Analysis of the optimal initial velocities and spins of soccer in Penalty Kicks

Problem B: FIFA Penalty Kicks

Team 480

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Abstract

Among the plentiful factors effecting the moving motion of the ball, gravity, air resistance, and Magnus force are the three most important factors. In the course of the goalkeeper's defense, the judgment of the direction of the shot, the defensive area and the start time of the attack are the main factors that affect the success or failure of the penalty kick saving. We divided the process into two parts: the soccer moving through the air and the defense of the goalkeeper.

The first part is to find initial velocities and spins that will result in a successful shot to an upper corner. During this process, the gravity, air resistance, and Magnus force working together, formulating the curve of the ball. Distinguish from the common theory, we take the multi-directional axis of rotation and air resistance moment into account and establish a physical model that can calculate and judge whether the input initial velocities and spins can match our request of distance and position. We assume that the distance from the top corner of the crossbar and goalpost from 1 soccer radius to 2 soccer radius is the upper corner, and the maximum velocity is 31m/s. So that we solve out the value range of initial velocities and spins that will result in a successful shot.

The second part is built on the model of part 1. To figure out the initial ball velocity and spin should the shooting player attempt to create to have the best chance of avoiding the goalkeeper and making the goal, among the three factors --- the judgment of the direction of the shot, the defensive area and the start time of the attack , supposing that the goalkeeper can always accurately judge the direction of the shot and react instantly to defend the goal and the defensive area is the only factor that needed to be considered. We simulate the maximum defending area of the goalkeeper and finally calculate the best initial velocity is 21.245 m/s with elevation of 0.2057 rad, declination of 1.4182 rad and the spin is 15.255 rad/s with elevation of 1.417 rad and declination of -0.752 rad, shooting towards the left bootom of the goal.

Key words: Penalty Kicks; initial velocities; spins; Magnus force

Contents

1. Introduction

According to the soccer competition rules, when a penalty kick is taken in the game of FIFA soccer, one player takes a single shot at the goal and is only defended by the opposing team's goalkeeper. It is taken with the ball at a point centered 11 meters from the goal, which is a rectangle 7.32 meters wide and 2.44 meters tall.

We divided the whole process into two parts: the process of the ball moving through the air and the defense of the goalkeeper. And we establish physical models to analyze and figure out the initial ball velocity and spin should the shooting player attempt to create to shot the upper corner of the goal as well as have the best chance of avoiding the goalkeeper and making the goal.

Players can choose to kick to different positions in the goal, top left, bottom left, top center, bottom center, top right, bottom right. To avoid the goalkeeper, the shooting player often aims for an upper corner of the goal. The impact factors of the ball's moving motion is complicated, after investigation and research, we have learned that gravity, air resistance, and Magnus force are the three most important factors in the curve motion of the ball. In order to find way of kicking the ball to have the best chance of avoiding the goalkeeper and making the goal, suppose that the goalkeeper can always accurately judge the direction of the shot and react instantly to defend the goal, thus the only main factor that need to be decided is the goal shooting area.

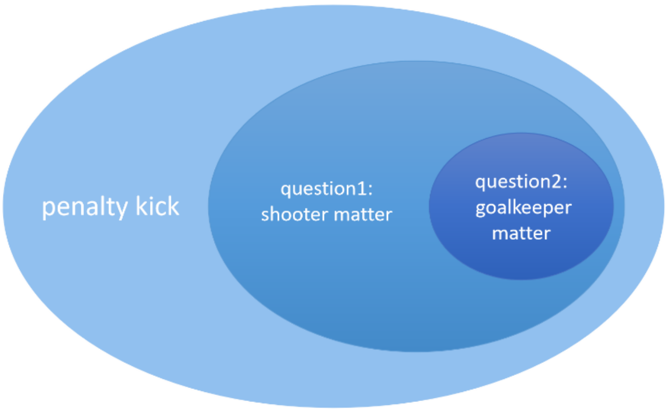

Figure 1 the process of solution

2. Restatement

This question is to consider the moving motion of soccer when a penalty kick is taken in the game of FIFA, and analysis the reaction of goalkeeper to find the best chance of scoring a goal. Finally, through investigation and solving systems of equations to find the initial ball velocities and spins that will result in a successful shot to an upper corner from the penalty mark, and also figure out the initial ball velocity and spin should the shooting player attempt to create to have the best chance of avoiding the goalkeeper and making the goal.

3. Assumptions and Notations

3.1 Assumptions

(1) The influence of psychological and other human factors are ignored.
(2) In the process of motion, the spinning soccer is only affected by gravity, air resistance, Magnus forces.
(3) Suppose the moment of air resistance on the soccer is proportional to the angular velocity of rotation.
(4) Suppose there is no effect of wind speed.
(5) The width of the goal frame is ignored.
(6) The value of air resistance is proportional to the value of velocity squared.

3.2 Notations

Variables	Explanations
$\vec{\omega}$	the angular velocity of the ball
$\vec{V}$	the velocity of the flow relative to the sphere
$\vec{f}$	Magnus force
ρ	the air density
r	the radius of the ball
I	the moment of inertia
t	time
$\vec{M}$	torque
m	the mass of the soccer
t_a	time of acceleration
k	proportional coefficient
a	accelerated speed
g	acceleration of gravity
V_0	the initial velocity
$x/y/z$	directions
$\alpha/\ \beta$	angle
$\vec{R}$	air resistance

4. Physical Analysis of Model

4.1 Moving Motion of the Ball

In the process of moving through the air, taking three forces into consideration: gravity, Magnus force $\vec{f}$, and air resistance.

The force that deflects a spinning soccer in flight is Magnus force. If the ball rotates, the combination of rotation and air viscosity creates a circulation in the boundary layer around the ball. As a result of the combination of the inflow and the current, the flow speeds up on the side of the inflow and the current in the same direction, and slows down on the opposite side. According to Bernoulli's principle, the difference between the pressure falling on the side where the flow is speeding up and the pressure rising on the other side where the flow is slowing down is called the Magnus force. The angular velocity of rotation is ω, the velocity of the flow relative to the sphere is $\vec{V}$.

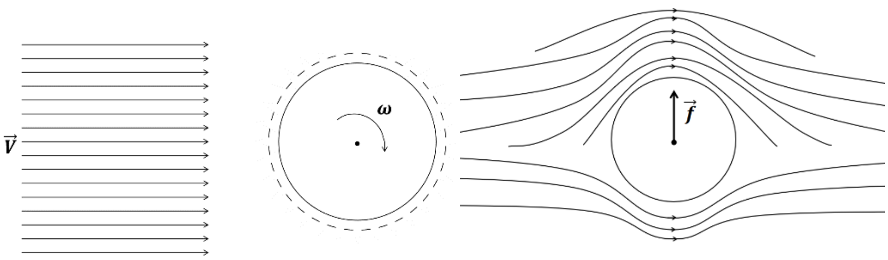

Figure 2 the cause of Magnus force

In the curve motion of the ball, the force analysis of the ball as figure 3:

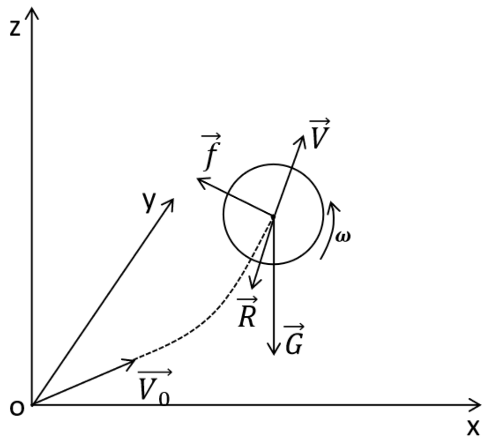

Figure 3 the force analysis of soccer

The magnitude and direction of Magnus force is given by the following equation :

$$\vec{f} = \frac{8}{3}\pi\rho r^3 \vec{\omega} \times \vec{V} \tag{1}$$

$\vec{V}$ is the speed of soccer relative to the air, $\vec{\omega}$ is instantaneous rotation angular velocity, r is the radius of the ball, ρ refers to the air density.

Suppose the moment of inertia of soccer about its central axis is I and the initial angular velocity is $\vec{\omega}$. According to the assumptions, the angular velocity of the ball at time t satisfies the following equation:

$$\vec{M} = I\frac{d\vec{\omega}}{dt} = -\eta\vec{\omega} \tag{2}$$

Which means:

$$\begin{cases} I\dfrac{d\omega_x}{dt} = -\eta\omega_x \\ I\dfrac{d\omega_y}{dt} = -\eta\omega_y \\ I\dfrac{d\omega_z}{dt} = -\eta\omega_z \end{cases} \tag{3}$$

Through equations 3,

$$\vec{\omega} = \vec{\omega}_0 e^{-\mu t} = \left(\omega_{0x}\vec{\imath} + \omega_{0y}\vec{\jmath} + \omega_{0z}\vec{k}\right)e^{-\mu t} \tag{4}$$

$\mu = \eta/I$, equation 1 can be expressed as:

$$\vec{f} = \frac{8}{3}\pi\rho r^3 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ V_x & V_y & V_z \end{vmatrix} = \frac{8}{3}\pi\rho r^3 e^{-\mu t} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_{0x} & \omega_{0y} & \omega_{0z} \\ x' & y' & z' \end{vmatrix}$$

$$= \frac{8}{3}\pi\rho r^3 e^{-\mu t}\left[\left(\omega_{0y}z' - \omega_{0z}y'\right)\vec{\imath} + (\omega_{0z}x' - \omega_{0x}z')\vec{\jmath} + \left(\omega_{0x}y' - \omega_{0y}x'\right)\vec{k}\right] \tag{5}$$

Taking air resistance into consideration, which is proportional to the square of the velocity and opposite to the direction of the velocity:

$$\vec{R} = -kV^2\vec{V} = -kV\vec{V} = -k\sqrt{x'^2 + y'^2 + z'^2}\left(x'^2\vec{\imath} + y'^2\vec{\jmath} + z'^2\vec{k}\right) \tag{6}$$

k is the proportional coefficient, which is determined by the size of the sphere, the density ρ of the air and the resistance coefficient c, $k = \frac{\pi}{2}c\rho r^2$.

Equations (5) and (6) and gravity are substituted into Newton's second law to get the equations of soccer dynamics:

$$\begin{cases} mx'' = -k\sqrt{x'^2 + y'^2 + z'^2}x' + \lambda\left(\omega_{0y}z' - \omega_{0z}y'\right)e^{-\mu t} \\ my'' = -k\sqrt{x'^2 + y'^2 + z'^2}y' + \lambda(\omega_{0z}x' - \omega_{0x}z')e^{-\mu t} \\ mx'' = -k\sqrt{x'^2 + y'^2 + z'^2}z' + \lambda\left(\omega_{0x}y' - \omega_{0y}x'\right)e^{-\mu t} - mg \end{cases} \tag{7}$$

m refers to the mass of the soccer; $\lambda = \frac{8}{3}\pi\rho r^3$.

Solving the system of equation (7), we can get the moving curve of the ball.

4.2 Initial Velocities and Spins of the Upper Corner Ball

System (7) is a complex nonlinear system of differential equations with no analytical solution. Considering the limits of human energy and strength, the maximum initial velocity is set to 31m/s, and have to make sure the soccer is kicked into the upper corner of the goal which is 11 meters away.

On the other hand, according to the actual situation, the partial velocity of the ball's initial velocity is slower in the x and z axis directions, while the partial velocity is larger in the y axis directions. According to some researches, the initial velocity in the x, y and z directions is set respectively 0~11m/s, 20~30m/s, 0~10m/s. By reading a lot of papers on curveball and looking through a lot of soccer videos, the value range of the angular velocity of the ball in the x,y and z direction is set respectively -8~8 rad/s, -5~5 rad/s and 0~15 rad/s.

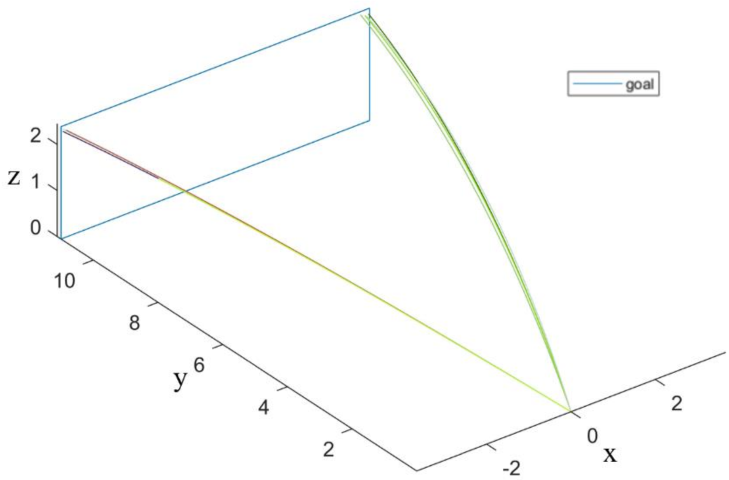

Figure 4 the simulation of shooting the upper corner

According to the value range, we substitute the numerical value into the system of differential equations (7) and select the initial velocity and spins that conforms to the system of differential equations and get more than 600 results.

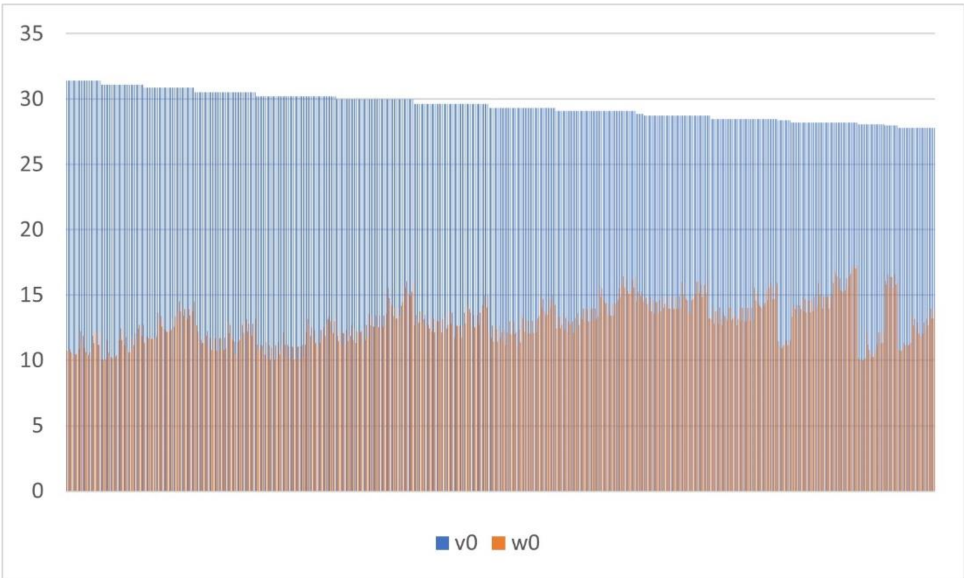

Figure 5 Statistics of data about v_0 and ω_0

We use x, y, and z coordinates to represent the angular velocity, the angular velocity elevation, and the angular velocity deflection, respectively. The size of the point is used to represent the velocity, the color of the point is used to represent the velocity deflection, and the color of the point frame is used to represent the elevation Angle of the velocity. The six-dimensional data can be represented in the three-dimensional coordinate system to complete data visualization.

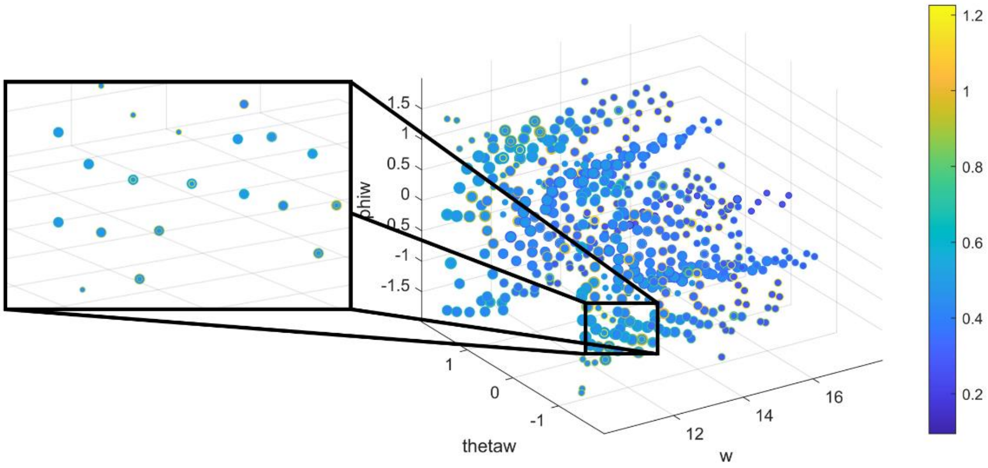

Figure 6 Data visualization of velocities and spins

As shown in table 1, we list the value range of velocities and spins:

Table 1 detailed data on velocities and spins

$v_0/(m/s^2)$	θ_v/rad	φ_v/rad	$type$	w_0/(rad/s)	θ_w/rad	φ_w/rad
27.82086	0.25434	1.19029	max	10.81665	$\pm$1.19029	$\pm$0.69474
			min	13.96424	$\pm$0.86039	0
27.96426	0.21624	1.15629	max	16.61325	$\pm$1.02285	$\pm$0.55860
			min	15.55635	$\pm$0.94304	$\pm$0.35877
28.08914	0.28881	1.19029	max	16.54734	$\pm$1.57080	$\pm$1.57080
			min		$\pm$1.07681	0
28.19574	2.50888	1.15629	max	17.29162	$\pm$1.28439	$\pm$1.03038
			min	13.34166	$\pm$1.05010	0
28.39014	0.32258	1.19029	max	11.57584	$\pm$1.15026	$\pm$1.03038
			min	10.95445	$\pm$1.0429	$\pm$0.46365
28.46050	0.28493	1.15629	max	15.93738	$\pm$1.57080	$\pm$1.57080
			min	12.68858	$\pm$1.14897	0
28.72280	0.24619	1.20362	max	13.19091	$\pm$0.86039	$\pm$0.62025
			min			
28.75761	0.31831	1.15629	max	16.09348	$\pm$1.28439	$\pm$1.03038
			min	13.56466	$\pm$0.94884	0
28.86174	0.20942	1.17056	max	15.26434	$\pm$0.93027	$\pm$0.55860
			min	14.49138	$\pm$0.86189	$\pm$0.46365
29.08610	0.24305	1.17056	max	16.43168	$\pm$1.21406	$\pm$1.03038
			min	12.08305	$\pm$0.98623	0
29.34280	0.27614	1.17056	max	15	$\pm$1.57080	$\pm$1.57080
			min	11.1803	$\pm$1.13673	0

29.63106	0.30861	1.17056	max	15	±1.30025	±1.19029
			min	11.74734	±0.90773	0
29.98333	0.235638	1.183921	max	16.06238	±1.1442	±0.89606
			min	11.18034	±0.90455	0
30.23243	0.267806	1.183921	max	13.34166	±1.48014	±1.57080
			min	10.04988	±1.09927	0
30.51229	0.299417	1.183921	max	13.15295	±1.29087	±1.19029
			min	10.04988	±0.90773	0
30.8869	0.22862	1.196463	max	14.49138	±1.07681	±0.55860
			min	11.35782	±0.81452	0
31.1288	0.25991	1.196463	max	12.72792	±1.47113	±1.03038
			min	10.04988	±1.00126	0
31.40064	0.290695	1.196463	max	12.24745	±1.29515	±1.37340
			min	10.39230	±1.07681	0

4.3 Defense of Goalkeeper

In the course of the goalkeeper's defense, the judgment of the direction of the shot, the defensive area and the start time of the attack are the main factors that affect the success or failure of the penalty kick saving [3].

In order to find the initial ball velocity and spin should the shooting player attempt to create to have the best chance of avoiding the goalkeeper and making the goal, suppose that the goalkeeper can always accurately judge the direction of the shot and react instantly to defend the goal, thus the only main factor that need to be decided is the goal shooting area. The average height of goalkeeper is 185 centimeters. The spread length of human arms is almost equal to its height. The distance from chin to shoulder of human body is 1/3 of the length of head, and the length of the whole body is 8 heads. Therefore, the 185-tall goalkeeper has an arm length of 69.5 centimeters. Professional athletes can jump to a height of more than 1 meter, take 1 meter as an example, the initial velocity of goalkeeper can be given by the following formula:

$$V_0 = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1} = 4.43\ m/s \tag{8}$$

When people take off, they need to lift the back root of their feet according to their physical conditions, 10 cm to 20 cm from the ground, 20 cm to 30 cm shallow squat, and take an average acceleration distance of S_a =40 cm, a is the average takeoff acceleration:

$$V_0{}^2 = 2(a-g)S_a \tag{9}$$

$$V_0 = (a-g)t \tag{10}$$

Thus, the average takeoff acceleration a and the acceleration time t_a can be calculate:$a = V_0{}^2 \times S_a + g = 34.33 m/s^2,\ t_a = V_0/(a-g) = 0.181s.$

When the goalkeeper takes off at an oblique angle, the force analysis to the goalkeeper's center of gravity is shown in Figure 6:

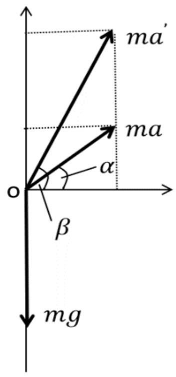

Figure 7 the accelerated speed analysis of goalkeeper

The process by which a goalkeeper dives for the ball can be regarded as a motion of uniform acceleration followed by free fall.According to the geometric relation and newton's second law:

$$\tan\beta = \frac{a \times \sin\alpha - g}{a \times \cos\alpha} \tag{11}$$

$$V_x = a \times \cos\alpha \times t_a \tag{12}$$

$$V_y = (a \times \sin\alpha - g) \times t_a \tag{13}$$

Thus, the displacement components in the direction of the two coordinate axes are obtained:

$$\begin{cases} x = \frac{1}{2} \times a \times \cos\alpha \times t^2 (t < t_a) \\ x = V_x \times (t - t_a) + \frac{1}{2} \times a \times \cos\alpha \times {t_a}^2 (t \geq t_a) \end{cases} \tag{14}$$

$$\begin{cases} y = y_0 + \frac{1}{2} \times (a \times \sin\alpha - g) \times t^2 (t < t_a) \\ y = y_0 + \frac{1}{2} \times (a \times \sin\alpha - g) \times {t_a}^2 - \left(V_y \times t_a - \frac{1}{2} g t^2\right) (t \geq t_a) \end{cases} \tag{15}$$

After get rid of t, s is the result of integrating with respect to x. Taking the derivative of α to find the maximum of s. It can be solved that when s is at its maximum $13.34m^2$, $V_x = 3.34m/s$, $V_y = 3.47m/s$, $\alpha = 57.52°$.

The goalkeeper's "control area" mainly refers to the place where the hands and arms can reach. The goalkeeper can skip the ball in any direction from the middle of the goal. In addition to diving outwards, a goalkeeper can also jump upwards to prevent the ball from going into the goal, which can formulate a three-dimensional defensive zone shapes like a ellipsoid.

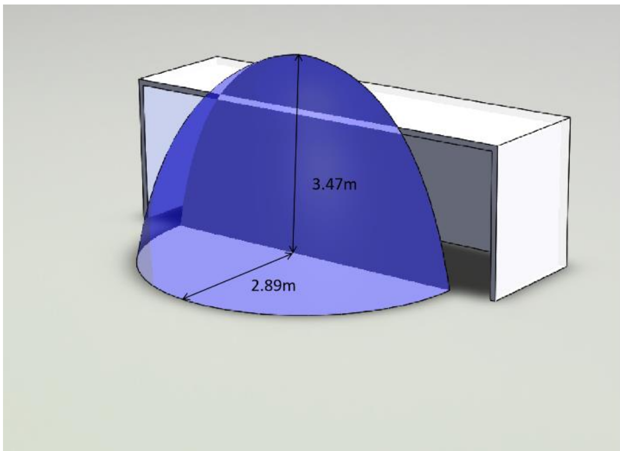

Figure 8 the area that goalkeeper can cover

Considering the goalkeeper's best case, any ball whose path overlaps with the goalkeeper's defensive area will be guarded by the goalkeeper. Through figure 8, if the shooting player attempt to create to have the best chance of avoiding the goalkeeper and making the goal, he has to aim at the four corners of the goal.

4.4 Initial Velocity and Spin of the Best Kick

Through the analysis above, we conclude that the easiest way to score is to shoot into the corner. However, we have to choose shooting upper or bottom corner. The shooting player should shot to an bottom corner for the following reasons:

Firstly, it is more possibly to score through the bottom corner than the upper one. We use the velocity and spin through regional center as an initial condition, then move the initial condition up and down 2rad/s or 2m/s. By counting the number that match condition of upper or bottom corner, we can distinguish the precise one.

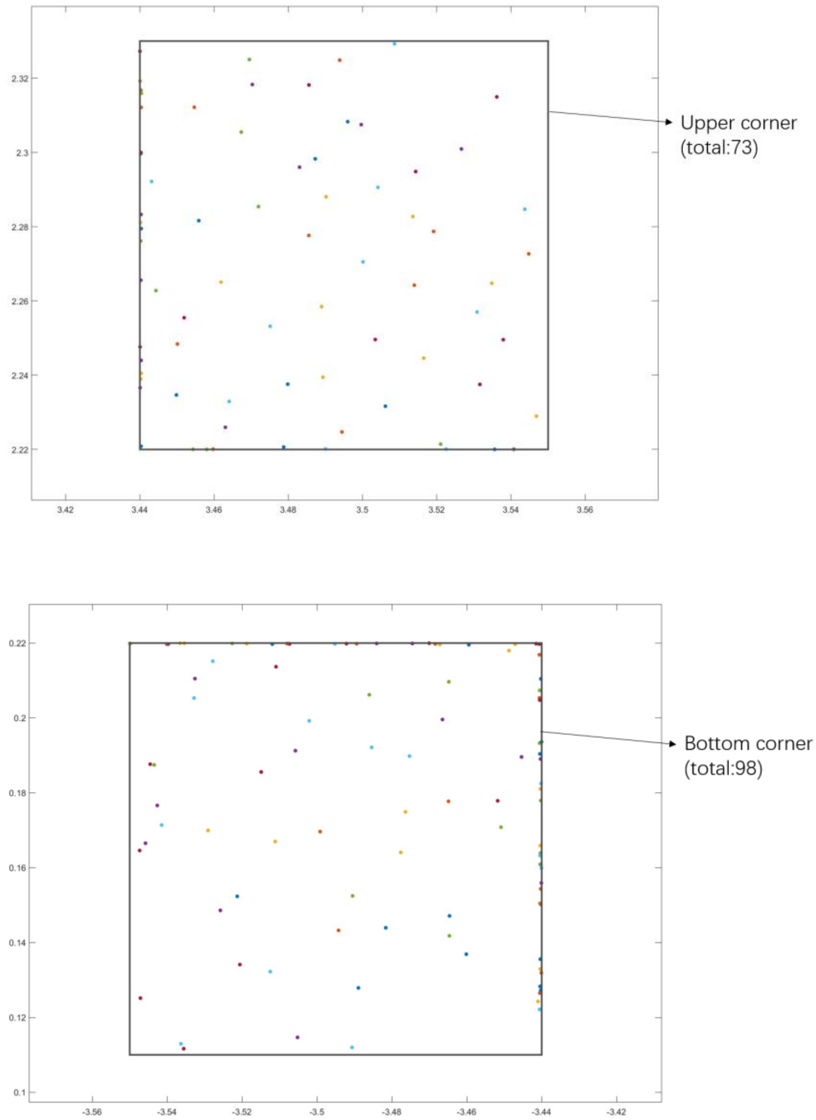

Figure 9 comparation of upper and bottom corner

Secondly, according to the World Cup statistics from 2002-2018, shooting bottom corner own higher goal and lower miss.

Table 2 penalty kicks statistics of the World Cup from 2002-2018

Corner and height	Goal	Miss	Saved	Post/Crossbar
Top left corner	83.53%	6.62%	0.17%	9.68%
Bottom left corner	82.95%	2.79%	8.15%	6.11%
To the center	71.51%	6.56%	1.30%	20.63%
Top right corner	77.59%	9.93%	0.43%	12.06%
Bottom right corner	83.35%	2.50%	8.07%	6.08%

Thirdly, there is higher risk to shot to upper corner than to bottom one. Because of the energy exhaustion and mental stress, shooting bottom corner should be a better choice.

Therefore, we conclude that the shooting player should shot to an bottom corner.

Applying the model proposed in 4.1, change the destination from upper corner to bottom corner, so that we solve out the initial velocity and spin of the best kick:

Table 3 the initial velocity and spin of the best kick

v_0/ (m/s^2)	θ_v/rad	φ_v/rad	w_0/ (rad/s)	θ_w/rad	φ_w/rad
21.245021	0.205708	1.418237	15.254751	1.416735	-0.751829

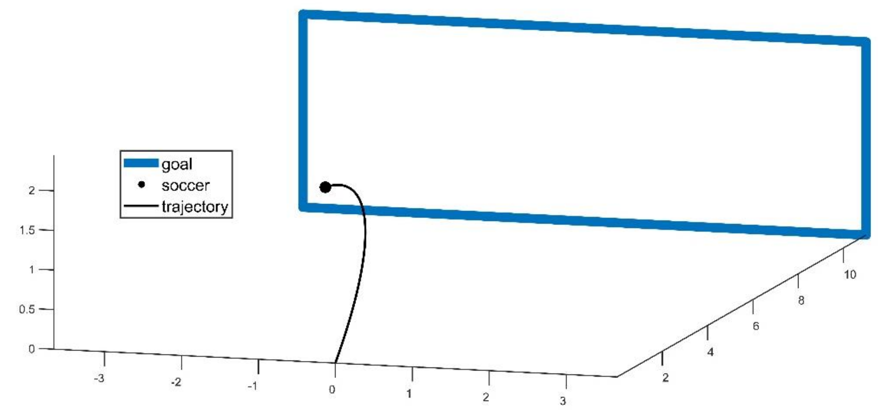

Figure 10 the initial velocity and spin of the best kick

We show the condition of shotting bottom left corner, because the other one could be the same.

5 Model Evaluation

5.1 Strengths

(1) In order to facilitate calculation, many papers make a series of simplifications, ignoring the air resistance moment and vertical air resistance, and limiting the axis of rotation to the vertical direction, which is quite different from the actual situation. We have improved this by taking into account the vertical air resistance and the multi-directional axis of rotation to make the physical model more realistic.
(2) The range of velocity is considered reasonably to ensure that it is achievable for athletes
(3) Three-dimensional coordinate system is uesd to represent six-dimensional data to enhance the readability of data.

5.2 Weaknesses

(1) The model is too ideal to satisfy all the situations .
(2) The range of angular velocity is not very clear. If the speed and angular velocity are to be further improved in rationality, it is necessary to make the range of angular velocity more accurate.

6 Reference

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Appendix

MATLAB code

```
clear all;
clc;
close all;
%% statistic
m = 0.43;                    %mass of soccer
a = 0.11;                    %radius of soccer
airv = 18.1*10^(-6);           %air viscosity coefficient
d = 1.29;                   %air density
A = pi*a^2;                   %the cross-section area of
soccer
I = 2/5*m*a^2;                 %rotational inertia of
soccer
c = 0.5;
k = pi/2*c*d*a^2;
lamda = 8/3*pi*d*a^3;
mu = 0.1;                    % mu = yita/I
g = 9.8;
%gate
width = 7.32; distance = 11; height = 2.44;
ballgate = [3.66,11,2.44;-3.66,11,2.44;-
3.66,11,0;3.66,11,0;3.66,11,2.44];
figure;
plot3(ballgate(:,1),ballgate(:,2),ballgate(:,3));hold
on;
%% start
w0x = -8:8;     v0x = [-10,-11];
w0y = -5:5;    v0y = [];
w0z = [-15:-10,10:15];    v0z = [];
range = [a,2*a];
for i = 10:11
    for j =25:30
        for kk = 6:9
            if(i^2+j^2+kk^2<=961)
                if(~ismember(i,v0x))
                    v0x = [v0x,i];
                end
                if(~ismember(j,v0y))
                    v0y = [v0y,j];
                end
                if(~ismember(kk,v0z))
```

```
                v0z = [v0z,kk];
            end
        end
    end
  end
end
szwx = size(w0x);  szvx = size(v0x);
szwy = size(w0y);  szvy = size(v0y);
szwz = size(w0z);  szvz = size(v0z);
detat = 0.0001;
t = 0:detat:1;
szt = size(t);
row =
szwx(2)*szwy(2)*szwz(2)*szvx(2)*szvy(2)*szvz(2);
line = szt(2);
x = [];
test = [];
y = [];
z = [];
record = [];
recordres = [];
flag = 1;
count = 1;
for wx = 1:szwx(2)
   for wy = 1:szwy(2)
      for wz = 1:szwz(2)
        for vx = 1:szvx(2)
           for vy = 1:szvy(2)
              for vz = 1:szvz(2)
                 xx = 0;
                 yy = 0;
                 zz = 0;
                 w0x0 = w0x(wx);w0y0 = w0y(wy);w0z0 =
w0z(wz);v0x0 = v0x(vx);v0y0 = v0y(vy);v0z0 = v0z(vz);
                 w0x0temp = w0x0;w0y0temp =
w0y0;w0z0temp = w0z0;v0x0temp = v0x0;v0y0temp =
v0y0;v0z0temp = v0z0;
                 for ti = 1:szt(2)
                    xx = v0x0*detat+xx;
                    yy = v0y0*detat+yy;
                    zz = v0z0*detat+zz;
                    if(yy<0||zz<0)
                       break;
                    end
```

```
                    if(yy>=10.9 && yy<= 11.1 &&
xx>=(width/2-range(2)) && xx<= (width/2-range(1)) &&
zz>=(height-range(2)) && zz<=(height-
range(1)))||(yy>=10.9 && yy<= 11.1  && xx<=-(width/2-
range(2)) && xx>= -(width/2-range(1)) && zz>=(height-
range(2)) && zz<=(height-range(1)))
                        x= [x,xx];
                        y= [y,yy];
                        z= [z,zz];
                        record = [record;t(ti),
w0x0temp, w0y0temp, w0z0temp,v0x0temp, v0y0temp,
v0z0temp];
                        v = (v0x0temp^2+ v0y0temp^2+
v0z0temp^2)^0.5;
                        w = (w0x0temp^2+ w0y0temp^2+
w0z0temp^2)^0.5;
                        thetav = asin(v0z0temp/v); phiv
= asin(v0y0temp/(v*cos(thetav)));
                        thetaw = asin(w0z0temp/w); phiw
= asin(w0y0temp/(w*cos(thetaw)));
                        recordres = [recordres; t(ti),
v, thetav, phiv, w, thetaw, phiw];
                        if(flag <= 6)
                          drawpic(detat, t(ti),
w0x0temp, w0y0temp, w0z0temp,v0x0temp, v0y0temp,
v0z0temp);
                          drawpic(detat, t(ti),
w0x0temp, -w0y0temp, -w0z0temp,-v0x0temp, v0y0temp,
v0z0temp);
                          flag = flag+1;
                        end
                        break;
                    end
                    v0x0tmp = v0x0;v0y0tmp =
v0y0;v0z0tmp = v0z0;
                    ax = (-k*(v0x0tmp^2 + v0y0tmp^2 +
v0z0tmp^2)^0.5*v0x0tmp + lamda*(w0y0 * v0z0tmp-w0z0 *
v0y0tmp))/m;
                    ay = (-k*(v0x0tmp^2 + v0y0tmp^2 +
v0z0tmp^2)^0.5*v0y0tmp + lamda*(w0z0 * v0x0tmp-w0x0 *
v0z0tmp))/m;
                    if(v0z0<=0)
                        az = (k*(v0x0tmp^2 + v0y0tmp^2
+ v0z0tmp^2)^0.5*v0z0tmp + lamda*(w0x0 * v0y0tmp-w0y0
```

```
* v0x0tmp)-m*g)/m;
                    else
                        az = (-k*(v0x0tmp^2 + v0y0tmp^2
+ v0z0tmp^2)^0.5*v0z0tmp + lamda*(w0x0 * v0y0tmp-w0y0
* v0x0tmp)-m*g)/m;
                    end

                    v0x0 = v0x0+ax*detat;
                    v0y0 = v0y0+ay*detat;
                    v0z0 = v0z0+az*detat;
                    w0x0 = w0x0temp*exp(-mu*t(ti));
                    w0y0 = w0y0temp*exp(-mu*t(ti));
                    w0z0 = w0z0temp*exp(-mu*t(ti));
                end
                count = count+1
            end
          end
       end
     end
  end
end
xlswrite('datarecord.xlsx',record);
xlswrite('datares.xlsx',recordres);

figure;
recorders = real(recordres);
szres = (recordres(:,2)-27)*10;
cres = (recordres(:,4)-1.1)*5;
alpha = (recordres(:,3)-0.2)*10;
scatter3(recordres(:,5),recordres(:,6),recordres(:,7)
,szres,cres,'filled');hold on;
scatter3(recordres(:,5),recordres(:,6),recordres(:,7)
,szres+3,alpha);hold on;
xlabel('w')
ylabel('thetaw')
zlabel('phiw')
colorbar
```

```
function drawpic(detat, t, wx0, wy0, wz0,vx, vy, vz)
%% statistic
m = 0.43;                    %mass of soccer
a = 0.11;                    %radius of soccer
airv = 18.1*10^(-6);          %air viscosity coefficient
d = 1.205;                   %air density
A = pi*a^2;                  %the cross-section area of
soccer
I = 2/5*m*a^2;                %rotational inertia of
soccer
c = 0.5;
k = pi/2*c*d*a^2;
lamda = 8/3*pi*d*a^3;
mu = 0.1;                    % mu = yita/I
g = 9.8;
%gate
width = 7.32; distance = 11; height = 2.44;
ballgate = [3.66,11,2.44;-3.66,11,2.44;-
3.66,11,0;3.66,11,0;3.66,11,2.44];
%% draw
tt = 0:detat:t;
szt = size(tt);
colorbox = rand(1,3);
x = [];y = []; z = [];
xx = 0;
yy = 0;
zz = 0;
w0x0 = wx0;
w0y0 = wy0;
w0z0 = wz0;
v0x0 = vx;
v0y0 = vy;
v0z0 = vz;
for ti = 1:szt(2)
   xx = v0x0*detat+xx;
   yy = v0y0*detat+yy;
   zz = v0z0*detat+zz;
   x = [x,xx];y = [y,yy];z = [z,zz];
   v0x0tmp = v0x0;v0y0tmp = v0y0;v0z0tmp = v0z0;
   ax = (-k*(v0x0tmp^2 + v0y0tmp^2 +
v0z0tmp^2)^0.5*v0x0tmp + lamda*(w0y0 * v0z0tmp-w0z0 *
v0y0tmp))/m;
   ay = (-k*(v0x0tmp^2 + v0y0tmp^2 +
v0z0tmp^2)^0.5*v0y0tmp + lamda*(w0z0 * v0x0tmp-w0x0 *
```

```
v0z0tmp))/m;
    if(v0z0<=0)
      az = (k*(v0x0tmp^2 + v0y0tmp^2 +
v0z0tmp^2)^0.5*v0z0tmp + lamda*(w0x0 * v0y0tmp-w0y0 *
v0x0tmp)-m*g)/m;
    else
      az = (-k*(v0x0tmp^2 + v0y0tmp^2 +
v0z0tmp^2)^0.5*v0z0tmp + lamda*(w0x0 * v0y0tmp-w0y0 *
v0x0tmp)-m*g)/m;
    end

    v0x0 = v0x0+ax*detat;
    v0y0 = v0y0+ay*detat;
    v0z0 = v0z0+az*detat;
    w0x0 = wx0*exp(-mu*tt(ti));
    w0y0 = wy0*exp(-mu*tt(ti));
    w0z0 = wz0*exp(-mu*tt(ti));
end
%     scatter3(xx,yy,zz)
    plot3(x, y, z, 'color',colorbox(1,:));hold on;
    axis equal
end
```