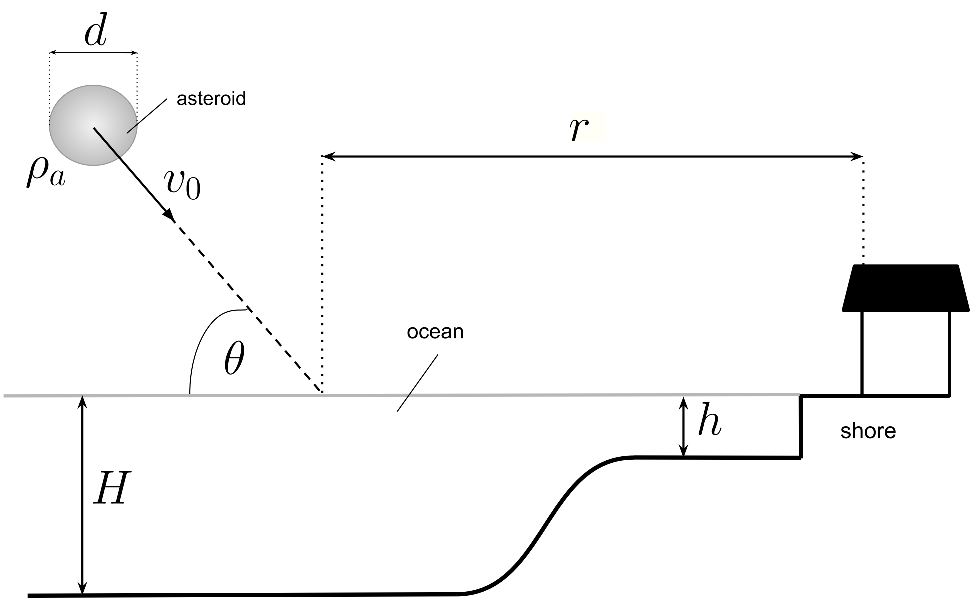

Figure 1: Model of an asteroid impact.

1 Notations used

Symbol	Meaning	Value
r	distance from the point of impact	1000 m
H	water depth at the point of impact	5000 m
ρ_w	density of water	1000 kg/m^3
g	gravity of Earth	9.81 m/s^2
C_D	drag coefficient of a sphere in air	0.47 [LL59]
C_{Dw}	drag coefficient of a sphere in water	0.877 [LL59]
R	universal gas constant	8.31446 J/mol K
T_0	sea level temperature	288.15 K
M	molar mass of air	0.0289652 kg/mol
β	temperature lapse rate	0.0065 K/m
p_0	atmospheric pressure at sea level	101325 Pa
v_{sound}	speed of sound in air	343 m/s
R_E	radius of the Earth	6400000 m
ρ_a	density of the asteroid	
d	diameter of the asteroid	
v_0	initial speed of the asteroid	
E_k	kinetic energy of the asteroid	
θ	angle of approach	
h	water depth at the continental shelf	
D_{tc}	transient crater diameter	
v_i	speed of the asteroid during impact with water	
v_e	speed of the asteroid during impact with bottom of the ocean	

Table 1: Used symbols and their values

2 Energy and velocity in the atmosphere

One of the most fundamental quantities we have to consider while analysing asteroid impacts is their kinetic energy. It can be written as:

$$E_k = \frac{1}{2} m v_a^2 = \frac{\pi}{12} \rho_a d^3 v_a^2 \tag{2.0.1}$$

assuming a spherical shape and uniform density. Later this energy will be used to assess the severity of earlier mentioned effects as it influences the amount of thermal, wind, seismic and other energy types released.

We assume that the asteroid has an initial velocity of v_0 (before entering the atmosphere). For our problem we can neglect the effect of gravitational field on the asteroid trajectory (because of the great speeds we are dealing with). This results in a straight line path with a constant angle of approach ([Bro83]). The change in velocity is then denoted by a drag equation ([CTZ93]):

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{3\rho(z) C_D}{4\rho_a d} v^2 \tag{2.0.2}$$

where $\rho(z)$ is the air density at height z above ground, which we assume to be ([BSBP97]):

$$\rho(z) = \frac{p_0 M}{R T_0} \left(1 - \frac{\beta z}{T_0}\right)^{\frac{gM}{R\beta} - 1}$$

for atmosphere with a vertical temperature gradient and constant temperature lapse rate given in Table 1.
Solving the equation for v and substituting $\rho(z)$ yields:

$$v_a(z) = v_0 \cdot \exp\left(-\frac{3C_D p_0}{4gd\rho_a \sin\theta}\left(1-\frac{\beta z}{T_0}\right)^{\frac{gM}{R\beta}}\right) . \tag{2.0.3}$$

Although if the calculated velocity is less than the terminal velocity near the surface:

$$v_t = \sqrt{\frac{2m_a g}{\rho_0 A C_D}} = \sqrt{\frac{4}{3}\frac{\rho_a d g}{\rho_0 C_D}} , \tag{2.0.4}$$

where ρ_0 is the density of air at sea level: $\rho_0 = \frac{p_0 M}{RT_0}$; the greater of the two figures is taken into consideration.
Velocity of the asteroid on impact with water can be calculated as:

$$v_i = v_a(0) = v_0 \cdot \exp\left(-\frac{3C_D p_0}{4gd\rho_a \sin\theta}\right) . \tag{2.0.5}$$

3 Thermal radiation on impact

Surface impacts produce thermal radiation as a result of a rapid compression of the target and the impactor material. Those violent events generate a rapidly expanding plume (called fireball) with very high pressure ($>$ 100 GPa) and temperature ($\approx 10,000$ K). As a result of the high temperature, the gas is ionized and appears opaque to thermal radiation due to the plasma's radiation absorption characteristics. Consequently, the plume expands adiabatically and only starts to radiate outwards when the plasma cools to the transparency temperature T_* ([ZR66]). [CMM05] includes an empirical relationship for the fireball radius R_f when it reaches its maximum heat radiation state (connected with temperature T_*):

$$R_f = 0.002E^{\frac{1}{3}} \tag{3.0.1}$$

To estimate if the thermal energy generated by the impact could threaten our city, we have to take into account the visibility of the fireball. To do that, we calculate the height of the impact center below horizon for the observer:

$$h = (1-\cos\theta)R_E . \tag{3.0.2}$$

Thermal effect is experienced at the place of interest only if this height is smaller than the radius of the fireball. Let's see what is the minimal diameter of the asteroid with typical impact velocity (17 $\frac{\text{km}}{\text{s}}$, [BNGK94]) and density for iron material $\left(8000\ \frac{\text{kg}}{\text{m}^3},\ \text{[Hil02]}\right)$ that would satisfy this condition:

$$h < R_f$$

Let us define Δ as the central angle between the impact and the observation points, $\Delta = \frac{r}{R_E}$

$$(1-\cos\Delta)R_E < 0.002E^{\frac{1}{3}}$$

$$\left(1-\cos\frac{r}{R_E}\right)R_E < 0.002d\left(\frac{\pi}{12}\rho_a v^2\right)^{\frac{1}{3}}$$

$$d > 500R_E\left(1-\cos\frac{r}{R_E}\right)\left(\frac{\pi}{12}\rho_a v^2\right)^{-\frac{1}{3}} \approx 4.61 \text{ km} \tag{3.0.3}$$

At this size other destructive effects would be far more devastating. The same applies for less dense asteroids, they need even bigger diameters to have an impact in therms of thermal radiation. For this reason we will treat the effect of the thermal radiation from impact at such a great distance as negligible and will be omitting them in further analysis. This aspect is discussed in greater detail in [CMM05] and [NSA+98].

4 Generation of tsunami waves

Wave mechanics is a complicated topic of study for many researchers. For the purpose of simplifying the model, we assumed one used in [CMM05]. Amplitude of the rim wave (formed by the material firstly ejected from the area of impact) is estimated to be:

$$A_t(r) = \min\left(\frac{D_{tc}}{14}, H\right) \cdot \frac{3D_{tc}}{4r} \text{ for } r > \frac{3}{4}D_{tc} \ , \tag{4.0.1}$$

where D_{tc} is the diameter of the transient crater also described in [CMM05]:

$$D_{tc} = 1.365 \cdot \left(\frac{\rho_a}{\rho_w}\right)^{\frac{1}{3}} d^{0.78} v_i^{0.44} g^{-0.22} \ \cdot \sin^{\frac{1}{3}}(\theta) \ , \tag{4.0.2}$$

where v_i is the velocity of the asteroid upon impact with the water which in our case would we equivalent to $v(0)$.

Collapse waves are generated by the oscillations in water height near the impact zone as the transient crater collapses after the initial impact. Waves of huge amplitude can be created by this process, which quickly dissipate with growing r. The equation showing the amplitude of collapse waves at distance $r > \frac{5D_{tc}}{2}$ is:

$$A_c(r) = 0.06 \cdot \min\left(\frac{D_{tc}}{2.828}, H\right) \cdot \left(\frac{5D_{tc}}{2r}\right)^q \ , \tag{4.0.3}$$

where $q = 3e^{-0.8\frac{d}{H}}$ for $\frac{d}{H} < \frac{1}{2}$. As Figure 2 shows, these waves at distances near 1000 km are substantially smaller than the ones generated by A_t. For this reason, mechanism of the creation of these waves has been deemed insignificant for the purpose of this research.

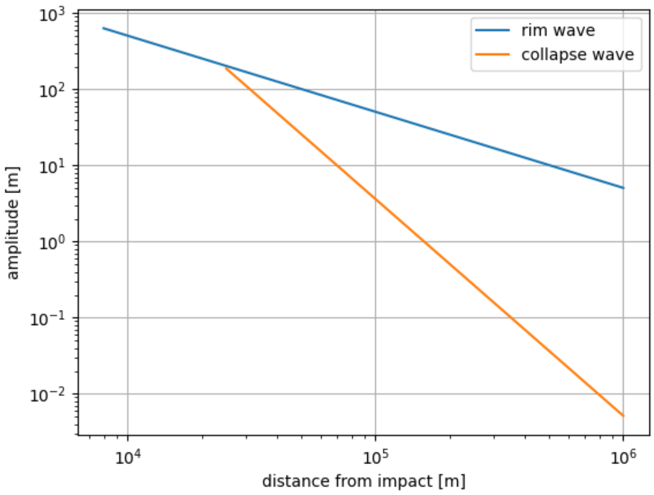

Figure 2: Function of rim wave and collapse wave amplitude. We can see that collapse wave amplitude shrinks much faster and at the distance of 1000 km is negligible.

We took into account the shoaling effect that waves go through when transferring through shallow water, amplifying the wave amplitude. We used the model presented in [DD84]. Thus, the shoaling coefficient is:

$$K_s = \frac{A_s}{A_d} = \left[\tanh(kh)\left(1 + \frac{2kh}{\sinh(2kh)}\right)\right]^{-\frac{1}{2}} , \tag{4.0.4}$$

where A_s is the wave amplitude near the shore, A_d is the amplitude before the shelf and k equals:

$$k = \frac{2\pi}{L_{sh}} , \tag{4.0.5}$$

where:

$$L_{sh} = \sqrt{gh} \cdot T . \tag{4.0.6}$$

Assuming that the initial wave amplitude L_0 is similar to D_{tc} [CMM05], we can estimate $T = \sqrt{\frac{2\pi L_0}{g}}$ and substitute into L_{sh}, getting:

$$L_{sh} = \sqrt{2\pi D_{tc} h} . \tag{4.0.7}$$

5 Air Burst

During an asteroid impact, a shock wave is induced which is often referred to as an air burst. The intensity of the shock wave depends on the energy released by the meteoroid and the height of the release. For our model, we assume that the energy is released at sea level (the meteor doesn't break up in the atmosphere). Hence, we can use the data deduced from nuclear blasts and equations used

$R[km]$ \ α	$0.01°$	$0.1°$	$1°$
65	2.59	1.49	1.00
200	2.17	1.37	1.15
750	4.56	3.10	3.27

Table 2: Shoaling coefficient depending on shelf slope (α) and shelf width (R). It can be seen that generally we have a great wave scaling for wider shelves. This can be due to the fact that wider shelves are more shallows thus resulting in bigger scaling. The same applies for the slope, flatter shelves are shallower at their end. On the other hand we can observer some inconsistencies, so further reserch on this topic might prove interesting.

in [CMM05] to estimate overpressure p at distance r:

$$p = \frac{p_x r_x E_{kt}^{1/3}}{4r}\left(1 + 3\left(\frac{r_x E_{kt}^{1/3}}{r}\right)^{1.3}\right) \ , \tag{5.0.1}$$

where $p_x = 75000$ Pa, $r_x = 290$ m and E_{kt} is the kinetic energy of an asteroid during impact in kilotons of TNT. We can also calculate the wind speed behind the shock wave and assess it's effect on the city:

$$v_{wind} = \frac{5p}{7p_0}\frac{v_{sound}}{(1+\frac{6p}{7p_0})^{1/2}} \tag{5.0.2}$$

Nevertheless, we have to keep in mind that this data has been extrapolated from explosions of magnitude of 1 kt. They also ignore the curvature of the Earth, which at a distance of 1000 km begins to play a role. As follows, these assumptions may not work for larger impacts which we are interested in. [CMM05] states that for impacts of energy > 10000 Mt, these effects may be overestimated by a factor of 2-5.

6 Earth vibrations after impact

In the case of low ocean depth level, a significant amount of asteroid's kinetic energy could be transferred into earth vibrations. Such vibrations could cause heavy damage in the nearby area. We've estimated the size of such damages as a function of angle θ and E_k by assuming that there is no substantial difference (other than kinetic energy lost in water) between hitting the surface and bottom of the ocean. Presented equations are an approximation for an asteroid impacting average ground [KSS18]. In the first step we've calculated the perpendicular component of energy E_k of falling asteroid to the ground during impact with the bottom of the ocean.

$$E_{kp} = \frac{\pi}{12} d^3 \rho_a v_e^2 \cdot \sin^2\theta \tag{6.0.1}$$

Here v_e is used as the velocity of the asteroid as it reaches the seafloor. We can calculate the value using a formula similar to the one used in finding the change of velocity in the atmosphere, assuming constant water density in the fluid column as it varies an insignificant amount ([NSBL16]):

$$v_e = v_a(0) \cdot \exp\left(-\frac{3\rho_w C_{Dw} h}{2\rho_a d \sin\theta}\right) \tag{6.0.2}$$

With that, using experimentally established dependencies between magnitude M of a spurious source, magnitude M_{eff} in distance r from the source (for r>700 km), and earthquake intensity I, we've obtained:

$$I = (\ln E_k - 2.49 \ln r - 8.42) \tag{6.0.3}$$

Using another experimentally established dependency, we've approximated Peak Ground Velocity P in the distance r from the source:

$$P = \begin{cases} \exp(0.29I - 0.68) & I >= 5 \\ \exp(0.48I - 1.62) & I < 5 \end{cases} \tag{6.0.4}$$

7 Damage assumptions

In this section we will make assumptions and define the meaning of "substantial damage" done to the coastal city. We take a look at previously inspected aspects of the asteroid impact. For each type of possible damage we asses its importance and define necessary levels of threat for it to be considered "substantial".

7.1 Thermal damage

As stated in Section 3 and seen in equation 3 we can neglect thermal wave effects in the analysis of minimal asteroids required for substantial damage. Other effects have a far greater impact at those distances.

7.2 Tsunami damage

As seen in [RLA17] we can expect tsunami to be the most impactful effect for large distances (1000 km for our purpose). [LAKK13] provides deep analysis of different structure classes and the damage sustained to them via a tsunami. According to the reference, buildings of class D (reinforced masonry, frames/walls with moderate level of earthquake-resistant design, steel/timber structures) will sustain damage of grade 3 or 4 (heavy / very heavy) with a tsunami height of about 7 m. We take this class into account because we assume a coastal city situated near the ocean to be prepared in some degree for heavy waves. Also, steel and reinforced concrete are the most common materials used in urban architecture, opposed to weaker wood or aluminum. Thus, we assumed our minimal tsunami height goal to be 7 m.

7.3 Air burst damage

Considering Table 5.145 in [GD97], to shatter windows in buildings, the overpressure must be equal to 0.5 - 1 psi (3447 - 6895 Pa) in nuclear explosions. To demolish a concrete wall (not reinforced) the additional pressure needed is around 1.5 - 5.5 psi(10342 - 37921 Pa). Considering this, the city structure assumed in subsection 7.2 and the table shown in [CMM05], we can estimate the overpressure needed to cause substantial damage to be around 20000 Pa. [GD97] also takes into account the damage caused by high winds created behind the initial shock wave. For our model we can assume that the "Moderate" level ('About 30 percent of trees blown down; (...) Area passable to vehicles only after extensive clearing.') is satisfactory for our needs. This is equivalent to winds of 90-100 mph (40 - 45 $\frac{\text{m}}{\text{s}}$). In spite of these facts, we have to remember that we are also taking the tsunami effects into account and the distance is about 1000 km. For an average asteroid($E_k \approx 10^{19} J$) the overpressure at 1000 km derived from equation 5.0.1 is approx. 1000 Pa, which is certainly below the damage threshold.

7.4 Seismic damage

Depending on empirical and simulation data for asteroid impacts (for example [RLA17]) we can expect the seismic effects to be minimal (especially at greater distances) compared to other effects such as tsunami. For earth impact scenario our computations turned out to match the result of other researchers [KSS18]. Using experimentally calculated scales [Wu03], we've managed to estimate Modified Mercalli Intensities from PGV values. Even assuming no velocity loss from the water depth, kinetic energy of an asteroid sufficient to make a high tsunami wave is not enough to cause significant seismic damage.

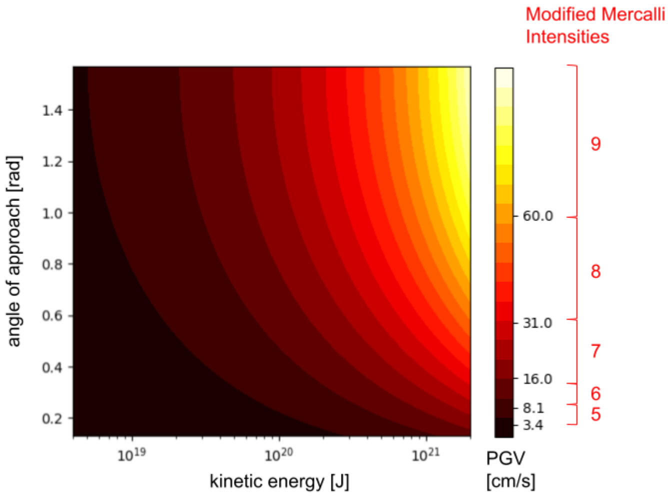

Figure 3: Seismic effect in PGV and Modified Mercalli Intensities depending on the angle of approach and the kinetic energy of the asteroid. Bigger impact for a larger angle of approach can clearly be seen as the effect depends mostly on the horizontal velocity of the impactor. A more straightforward relation applies to kinetic energy in the same way. The values are calculated without taking water drag into consideration, but for energies that we are interested in the seismic effect at the great distance ($PGV < 3\ \frac{cm}{s}$) is minimal and we can neglect it entirely.

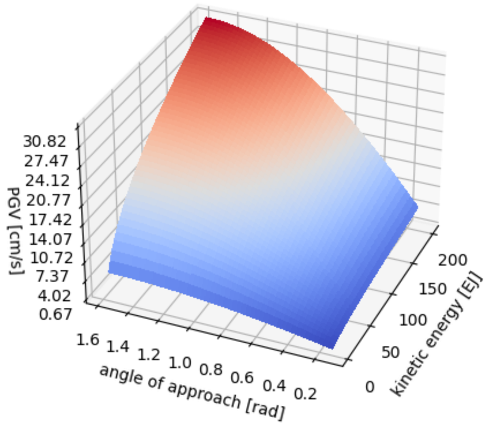

Figure 4: Seismic effect measured in PGV as a function of the angle of approach and the kinetic energy. Similar reasoning as in the Figure 3 applies.

8 Calculating the minimum mass

Using the model presented, we can estimate the effect of an asteroid impact on a coastal city situated 1000 km away. Because of data presented in the subsection 7, we can assume that the creation of tsunami will be of utmost importance in calculating the minimal mass of an object required to cause damage. We conducted our calculations for two most interesting densities: $\rho_1 = 3000 \, \frac{\text{kg}}{\text{m}^3}$, which is the upper estimate for rock density, and for: $\rho_2 = 8000 \, \frac{\text{kg}}{\text{m}^3}$, which is the density of iron [Hil02]. We also assumed five spaced out approach angles: $\theta_1 = 10°, \theta_2 = 20°, \theta_3 = 45°, \theta_4 = 60°, \theta_5 = 90°$, with $\theta_3 = 45°$ being the most likely for asteroid impacts [M.62]. The depth of water at the impact site was assumed to equal $H = 4000$ m, which is the average depth of the Pacific Ocean [Adm18]. The velocity used is $v_0 = 17$ km/s as it's the most common for terrestrial asteroid impacts [BNGK94]. With these assumptions, Figures 6 and 7 show the amplitude of the wave near the shore (for average shelf depth $h = 113$ m [Wik] and average $\alpha = 0.1°$ [Bri]) with regards to d for ρ_1 and ρ_2. We can deduce that the higher the density and the approach angle, the higher the wave will be. Thus, assuming the most positive conditions ($\theta = 90°, \rho_a = \rho_2$), the minimal required mass to create 7 m waves equals $1.7 \cdot 10^{11}$ kg.

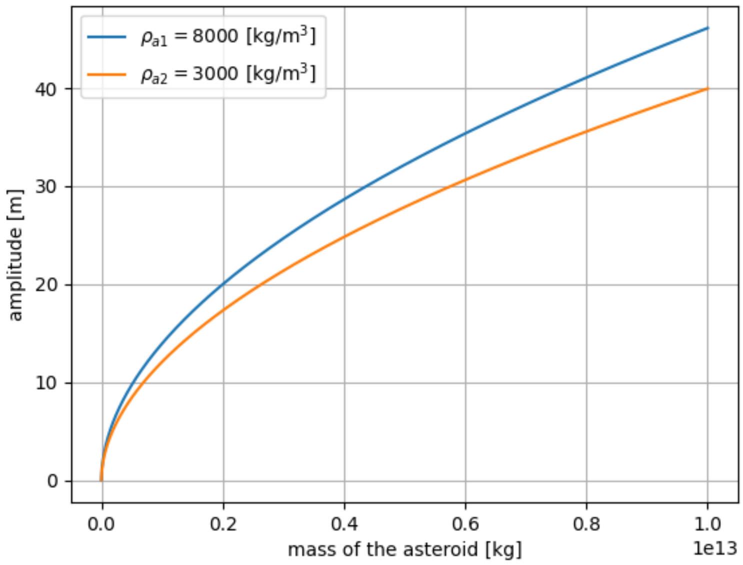

Figure 5: Function of wave amplitude 1000 km from the impact (sea depth 5 km). We can see that for two asteroids with the same mass, the amplitude generated by the one with higher density is bigger.

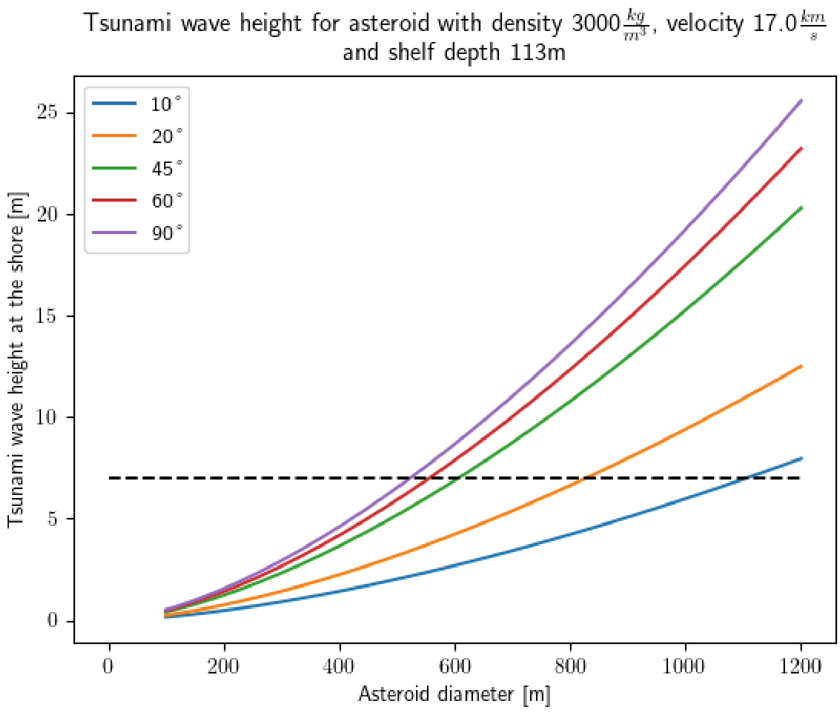

Figure 6: Function of tsunami amplitude with regards to asteroid diameter for different θ. The dashed line represents the threshold of 7 m.

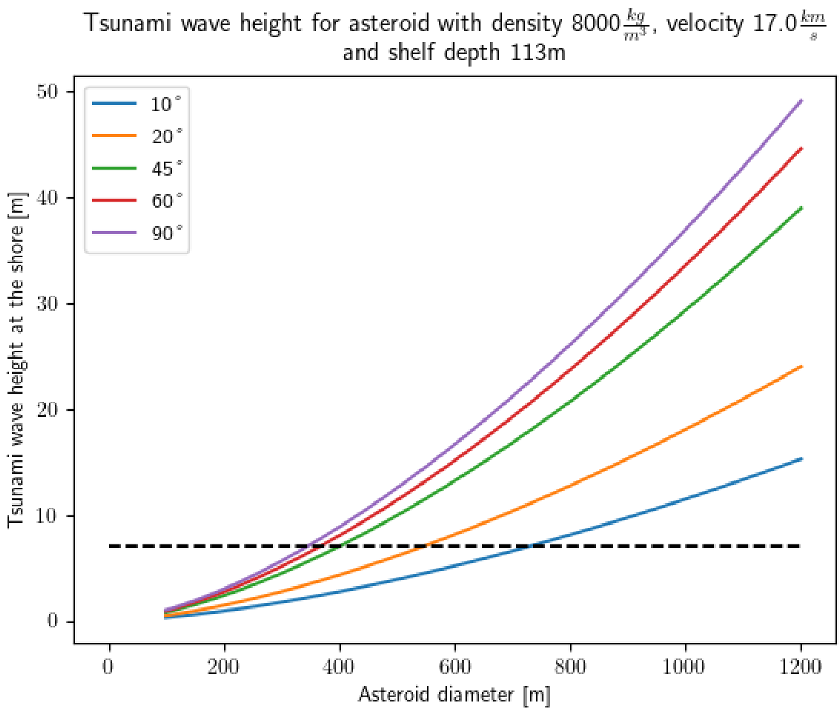

Figure 7: Function of tsunami amplitude with regards to asteroid diameter for different θ. The dashed line represents the threshold of 7 m.

9 Conclusion

The model presented in this work, although simplified, has allowed us to estimate the minimal mass of a destructive asteroid to be about $1.7 \cdot 10^{11}$ kg. With the constraints of the simplified model and a lack of exact studies conducted in this area, we cannot assure that the value presented is optimal. Theoretical behavior and simulation of oceanic waves is an intricate topic, still being discussed by experts. The exact computation, including solutions to difficult equations (e.g. for changing θ while falling) is beyond the scope of our capabilities. What is more, currently published research doesn't agree whether the waves created by singular impacts can pose any threat to coastal regions (the "Van Dorn" effect). Also, we omit the effects of breaking up and/or "pancaking" of the asteroid in the atmosphere or in the ocean, which would have affected the velocity and kinetic energy of the object. Such effects would allow for the shock wave to reflect off the surface, causing interference and thus possibly changing the overpressure.

On the other hand, the model can be accurate enough in the regions up to a few thousand kilometers away from the collision site. It also greatly simplifies the calculations, while being favorable with real-life observations and computer simulations. Considering possible future improvements, we could have taken into account some previously mentioned factors which would have improved the accuracy of our work. For example, when contemplating a big enough object in a shallower body of water, the seismic effects could play a bigger role. We could also take into account the ejecta released from the impact site, although it is supposed that their size (and the damage they can cause) decays much more rapidly with distance than tsunami or seismic shaking [CMM05].

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