Deflecting an Asteroid

Team 346, Problem A

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Abstract

In this paper, we find the minimum time before impact with the Earth at which an asteroid with a 100 meter diameter can be deflected by a 20 000 kg spacecraft. We investigate the feasibility and effectiveness of several different deflection methods through theoretical physical approaches to determine the shortest distance from the Earth, and thus the shortest time, achievable. We show, using conservation of energy, conservation of momentum, the Ideal Rocket Equation, and Newton's Law of Universal Gravitation, that deflection through kinetic impact is achievable only at large distance from the Earth and time frame of several years, even with a high momentum transfer efficiency. For a C-type asteroid of average density, liquid hydrogen fuel, and a momentum transfer efficiency of unity, the distance required would be about $3.8 \cdot 10^9$ km, with a corresponding time of 4.8 years. Therefore, we also consider the approach of splitting the asteroid through impact with the spacecraft. The scenario of symmetrical fragmentation into two pieces yields a much smaller distance from Earth and time frame of approximately $1.3 \cdot 10^6$ km and 15 hours, respectively. Though this approach allows for deflection within a shorter time frame, we risk some fragments hitting the Earth rather than being deflected, with that probability increasing as the collision occurs closer to the Earth.

Contents

1 Introduction and Problem Restatement

Asteroid impacts may one day threaten the survival of humanity. Recently, organisations such as NASA have started testing planetary defense capabilities against asteroids, such as through the DART mission [1]. Within this paper, we aim to find the minimum time before impact with Earth at which the asteroid can still be deflected by a spacecraft. This paper follows an engineering design process: first we frame the problem through restating and listing assumptions, then we devise several possible deflection approaches, and finally assess each for feasibility, and if feasible, for how much time before Earth impact is required in order to determine the minimum amount of time at which an asteroid can still be deflected.

2 Assumptions

Assumptions are acknowledged when they are made throughout the paper, but a list of key assumptions is also provided here.

1. The asteroid is a sphere for the purpose of estimating mass. In reality, the asteroid can take a myriad of shapes which would give it a large range of masses. This assumption is also made again when we consider the gravitational energy for splitting a uniform sphere.
2. The asteroid has a uniform density, which we take to be that of an average density for a C-type asteroid (see Section 4, "Solving for Collision Distance from Earth and Time before Earth Impact"). Once again, this assumption is made only for calculating mass and gravitational energy of splitting, and does not effect the collision mechanism beyond these.
3. Though we first develop a general formula, we assume that the asteroid is initially heading directly towards the center of the Earth for later calculations.
4. Though we also simulate other types of fragmentation, our calculations for fragmentation consider the highly idealized case of symmetric separation into two parts.

3 Two Failed Approaches for Asteroid Deflection

We continue to develop our solution through an engineering design processes by considering and evaluating the feasibility of several different approaches. In this section, we will demonstrate why certain approaches are not feasible, before we move onto optimizing and evaluating successful approaches.

Landing on the Asteroid Landing on the asteroid and then burning fuel while on the asteroid to move the it may be a more efficient use of fuel than colliding with the asteroid. However, in order to land, we must bring the spacecraft to the speed of the asteroid. Letting v_∞ represent the initial velocity of the asteroid and v_e represent the effective exhaust velocity, the Ideal Rocket Equation yields

$$v_\infty = v_e \ln\left(\frac{m(t_0)}{m(t)}\right) \Rightarrow m(t) = m(t_0)\exp\left(-\frac{v_\infty}{v_e}\right) \tag{1}$$

Substituting a value of 4.5 km/s for v_e, as is typical for liquid hydrogen fuel (see Section 4, "Solving for Collision Distance from Earth and Time before Earth Impact"), we get that the mass on landing is only about 40 kg, or that 99% of the mass has been lost. Therefore, there would not be enough fuel left to divert the asteroid.

Tethering We can try to eliminate the expenditure of fuel in bringing the spacecraft to the speed of the asteroid by launching a cable to connect the spacecraft to the asteroid. A cursory calculation,

$$m_{\text{cable}} = \rho_{cable} LA = \rho_{cable}(\frac{1}{2}at^2)(\frac{ma}{\sigma}) = \frac{m\rho_{cable}}{2\sigma}a^2t^2 = \frac{m\rho_{cable}}{2\sigma}v_{asteroid}^2 \tag{2}$$

where L is the length of the cable, A is the cross sectional area of the cable, a is the acceleration acting on the spacecraft, ρ_{cable} is the cable density, m is the spacecraft mass, and σ is the ultimate yield strength of the cable material, shows the flaw in this approach. Substituting for the known mass of the craft and speed of the asteroid, as well as $\sigma = 420$ MPa (ultimate) and $\rho = 7800\ \frac{kg}{m^3}$, which are typical values for steel, we obtain a cable mass of approximately $1 \cdot 10^8$ kg, or 5000 times the mass of the spacecraft!

4 Theoretical Analysis of Kinetic Impact Deflection

The most straightforward approach is to collide the spacecraft with the asteroid in order to change its course.

Momentum Required for Deflection Figure 1. shows key parameters of the problem, where D represents the distance from the asteroid to the axis passing though the Earth's centre in the direction of the asteroid's initial velocity.

The initial angular momentum of the asteroid is $L = M_a v_\infty D$, where M_a is the mass of the asteroid. Due to the conservation of energy $(K_1 + U_1 = K_2 + U_2)$,

$$\frac{M_a v_\infty^2}{2} = \frac{L^2}{2R^2 M_a} - \frac{GM_a M_{\text{Earth}}}{R} \tag{3}$$

as the asteroid approaches but does not hit the Earth, where R is the closest distance

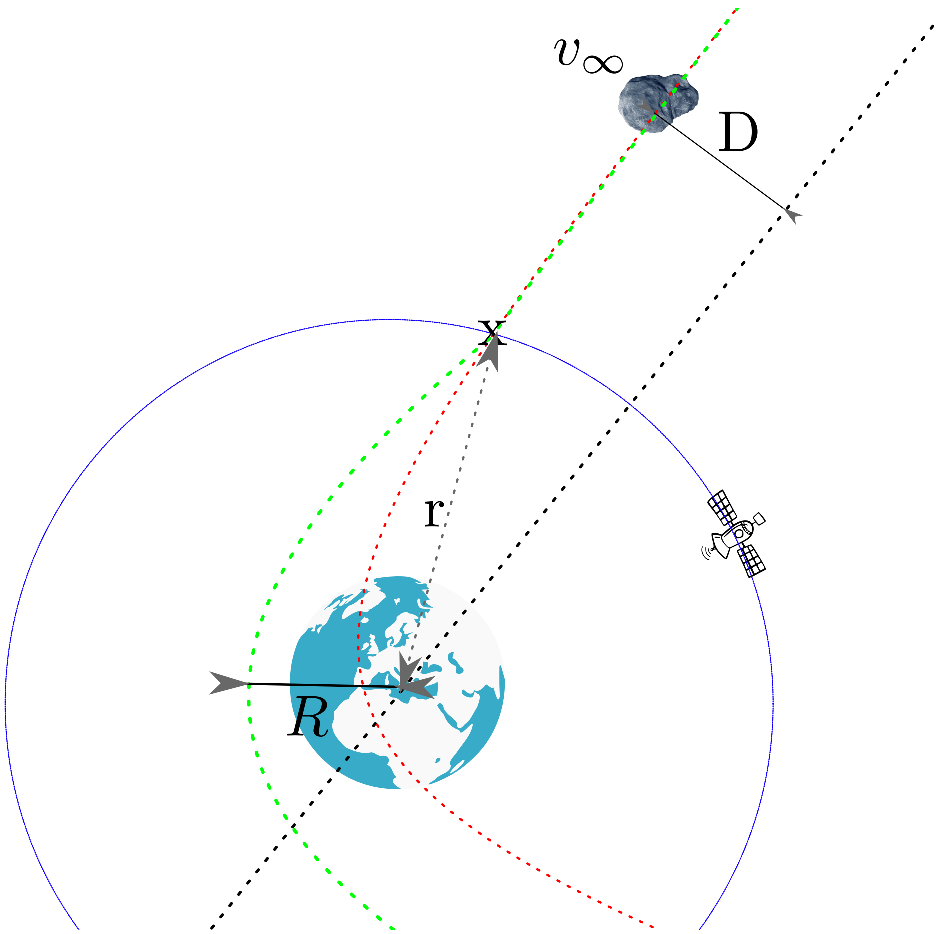

Figure 1: A schematic of the asteroid's path. The red line represents the path taken when there is no collision with the spacecraft, while the green line represents the path taken when there is collision with the spacecraft.

from the path of the asteroid to the centre of Earth. Rearranging this equation yields

$$L^2 = (M_a R v_\infty)^2 + 2GM_a^2 R M_{\text{Earth}} \tag{4}$$

For impact with the Earth, $R = R_{\text{Earth}}$, so to avoid impact we need

$$L_c^2 > (M_a R_{Earth} v_\infty)^2 + 2GM_a^2 R_{Earth} M_{\text{Earth}} \tag{5}$$

where L_c is the asteroid's angular momentum after impact. We therefore aim to maximize L_c through the spacecraft collision.

Following the inelastic collision between the spacecraft and the asteroid, the asteroid's angular momentum is

$$L_c = M_a v_\infty D + \vec{r} \cdot \vec{p}_{sat} = M_a v_\infty D + p_{\text{sat}} r \sin\phi \tag{6}$$

where $\vec{r}$ is the radius vector from the Earth to the point of collision, $\vec{p}_{sat}$ is the momentum vector of the spacecraft, and ϕ is the angle between these vectors, with an optimal value of 90°.

The worst case scenario is when the asteroid impacts Earth head on, or, $D = 0$, as all the angular momentum required to miss the Earth must come from the spacecraft. Within this scenario we need

$$p_{\text{sat}}^2 r^2 > (M_a R_E v_\infty)^2 + 2GM_a^2 R_{Earth} M_{\text{Earth}} \tag{7}$$

Maximizing Spacecraft Momentum prior to Impact The spacecraft may by accelerated prior to impact with the asteroid to maximize momentum. According to the ideal or Tsiolkovsky rocket equation,

$$u_{\text{sat}}(t) = u_{\text{sat}}(t_0) + v_e \ln\left(\frac{m(t_0)}{m(t)}\right) \tag{8}$$

where v_e, $u_{\text{sat}}(t)$, m, and t_0 are the effective exhaust velocity, spacecraft velocity, spacecraft mass, and time when the acceleration commences, respectively.

Substituting Eq. (9) into the equation for momentum, $p_{\text{sat}} = m(t) u_{\text{sat}}(t)$, and differentiation with respect to $m(t)$ yields

$$\frac{dp_{sat}(t)}{dm(t)} = (u_{\mathrm{sat}}(t_0) + v_e \ln\left(\frac{m(t_0)}{m(t)}\right)) - v_e \tag{9}$$

Setting this to zero and substituting for u_{sat} yields that maximum momentum is achieved when $u_{\mathrm{sat}}(t) = v_e$. This has an intuitive physical explanation. When the velocity of the spacecraft exceeds the exhaust velocity the ejected gases move in the direction of the spacecraft respect to the stationary frame, hence, reducing the momentum of the spacecraft due the momentum conservation. When the spacecraft moves slower than the exhaust speed the jet engine adds the momentum of the spacecraft as the exhaust gasses move backwards with respect to the stationary frame.

Therefore, the maximum spacecraft momentum that can be obtained using a jet engine is

$$p_{\mathrm{opt}} = v_e m = v_e m(t_0) \exp(u(t_0)/v_e - 1) \tag{10}$$

where the initial velocity is given by the equation for centripetal acceleration

$$u(t_0)^2 = G\frac{M_{\mathrm{Earth}}}{r} \tag{11}$$

where r is the orbital radius.

Time to Impact with Earth Rearranging Eq. (4) yields

$$v(r) = \sqrt{v_\infty^2 + \frac{2GM_{\mathrm{Earth}}}{r}} \tag{12}$$

where we ignore atmospheric drag as the asteroid approaches Earth due to the asteroid's large size. Therefore, the before impact with Earth is

$$T_{\mathrm{impact}} = \int_{R_{Earth}}^{r} \frac{ds}{v(s)} \tag{13}$$

where r is the distance from Earth at the moment of spacecraft-asteroid collision. The integration can be carried out explicitly using change of variable

$$\sinh^2(\zeta) = \frac{v_\infty^2}{2GM_{\text{Earth}}} \tag{14}$$

The integration then yields

$$T_{\text{impact}} = \frac{1}{v_\infty}(\sqrt{s^2+2as} - a\ln(a+s+\sqrt{2as+s})\ |^{r}_{R_{Earth}} \tag{15}$$

where

$$a = \frac{GM}{v_\infty^2} \tag{16}$$

Solving for Collision Distance from Earth and Time before Earth Impact
According to equation (10) we conclude the effective exhaust velocity of the jet engine is one of the most critical parameters that influence the efficacy of the asteroid deflection. NASA typically uses liquid hydrogen, which can have a specific impulse in a vacuum of 460 [s] on the higher end [2], which we multiply by a specific gravity of 9.8 $[\frac{m}{s^2}]$ to get an effective exhaust velocity of 4.5 $[\frac{km}{s}]$.

Another critical parameter is to consider is the density of the asteroid. For the three types of asteroid composition, C, S, and M, the average densities are 1380 $[\frac{kg}{m^3}]$, 2710 $[\frac{kg}{m^3}]$, and 5320 $[\frac{kg}{m^3}]$, respectively [3]. C type asteroids are the most common [4], so they are the type we will consider when we solve the problem numerically.

As per the problem statement, we wish to minimize the distance from the Earth to the collision, or in other words consider the case for equality in Eq. (7) when we use the optimal momentum from Eq. (10). We thus obtain a distance of about $3.8 \cdot 10^9$ km – an order of magnitude greater than the distance to Mars and a corresponding time of **4.8 years**. This large result is not completely unexpected, as previous studies suggest that asteroids should be deflected years before they reach earth [7, 6, 8]. We could gain a better result by including the effect of ejecta, parts of the asteroid that are sent in the opposite direction after collision. These ejecta would increase our momentum transfer efficiency from the case of $\beta = 1$, to possibly $\beta \approx 3$. [5]. The revised equation for momentum would be

$$p_{\text{opt}} = \beta v_e m = \beta v_e m(t_0) \exp(u(t_0)/v_e - 1) \tag{17}$$

However, this is not enough, as r is only inversely proportional to β, so the distance would not decrease by the several factors of 10 required to deflect at low Earth orbit.

5 Theoretical Analysis of Deflection through Kinetic Impact with Asteroid Fragmentation

To be able to deflect an asteroid at a closer distance, a more explosive approach must be taken, as spacecraft momentum is too small to deflect asteroids unless they are sent several years before impact [7, 6, 8].

Before we move forward we establish the following result:

$$U_{gravitational,asteroid} = \frac{3}{5}\frac{GM_{asteroid}^2}{radius} = 4.2 \cdot 10^5 J \tag{18}$$

Meanwhile, the kinetic energy of the spacecraft impact is

$$K_{collision} = \frac{1}{2} m_{sat} v_\infty^2 = 6.0 \cdot 10^{12} J \tag{19}$$

Since the energy of the impact is several orders of magnitude greater than the gravitation energy within the asteroid, we do not not expect gravitational forces between asteroid fragments to play a role in its subsequent dynamics.

Unlike momentum, the kinetic energy of the impact provides favorable square root scaling of the ratio of asteroid to spacecraft masses, which may allow for the explosive separation of the asteroid. We consider symmetric fragmentation which results in two objects being deflected in opposite direction with velocity U. For small mass spacecrafts the kinetic energy balance in the reference frame of the centre of mass is written as

$$\frac{1}{2} m \left((u_{\text{sat}}^{\parallel} + v_\infty)^2 + (u_{\text{sat}}^{\perp})^2 \right) = \frac{1}{2} M U^2 \tag{20}$$

where

$$U = \sqrt{\frac{m}{M}}\sqrt{(u_{\text{sat}}^{\parallel} + v_\infty)^2 + (u_{\text{sat}}^{\perp})^2} \tag{21}$$

and $u_{\text{sat}}^{\parallel}$ and $u_{\text{sat}}^{\perp}$ are parallel and perpendicular components of the spacecraft velocity with respect to the asteroid.

Maximizing the resulting velocity U with respect to the incoming velocity and mass of the spacecraft using the ideal rocket equation gives the following constraint on the relative velocity

$$u_{\text{sat}} + v_\infty = 2v_e \tag{22}$$

As v_∞ is far larger than the effective exhaust velocity it is disadvantageous to accelerate the spacecraft towards the asteroid.

The division of the mass into fragments then transforms condition (7) into

$$\begin{aligned} U^2 r^2 &> (R_{\text{Earth}} v_\infty)^2 + 2GR_{\text{Earth}} \\ \Rightarrow r^2 &> \frac{M}{m_{\text{sat}}} \frac{(R_{\text{Earth}} v_\infty)^2 + 2GR_{\text{Earth}} M_{\text{Earth}}}{(u_{\text{sat}} + v_\infty)^2} \end{aligned} \tag{23}$$

Though we will use Eq. (23) for calculations, we may also drop low order terms to get the simplified equation

$$r > \sqrt{\frac{M}{m_{\text{sat}}}} R_{\text{Earth}} \tag{24}$$

Evaluation of the Eq. (23) gives value $r_{\text{impact}} = 208 R_{\text{Earth}}$. Substituting the limits R_{Earth} and r_{impact} into Eq. (15) using integration gives a time before impact of $T_{\text{impact}} = 5.26 \cdot 10^4[\text{s}] \approx 14.6[\text{hours}]$.

A Simulation of Asteroid Fragmentation The impact of a collision on the structure of an asteroid has many complexities [9], with results varying due to the size and composition of an asteroid. We investigate the outcome of a collision that results in the fragmentation of an asteroid into a large number of fragments that move as a spherical front from the impact point. In the frame of reference of the asteroid, the fragments have the same velocity given by Eq. (20). We model the distribution of the orthogonal components of the asteroid fragments. In Fig. 2 the distribution of the velocity in the direction orthogonal to the asteroid velocity is shown. Notably,

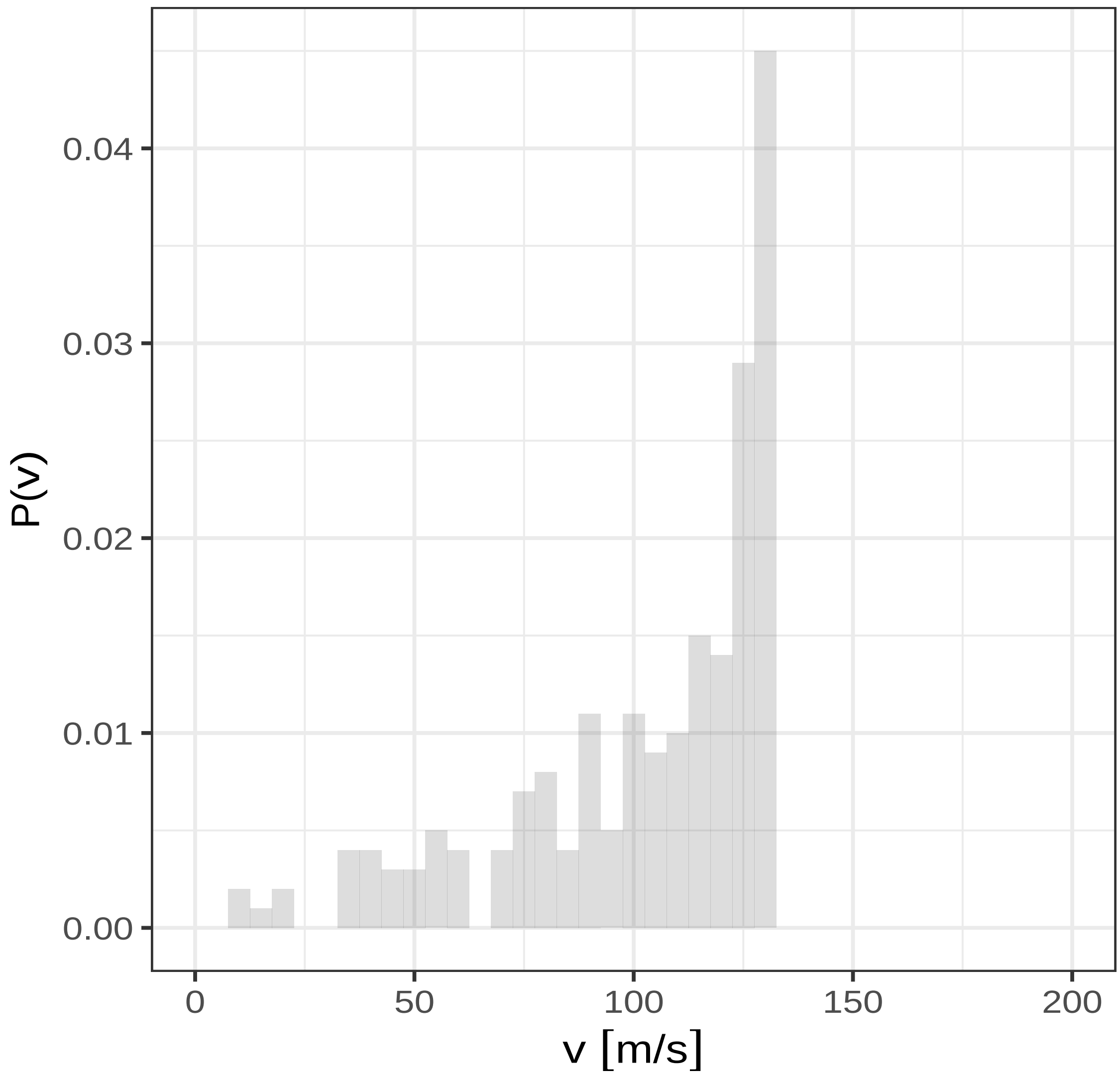

Figure 2: $u^{\perp}$ distribution of the asteroid fragments. We imposed that the asteroid is split into 200 fragments and all the kinetic energy of the satellite impact has been redistributed into kinetic energy of the fragments.

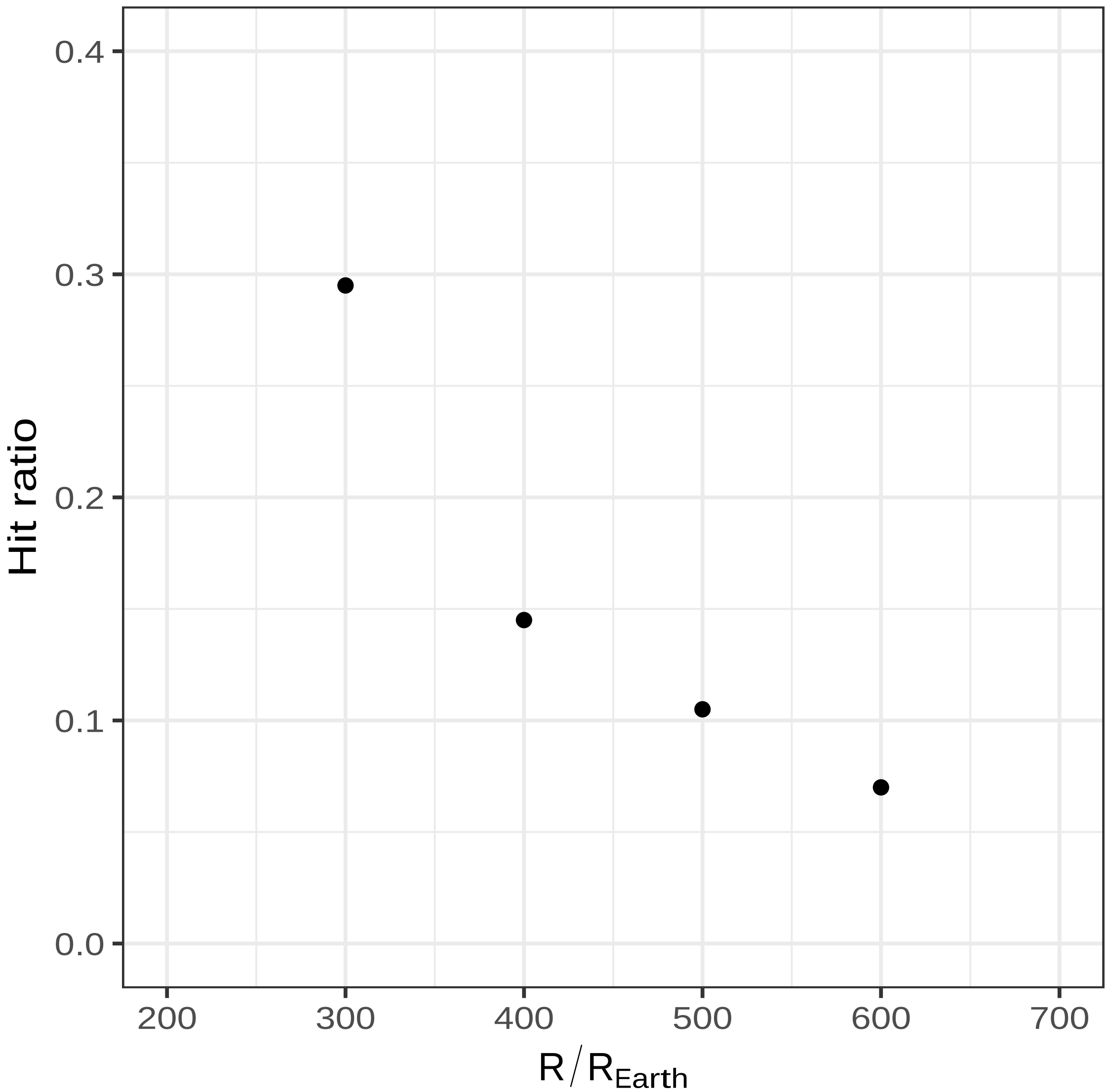

Figure 3: Probability of the asteroid fragments to impact Earth as a function of the satellite strike distance.

the velocities are clustered towards the edge of the distribution. This is explained by the fact the main contribution to the area of a sphere comes from its sides rather than the visible front area.

We counted the number of fragments that satisfy the inequality (23) for different distances of the satellite strike. At the distance that exceeds the minimal strike distance by 50% the number of fragments that impacted Earth was 30%. Doubling the distance again decreased the number of impacting fragments to 7%. The results of our simulation show that when we move away from the ideal case of symmetrical fragmentation into two pieces, we expose ourselves to the risk of asteroid fragments striking the Earth. Therefore, deflection through asteroid fragmentation should only be used when there is not enough time before collision left to achieve kinetic impact deflection.

6 Discussion

Model Strengths

1. The model considers multiple approaches to deflecting an asteroid in order to compare the minimum times and distances at which deflection is still possible.
2. The model is relatively simple and uses concepts known by students that have completed a year of physics.
3. The model can be solved explicitly at most stages, which avoids the need for numerical approaches.

Model Shortcomings

1. The impact of ejecta, which typically increases the momentum transfer efficiency, is not analyzed in detail due to its complexity and the uncertainties involved. In fact, it is possible that the ejacta could in some cases slightly decrease the momentum transfer efficiency if they are released in the forward direction [5].
2. The approach of fragmenting an asteroid is highly chaotic, with complete deflection achievable only in highly idealized scenarios, as is shown by our simple simulation.
3. For the analysis of deflection purely through kinetic impact, we do not consider the effect of celestial bodies apart from the Earth, which would have some effect given

the large distance between the Earth and the point of collision.

7 Conclusion

We conclude that the shortest time before Earth impact required to reflect an asteroid with a kinetic impact approach is 4.8 years. When using a fragmentation approach, an ideal symmetrical fragmentation allows deflection at a time to impact of 15 hours. However, in reality, the second approach is difficult to predict and may lead to asteroid chunks hitting the Earth. The other two possible approaches of landing and the tethering are shown to be impossible for the given spaceship mass. The results of this paper emphasize the need for early detection systems as a significant amount of time before Earth impact is needed to ensure safe deflection.

References

[1] Josh Handal, Justyna Surowiec, Michael Buckley (2022) *NASA's DART Mission Hits Asteroid in First-Ever Planetary Defense Test*, NASA.

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[6] T.J. Ahrens and A.W. Harris. Deflection and fragmentation of near-earth asteroids. *Nature*, 360(6403):429–433, 1992.

[7] A. Carusi, G.B. Valsecchi, G. DAbramo, and A. Boattini. Deflecting neos in route of collision with the earth. *Icarus*, 159(2):417–422, 2002.

[8] G. Vardaxis and B. Wie. Near-earth object intercept trajectory design for planetary defense. *Acta Astronautica*, 101(1):1–15, 2014.

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Appendices

Maple Calculations for Distance and Time

>
> $with(ScientificConstants):$
> $with(Units[Standard]):$
> $Units\text{:-}UsingSystem();$

$$SI \quad (1)$$

> $vinf := 25000 \cdot [[\frac{m}{s}]];$

$$vinf := 25000 \, [[\frac{m}{s}]] \quad (2)$$

> $a := evalf\left(\frac{Constant(G, units) \cdot Constant(M[Earth], units)}{vinf^2}\right);$

$$a := 6.374049600\ 10^5 \, [[m]] \quad (3)$$

> $R := 6371.0[[km]];$

$$R := 6371.0 \, [[km]] \quad (4)$$

> $ve := 4500.0[[\frac{m}{s}]];$

$$ve := 4500.0 \, [[\frac{m}{s}]] \quad (5)$$

> $mm := 20e3[[kg]];$

$$mm := 20000. \, [[kg]] \quad (6)$$

Asteroid mass

> $MA := evalf\left(\frac{1.38[[\frac{g}{cm^3}]] \cdot \text{Pi} \cdot (100[[m]])^3}{6}\right);$

$$MA := 7.225663104\ 10^8 \, [[kg]] \quad (7)$$

Orbital velocity

> $vvv := evalf\left(\text{sqrt}\left(\frac{Constant(G, units) \cdot Constant(M[Earth], units)}{R}\right)\right);$

$$vvv := 1768.189990 \, [[\frac{m}{s}]] \quad (8)$$

Rocket equation

> $ps := r \rightarrow evalf\left(mm \cdot ve \cdot \exp\left(\frac{\text{sqrt}\left(\frac{Constant(G, units) \cdot Constant(M[Earth], units)}{r}\right)}{ve} - 1\right)\right);$

$ps := r$ (9)

$\rightarrow evalf\Big(mm\, ve$

$e^{(\mathit{ScientificConstants{:-}Constant}(G,\ \mathit{units})\ \mathit{ScientificConstants{:-}Constant}(M_{Earth},\ \mathit{units})\ \mathit{Units{:-}Standard{:-}`/`}(r))^{1/2}\ \mathit{Units{:-}Standard{:-}`/`}(ve)\ +\ (-1))}$

> $ps := r \rightarrow evalf(mm \cdot ve \cdot \exp(-1));$

$$ps := r \rightarrow evalf(mm\, ve\, e^{-1}) \quad \textbf{(10)}$$

>

> $evalf(ps(5.0e11[\![m]\!]));$

$$3.310914971\ 10^{7} \left[\!\left[\frac{kg\, m}{s}\right]\!\right] \quad \textbf{(11)}$$

> $evalf\left(\mathrm{sqrt}\left(\frac{Constant(G,\ units) \cdot Constant(M[Earth],\ units)}{5.0e11[\![m]\!]}\right)\right);$

$$28.22687016 \left[\!\left[\frac{m}{s}\right]\!\right] \quad \textbf{(12)}$$

> $val1 := evalf((MA \cdot R \cdot vinf)^{2} + 2 \cdot Constant(G,\ units) \cdot Constant(M[Earth],\ units) \cdot R \cdot MA^{2});$

$$val1 := 1.589521977\ 10^{40} \left[\!\left[\frac{kg^{2}\, m^{4}}{s^{2}}\right]\!\right] \quad \textbf{(13)}$$

> $\frac{fsolve((r[\![m]\!] \cdot ps(r[\![m]\!])) = \mathrm{sqrt}(val1),\ r)}{R};$

$$5.976922657\ 10^{5} \left[\!\left[\frac{1}{m}\right]\!\right] \quad \textbf{(14)}$$

>

> $\frac{mm}{MA};$

$$0.00002767912054 \quad \textbf{(15)}$$

> $evalf\left(\frac{Constant(G,\ units) \cdot MA \cdot MA}{50[\![m]\!]}\right);$

$$6.967974266\ 10^{5} [\![J]\!] \quad \textbf{(16)}$$

> $evalf\left(\mathrm{sqrt}\left(\frac{MA}{mm}\right)\right);$

$$190.0744999 \quad \textbf{(17)}$$

Satellite velocity

> $vel := r \rightarrow evalf\left(\mathrm{sqrt}\left(vinf^{2} + \frac{2 \cdot Constant(G,\ units) \cdot Constant(M[Earth],\ units)}{r}\right)\right);$

$vel := r$ **(18)**

$\rightarrow evalf((vinf^{2} + 2\ \mathit{ScientificConstants{:-}Constant}(G,\ \mathit{units})\ \mathit{ScientificConstants{:-}Constant}(M_{Earth},\ \mathit{units})\ \mathit{Units{:-}Standard{:-}`/`}(r))$

$^{1/2})$

>

Collision time

> $int\left(\frac{1}{\mathrm{sqrt}\left(1+\frac{1}{u}\right)}, u\right);$

$$-\frac{1}{2}\,\frac{\sqrt{\frac{u+1}{u}}\;u\left(-2\sqrt{u^2+u}+\ln\left(\frac{1}{2}+u+\sqrt{u^2+u}\right)\right)}{\sqrt{u\,(u+1)}} \quad \textbf{(19)}$$

>

> $\frac{int\left(\frac{1}{vel(r[\![m]\!])}, r=\frac{R}{[\![m]\!]}\,..\,\frac{5.97\mathrm{e}5\cdot R}{[\![m]\!]}\right)}{3600\cdot 24\cdot 365};$

$$4.824292542\left[\!\left[\frac{s}{m}\right]\!\right] \quad \textbf{(20)}$$

> $evalf\left(\mathrm{sqrt}\left(\frac{mm\cdot vinf^2}{MA}\right)\right);$

$$131.5273749\left[\!\left[\frac{m}{s}\right]\!\right] \quad \textbf{(21)}$$

>

>

>

>

>

FragmentationSimulation.R

```
initpath <- getwd()
PWD <- setwd('/tmp/')

library(tidyverse)
library(latex2exp)

xx <- rnorm(200)
yy <- rnorm(200)
zz <- rnorm(200)

val = 131.5273749

pp <- val*sqrt(xx^2+yy^2)/sqrt(xx^2+yy^2+zz^2)

velp <- data.frame( cbind(v=pp) )

ggplot(velp) +
  geom_histogram(aes(x=v, y=..density..), alpha=0.2, binwidth = 5.0, show.legend = FALSE) +
  labs(y=TeX("P($v$)"), x=TeX("$v$ $\\lbrack$m/s$\\rbrack$")) +
  theme_bw(base_size = 24) + theme(aspect.ratio = 1.0) +
  xlim(0.0,200.0)
ggsave("distr-vperp.svg", width = 4, height = 4, units = "in", dpi=600, scale = 2.4)



a1 <- sum(pp<91.20)/length(pp)
a2 <- sum(pp<68.46)/length(pp)
a3 <- sum(pp<54.77)/length(pp)
a4 <- sum(pp<45.64)/length(pp)

AA <- list(r=c(300,400,500,600),P=c(a1,a2,a3,a4))

df <- as.data.frame(do.call(cbind,AA))

ggplot(df) +
  geom_point(data=df, aes(x=r, y=P)) +
  stat_smooth(aes(y=P, x=r), formula = y ~ s(x, k = 2), method = "gam", se = FALSE) +
  labs(y=TeX("Hit ratio"), x=TeX("$R/R_{Earth}$")) +
  theme_bw(base_size = 24) + theme(aspect.ratio = 1.0) +
  xlim(200.0,700.0) + ylim(0.0, 0.4)

ggplot(df) +
  geom_point(data=df, aes(x=r, y=P), size=4.0) +
  labs(y=TeX("Hit ratio"), x=TeX("$R/R_{Earth}$")) +
  theme_bw(base_size = 24) + theme(aspect.ratio = 1.0) +
  xlim(200.0,700.0) + ylim(0.0, 0.4)
ggsave("p-impact.svg", width = 4, height = 4, units = "in", dpi=600, scale = 2.4)


setwd(initpath)
```