Maximum Height of Space Diving

Team 131, Problem A

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Abstract

In our study, we investigate the critical initial altitude for safe skydiving operations. Employing a rigorous set of assumptions, our focus centers on pinpointing the maximum altitude that can induce hazardous levels of acceleration leading to potential harm. We construct a comprehensive model, accounting for supersonic phenomena, and rigorously validate its accuracy using empirical data gathered from Felix Baumgartner's remarkable 39-kilometer jump. Through our model, we derive intricate relationships between velocity, acceleration, and the corresponding descent distances. Subsequently, we assess the upper bounds of acceleration associated with different initial altitudes, thereby determining the maximum altitude at which a skydiver can descend without facing life-threatening levels of acceleration. Considering these insights in conjunction with other pertinent factors, we unveil our ultimate findings. Our analysis culminates in the critical revelation that the safety threshold for initial jump height in skydiving must not exceed 94 kilometers to ensure the successful return of the diver to Earth's surface, thus mitigating the risks associated with excessive acceleration.

Key Words: Supersonic, Acceleration, Initial height, skydive

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1 Introduction

1.1 Background

1.1.1 History

In 2012, Austrian parachutist Felix Baumgartner broke the highest skydive records since 1960. Fallen from an altitude of 38.969 kilometers, he became the first person to reach one Mach in a skydiving. This remarkable achievement ignites people's passion for exploring the limits of humans. Following in his footsteps, we want to explore whether a space dive is achievable theoretically. And furthermore, what is the highest limit for space diving? What factor will constrain the maximum altitude for a safe space dive?

1.1.2 Supersonic speed

The sound barrier or sonic barrier is the large increase in aerodynamic drag and other undesirable effects experienced by an aircraft or other object when it approaches the speed of sound. When aircraft first approached the speed of sound, these effects were seen as constituting a barrier, making faster speeds very difficult or impossible. However, reaching enough height, the atmospheric temperature and the air density can no longer be approximated as constants and should change accordingly. This can lead to possibility of a long duration of supersonic flight.

1.2 Problem Restatement

According to the question, we are supposed to confront with a physical scenery, that a space diver is carried to a certain height, jumping down toward the earth. We should take into account possible impacts and dangers the space diver will encounter during different periods in the entire process, modelling this physical scenery and solve the model to gain the maximum height from which the space diver can land successfully.

In this article, we are going to explore the theoretical maximum altitude of a safe space dive. We will first check the potential dangerous factors in a long space dive and bring out specific safety criteria. Then, we will establish a physical model of the space dive process from kinematic and fluid mechanic formulas. We can then get the velocity and the acceleration during the falling process. Based on the physical model and the safety criteria, we are able to explore, under extreme circumstances, whether vertical acceleration, flat spin, and temperature will hurt the skydiver. Finally, through careful consideration, we can assess the maximum altitude from which the skydiver could successfully descend to the surface.

2 Notations

Symbols	**Description**
ρ	Density of air
C_s	speed of sound
γ	Adiabatic index
R^*	molar gas constant
T_M	atmospheric temperature
M_0	molar mass taken as $29g/mol$
C_D	Drag coefficient
A	cross section area of human body
h	Initial height
M_e	Mass of the earth
R_e	Average radius of the earth
T	Temperature of the air
M_A	Average molar mass of the air
x	Space diver free-fall displacement
v	Velocity of the space diver
a	Acceleration of the space diver
s	Shock wave angle
M	Mach
P_{t0}	Total upstream pressure
P_{t1}	Total downstream pressure
R	Cylindrical human model radius
ω_d	Dangerous spinning angular velocity
t_r	Reaction time of human

Here the main notations are defined while their specific values will be discussed and given later.

3 Assumptions

Note: In order to simplify the space-diving process and to get an abstract physical model without losing practical meaning, we will make some assumptions about the space-diving process in this subsection.

We assume the weather at the time of the space dive is clear, meaning that there will be no severe airflow or even storm in the troposphere.

3.1 Temperature

Once the skydiver is in the space, he/she has to wear a space suit to protect him/her from low pressure, extreme temperature and cosmic rays. But the space suit itself has a tolerance of temperature. Based on an article published by NASA in 2019 [1], a spacewalk space suit can withstand the temperature range from $-250°F$ ($-156°C$) to $250°F$ ($121°C$). Therefore, the atmospheric temperature can not exceed that range.

3.2 Acceleration and Body Position

During the falling process, the most dangerous factor to the human body is the acceleration. In astronautics, we usually use G_x, G_y, G_z to describe the acceleration the human body endures. G value is the ratio between the acceleration in one direction and the gravitational acceleration.

$$G = \frac{a}{g}$$

Positive G_x is pointing from chest to back, positive G_y is pointing from right shoulder to left shoulder and positive G_z is pointing from head to toe.

As recommended by James M. Pattarini et al. (2013) [2], the most stable position in a high-altitude free fall is the "Delta" position, with the skydiver's head downwards. So, the acceleration in the transitional motion in the space dive should be majorly G_z.

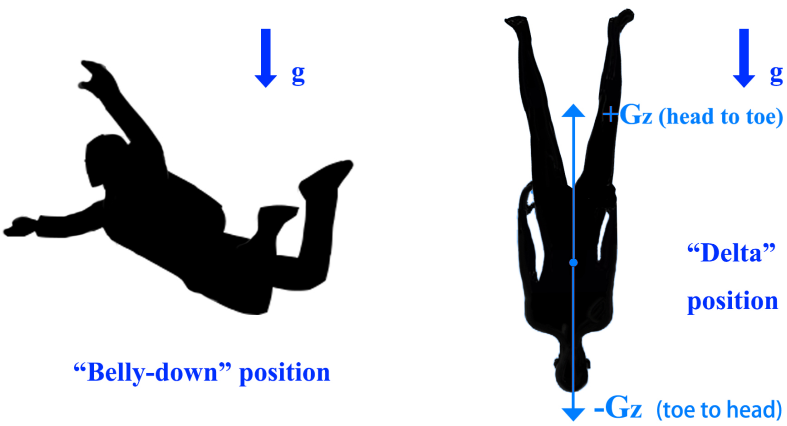


Figure 1: Belly-down" and "Delta" position.

Based on extensive studies on human acceleration tolerance done by NASA in 1992 [3], the human acceleration tolerance limit for $+G_z$ acceleration is $4G$ for 5 seconds and $10G$ for 0.1 seconds. $-G_z$ acceleration tolerance is $-5G$ for 5 seconds and $-7G$

for 0.1 seconds. So we assume the transitional motion in a "safe" space dive should not exceed $\pm 5G_z$.

In order to complete a space dive mission, we assume the space diver has to adjust his/her body position from "Delta" position to "Belly-down" position at a certain point. Also, we denote the cross-sectional area of the diver at "Delta" position with A_0, A_1.

3.3 Flat Spinning Prevention

Based on M. Nakashima and A. Aoyama's simulation on the stability in skydiving [5], even a small interference on a skydiver will lead to rolling and spinning.

It is proved that spinning can increase the effect of G_z[1], which is deadly to the space diver. Pattarini et al. provided that spinning with 50 rpm is approximately equal to $-0.9G_z$ and 100 rpm approximately $-3.5G_z$ [2].

Because an important factor in skydiving (not only space diving!) is to avoid flat spinning, an expert in skydiving must know how to adjust body position to correct flat spinning generated by small interference. Besides, there is some other equipment like drogue parachute to avoid deadly flat spin. Therefore, we will not consider spinning in our physical model.

3.4 Terminal Velocity

The space diving process can be divided into mainly two parts: free fall and parachute time. At a certain height from the ground, the diver shoots the parachute.

Because the main focus of this essay is to explore the maximum altitude a safe space dive can achieve, we will not discuss the parachute time in the essay. This parachute time includes position adjustment and landing point navigation.

However, a safe parachute opening should depend on the velocity when the diver shoots the parachute. If the velocity is too fast, the diver cannot land safely to the ground.

3.5 Exception of other dangers

The equipment is fine made and protective that it could withstand high pressure and resist high energy waves.

We assume the weather condition during the dive is ideally good.

3.6 Premises of Supersonic fall

We want to build a physical model of the supersonic fall during a space dive. We first assume that the space diver has no initial velocity with respect to the ground.

3.6.1 Speed of sound

- non-dispersive medium
- idea gas
- adiabatic process
- no ultrasonic effects like a(1 Hz and 5 MHz)

Formula:

$$C_s = (\frac{\gamma \cdot R^* \cdot T_M}{M_0})^{1/2}$$

3.6.2 Neglect Coriolis force

After some calculaion we found that Coriolis force can be neglected.

$$\vec{F}_{cor} = -2m(\vec{\omega} \times \vec{v})$$

Here we assume it falls in plane of equator and neglect the parallel velocity. The acceleration due to coriolis force is

$$a_{cor} = 2\omega v \approx 0.15m/s^2 \ll g$$

Thus it can be neglected.
Discussion:
the shock wave is detached from the human body, thus, the its impact to the space diver can reasonably be neglected.

4 Modelling

4.1 Earth's Atmosphere

4.1.1 Atmospheric temperature

Because we are exploring a "space dive," the atmospheric environment is quite different from that of sea level. The Earth's atmosphere varies in height. Generally, air pressure and air density decrease with height, but temperature has a more complicated pattern with regard to height.

Knowing from NASA's definition in 2015 [4], the earth's atmosphere has four primary layers:

- Thermosphere 53-375 miles (84.8-600 km)
- Mesosphere 31-53 miles (49.6-84.8 km)

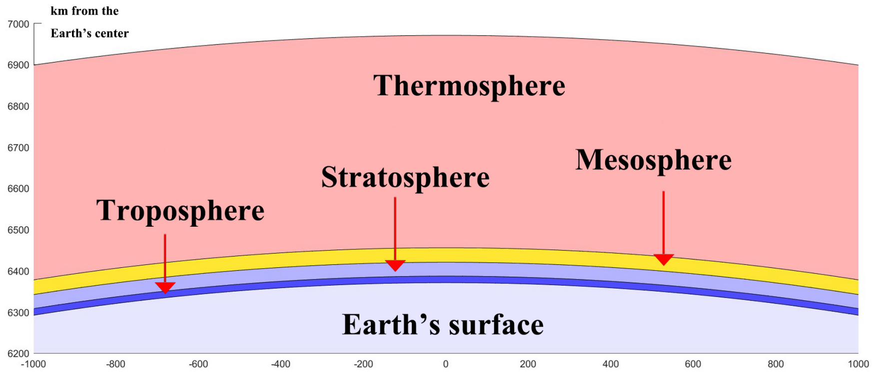


Figure 2: Atmosphere of Earth (layer illustration).

- Stratosphere 10-31 miles (16-49.6 km)
- Troposphere 0-10 miles (0-16 km)

Because the temperature of the earth has a complex relationship with height above the Earth surface. There is neither usable experimental data of the atmospheric temperature at different height nor simple formula to calculate their relationship. Therefore, we use *NRLMSIS (2.0) Atmosphere Model* to simulate the atmospheric temperature at the height from 0-150km above the surface, with an interval of 1 km. The result we access from simulation is shown in Figure 3.
From the temperature data, we know that if the space diver enters thermosphere (above 84.8 km). The atmospheric temperature will raise sharply. At a height of 121 km above the Earth's surface, the temperature will raise to $402.2K$ ($129.05°C$). The temperature continues to increase above. This temperature exceed the temperature tolerance of the space suit. Therefore, the maximum altitude of a space dive should not exceed 120 km.

4.1.2 atmospheric density

In the calculation of atmospheric density, we assume that the air at high altitude is ideal gas.
If we assume the cumulative weight of atmosphere should equal to the pressure at the distance r to the center of the earth

$$\int_{\Omega} \rho(r) \frac{GM}{r^2} d\tau = P(r)A(r)$$

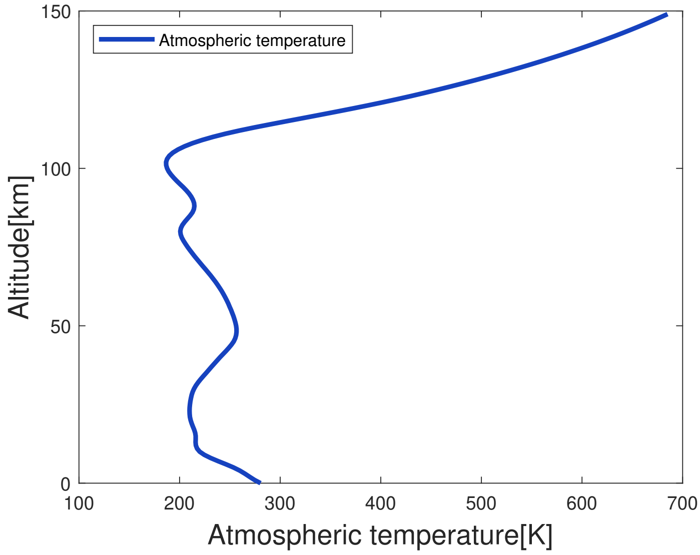


Figure 3: The relationship between atmospheric temperature and altitude. The data is acquired on NRLMSIS (2.0)

taking derivative of both sides:

$$\rho(r) = -\frac{2rP(r) + r^2P'(r)}{GM}$$

Assume the temperature is constant.Let $P(r) = k\rho(r)$ with initial value $P(R) = P_0$, substituting $r = R + s$

The density of the atmosphere of Earth at a certain height s from the Earth's surface is calculated by:

$$\rho(s) = \frac{\rho_0 R_e^2 e^{-\frac{GM\rho_0 s}{P_0 R(R_e+s)}}}{(R_e + s)^2} \tag{1}$$

4.2 Terminal Safety Verification

Newton's second law

$$F = mg - F_D$$
$$F_D = \frac{1}{2}\rho C_D A v^2$$

The total decending height is relatively little, therefore we can assume a quadratic relationship of F_D and v^2

$$F_D = kv^2$$

where $k = \frac{1}{2}\rho C_D A \approx 0.5 \times 1.23 \times 1 \times 0.2 \approx 0.5$

$$\frac{dv}{dt} = g - \frac{kv^2}{m}$$

$$\frac{dv}{dx}v = g - \frac{kv^2}{m}$$

Let v_t and v_0 denote terminal speed and the initial speed entering troposphereThe height of deceleration is

$$h = \int_{v_0}^{v_t} \frac{vdv}{g - \frac{kv^2}{m}}$$

$$h = -\frac{m}{2k} ln(\frac{g - \frac{kv_t^2}{m}}{g - \frac{kv_0^2}{m}})$$

Assume the terminal speed is $v_t = 100m/s$

$$h \approx 190ln(\frac{0.5v_0^2 - 1900}{3100})$$

It is hard to maintain a speed larger than sound, so here we simply assume $v_0 = 400m/s$

$$h \approx 613m$$

Therefore, the distance is so little that there should be enough space for the skydiver to decelerate. And we have also shown that the former assumption is valid.

4.3 Supersonic Fall

In Felix's case, he experienced a short supersonic wave. From our point of view, the reason is that when he reaches the supersonic stage, the resulting force brought by the sharp increase in C_D is large enough to stop him in seconds. In our hypothesis, however, reaching a certain height, the diver is able to accelerate on supersonic condition that could result in a hazardous speed. Before diving into more details, we first review some basic assumptions.

4.3.1 Basic assumptions

- non-dispersive medium
- idea gas
- adiabatic process
- no ultrasonic effects like a(1 Hz and 5 MHz)
- Neglect coriolis force. All the v in following discussion is pointing directly toward center of the earth
- The composition of atmophere doesn't change much.

Formula for speed of sound:

$$C_s = (\frac{\gamma \cdot R^* \cdot T_M}{M_0})^{1/2}$$

4.3.2 Aerodynamic drag

The general aerodynamic drag force formula is given by

$$F_D = \frac{1}{2}\rho C_D A v^2$$

Note that ρ is a function of height due to relatively large change in height, namely equ.(1).

In our model, we will assume the difference between stages will only result in the change in coefficient of air drag, specifically C_D. We thus make the assumption that in transonic stage, $C_D = 5$, while in other subsonic stages except the last one C_D remains 1. Since when space diver enters into the last subsonic stage or upper troposphere, assuming to have height 25 kilometers from the ground, he has been relatively close to the ground layer where air density maintains a relatively high value, the air drag coefficient will be affected by this issue and increase as a result, then we will assume that $C_D = 1.2$ when the distance between the diver and the ground is shorter than 25 kilometers. What's more, the value of cross section area will remain $0.18\mathrm{m}^2$ due to the necessity of maintaining stable position in all subsonic regions except the last one, while in supersonic region, it will be hard for space diver to control his body postures, leading to cast larger projected area, so we assumed $A = 0.2\mathrm{m}^2$ in supersonic region. For the last subsonic stage or distance shorter than 25 kilometers, space diver is supposed to shift his body posture to prepare for parachute-opening and landing, that he should have his belly pointing down to the ground. Under this circumstance, his sectional area below height of 25 kilometers will be $0.8\mathrm{m}^2$ These assumptions correspond to the result shown in Felix's 39km jump [7].

we can establish a model of dynamics in transonic-supersonic phase.

4.3.3 Base model

Newton's second law gives

$$ma = G - F_D$$

Let h be the initial height

$$a = \frac{GM_e}{(R_e + h - x)^2} - \frac{1}{2m} C_D A \rho(h-x) v^2$$

we then obtain the differential equation

$$\frac{d^2x}{dt^2} = \frac{GM_e}{(R_e + h - x)^2} - \frac{1}{2m} C_D A \rho(h-x) (\frac{dx}{dt})^2$$

by chain rule and substitution we have

$$\frac{d(\frac{v^2}{2})}{dx} = \frac{GM_e}{(R_e + h - x)^2} - \frac{1}{2m} C_D A \rho(h-x) v^2$$

Plugging in ρ

The numeric form is

$$\frac{d(\frac{v^2}{2})}{dx} = \frac{3.979 \times 10^{14}}{(6.371 \times 10^6 + h - x)^2} - \frac{1}{190} \times C_D \times 0.18 \times 5.236 \times 10^{13} \times \frac{e^{\frac{-805.525 \times (h-x)}{(6.371 \times 10^6 + h - x))}}}{(6.371 \times 10^6 + h - x)^2} \times v^2$$

C_D is left undetermined due to its variation across different phases.

4.3.4 different stages

In the first stage of falling, the diver is assumed to be in "Delta" position (Figure 1). The cross section of the diver is then set as 0.18 [7].

According to the formula for speed of sound, the value varies according to the height. We can treat it as another function $v_s(x)$

After calculating speed in each time interval, a comparison will be made between the velocity of the diver and the speed of sound in corresponding height. In this model, we set the entering and leaving speed in transonic region to be 0.8 mach (Figure 4), consistent with the data from the 39km jump [7].

If the velocity is indeed greater than the speed of the sound at that stage, the program will enter "transonic phase" in motion. In such a phase, the value of C_D will abruptly be adjusted to 5. Apart from this, the motion obey the base model.

Solving the ODE numerically, we then have the relationship between velocity and falling distance. One result is plotted using Matlab (Figure 5).

Now we could have a function of acceleration a over falling distance x and initial height h.

$$a(x,h) = \frac{GM_e}{(R_e + h - x)^2} - \frac{1}{2m} C_D A \rho (h - x) v(x)^2$$

By and we can have the maxima of a during the fall as $a_m(h)$. then we could get the limit height s.t. $a_m(h_m) = 5g$.

For a more intuitive displacement, the relationship between maximum acceleration in diving process and the initial height is graphed in Figure 6.

5 Model Verification

5.1 Atmosphere Density

We plot the using two sets of data, the one we obtain from the NRLMSIS model and the other calculated with the formula derived. The result is shown below. We can then conclude that our density model is appropriate.

5.2 Supersonic Fall

Based on the falling model we formed in previous section, we obtain different sets of statistics of velocity and acceleration of the skydiver with corresponding free-fall displacement, when entering different values of initial height. To verify the correctness of the model and the generated graphs, we are supposed to test and compare the model with real circumstances, and Felix Baumgartner's space diving can serve as a perfect contrast.

According to videos, provided statistics of research article ([7]) and graphs plotted in previous research, we collect the following information about Felix Baumgartner's space diving. Initially, he had his initial height before diving as roughly 39 kilometers away from the ground, without velocity parallel to the ground relative to the earth's motion in general as assumption. Then after his jumping, he went through three processes: first subsonic stage with his head pointing down toward the earth surface, transonic-supersonic stage where Baumgartner's velocity was roughly between 0.8 Mach and 1.25

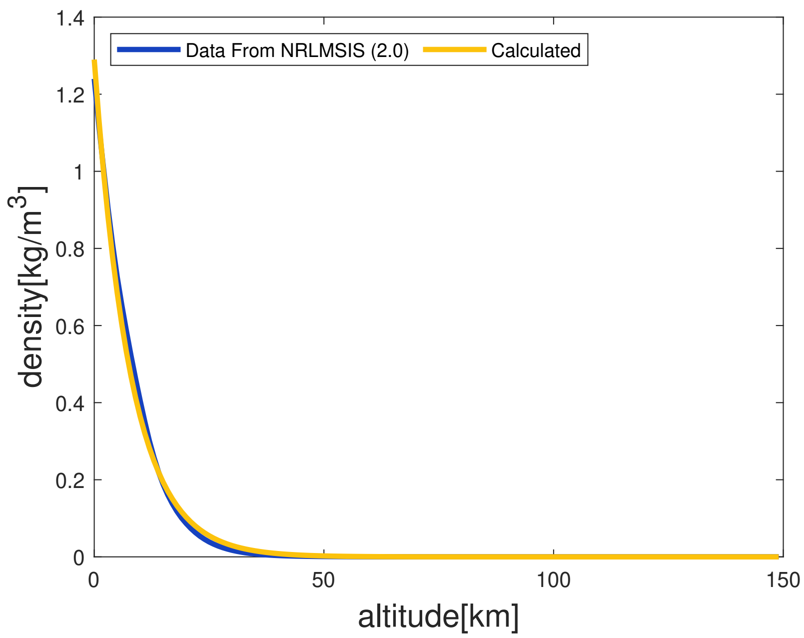
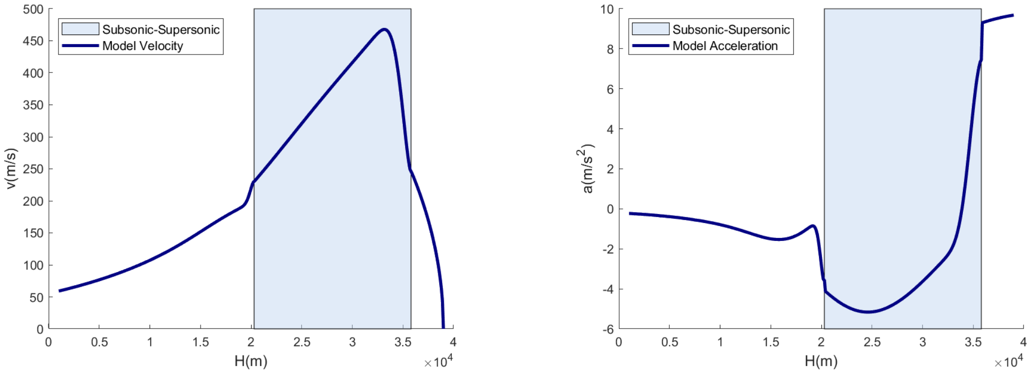


Figure 4: The relationship between density of air and altitude. The graph contains two curves. The blue one is generated with data from empirical model while the yellow one is from formula derived

Mach, and the second subsonic stage with his belly down ([7]). According to video and Colino's report, the first circumstance maintained from 0s to 25s approximately counting from his jumping, while the supersonic stage occurred between 25s and 75s, and he didn't reach to velocity close to speed of sound again after this period. The information of Baumgartner's acceleration during this process was also fitting by previous research, with maximum absolute value of his acceleration being about $10\mathrm{m/s}^2$, 1G when he entered the supersonic stage for the first time.



(a) Model velocity (v-x plot)

(b) Model acceleration (a-x plot)

Figure 5: Supersonic model with initial height 39km

Then we start to evaluate our model and results. Inputting initial height 39000m into our program, we plot out the graph regarding the relation between velocity and free-fall displacement as follows. According to our graph, we claim that the space diver enters the supersonic region, the shadow area in the graph, when he is 35.8 kilometers above the ground. He then reaches his largest velocity of 467.7343m/s, then has velocity reduction and exits the supersonic region at the height of 20.3 kilometers. He has terminal velocity of 59.1172m/s when he is one kilometer above the ground, certainly enough for him to travel back to ground under safety velocity with parachute. When we come to the acceleration, we find that the man has maximum acceleration of $9.6841\mathrm{m/s}^2$ and minimum acceleration $-5.1594\mathrm{m/s}^2$. For the entire process, we conclude that the diver has a relatively steady acceleration in the first subsonic region which is closed to gravity acceleration, then experiences a sharp decrease in acceleration when entering the subsonic region for about 0.5 kilometers. After that, he has a slower rate of the change of acceleration, and his acceleration increases slowly after he has his minimum acceleration and before stepping into the second subsonic area. When he enters the second subsonic area, he has an abrupt increase in acceleration initially, then this physical quantity becomes steady again and converges to 0 after diver shifts his posture.

Compared to the previous research, we can state that we have achieved a good approximation to Baumgartner's real circumstance in the relation between velocity and the height. For the positions where diver enters the supersonic region, the standard error between previous researches value and ours is only $\frac{36-35.8}{36} = 0.56\%$, while the one for the position where diver exits the region is $\frac{20.3-30}{20} = 1.5\%$, both of which are acceptable. For the acceleration, though according to the graph of other research, the minimum acceleration of Baumgartner was roughly equal to the gravity acceleration ([7]), having discrepancies with our value, the general shape of the acceleration plotted by us is correct, verifying our selection and assumptions for air drag coefficients C_d and projected sectional areas A for each stage are quite reasonable.

6 Strength and Weaknesses

6.1 Strength

1. In our modeling, we did a verification utilizing the database of the experimental space dive of red bull.

2. We utilize the newest meteorologic model to analyse and verify our model.

6.2 Weakness

1. One of our weaknesses is that we didn't dive into the factors of C_D but rather simply chose two constants in different phases. This may lead to absurd discontinuity in $v - x$ or $a - x$ relationship just like Figure 8 has shown. This can be attributed to the assumed sharp increase in C_D cause the discontinuity in the acceleration.

2. Due to our way of modeling, we didn't obtain the relationship between acceleration and time. This may bring inconvenience in analysing the duration of the large acceleration in supersonic status.

7 Discussion

So far, our model about supersonic fall is based on a "best" condition. Namely, we assume no spinning will happened during the "free fall" period. But only consider the two models above is not enough, for safety reasons, we have to consider more danger threats and based our final results on the "worst" case.

7.1 Shock Wave

We know that when an object moving in the air surpasses the speed of the sound, the gas molecules' deflection from the object becomes so intensive that the distances between gas molecules are shortened. In such a sense, we consider the space diver to surpass the speed of sound. The air around him is so compressed that a shock wave is generated.

According to NASA [6], if the space diver has a supersonic velocity, shock wave is generated in front of his/her moving direction. In order to get an estimation about what could happen at supersonic velocity, we assume the shock wave to be oblique shock wave.

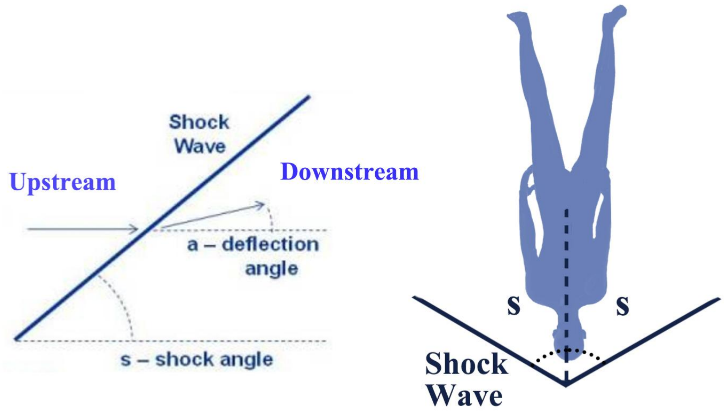


Figure 6: Oblique shock wave illustration.

We will estimate the pressure ratio between upstream area and downstream area. This pressure ratio changes significantly with Mach.

$$\frac{P_{t1}}{P_{t0}} = [\frac{(\gamma+1)M^2\sin^2 s}{(\gamma-1)M^2\sin^2 s+2}]^{\gamma/\gamma-1}[\frac{\gamma+1}{2\gamma M^2\sin^2 s-\gamma+1}]^{1/\gamma-1} \tag{2}$$

γ stands for adiabatic index of the air. s is the shock angle of the shock wave, we now assume it to be $60°$. P_{t1} and P_{t0} are the total pressure in downstream area and upstream area, respectively.

Mach	Pressure ratio P_{t1}/P_{t0}
1	1.004
2	0.842
3	0.461
4	0.220
5	0.105

Table 1: Pressure ratio between upstream and downstream at different Mach

From the table below, we can have an estimation that if the speed of the diver reaches 5 Mach, the Pressure ratio will increase to dangerous $P_{t0} = 9.52P_{t1}$!

7.2 Flat Spin Assessment

There are very few quantitative research or simulation on spinning problem in skydive, and even fewer article discuss about flat spin in the upper atmosphere. Therefore, we can only assess the flat spinning effect in a rough way.

We assume diver's body, when descending in "Delta" position is a cylinder, with a radius $R = 0.24m$ and height $1.9m$. Suppose the cylinder will rotate by its central axis (correspond to spinning), the moment of inertia I is given by:

$$I = \frac{1}{2}mR^2 = 5.472kg \cdot m^2$$

Now we can build a spinning modal based on some rough assumptions below:

- The torque that leads to a deadly spinning is cause by pressure difference between upstream and downstream areas.
- The pressure difference is calculated by Equ. (2), which means that we assume that it is still valid even under hypersonic speed.
- The pressure difference acts on an area of $S = 0.01m^2$ of human body.
- The shape of the space diver is a cylinder. The space diver has 1 second of reaction time t_r. Dangerous spinning speed for the space diver is 60 rpm, $\omega_d = 2\pi$.
- The downstream pressure is estimated by ideal gas formula, where we assume $\hat{\rho} = 1.237 \times 10^{-3}$, $\hat{T} = 252K$ are both constant.

So we can build the relationship between the maximum Mach during a space dive and the

$$I\omega_d = L = R \times (t_r F)$$

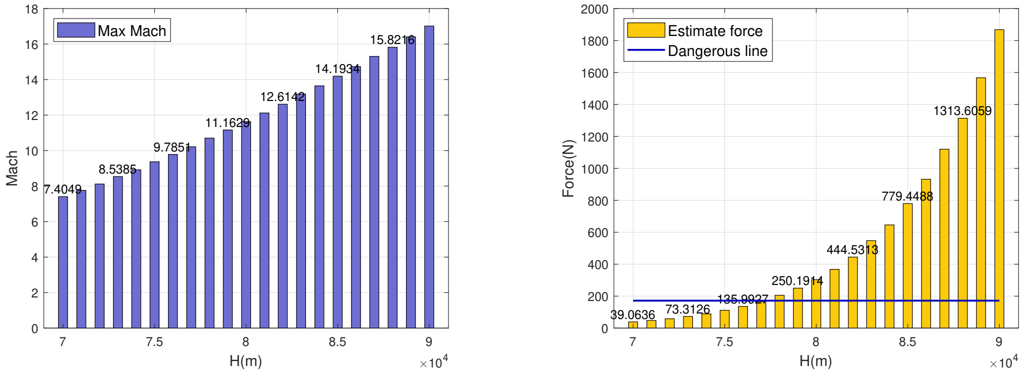
So we can calculate that dangerous Force $F_d = 171.9N$

$$F = p_{dif} S$$

The pressure difference p_{dif} can be approximated by:

$$p_{dif} = (\frac{P_{t0}}{P_{t1}} - 1)p_0 = (\frac{P_{t0}}{P_{t1}} - 1)\frac{\hat{\rho}R\hat{T}}{M}$$

From 70 km to 90km, we calculate the maximum velocity and sound speed ratio (Mach) based on Supersonic Fall model. Then using the equations above we can check whether it is possible to have enough pressure difference to generate dangerous force.



(a) Max Mach vs. initial height

(b) Estimate force caused by pressure difference at supersonic speed

Figure 7: Flat spin assessment

Under our assumptions, we can directly see that when the initial height is larger that **78 km**, the pressure difference caused by shock waves will be large enough to create dangerous force.

In conclusion, we assess the danger of shock waves causing potential deadly spin. We find out that when the initial height is larger than 78 km, the shock wave at supersonic speed may cause deadly flat spin to the space diver.

7.3 Graph of max acceleration vs. initial height and discontinuity explained

To further explain the relationship of initial height and maximum acceleration, we graph the two variables. From the graph we observe a peak at around 16km, which means that the diver experienced a relatively large acceleration starting at 16km above the ground. This seems to be a unreasonable discontinuity in the result. Exporting the velocity diagram, we find that he exceeds the speed of sound during the diving process. And the acceleration experiences an impulse around the supersonic point.

This is reasonable. When he reaches the speed of sound, although his velocity is not that large compared to that diving from 95km, he somehow experienced a relatively large drag force due to large atmospheric density. The same analysis goes for the area around the peaking point. In that sense, this region forms a danger zone that require caution of the skydivers. Falling from this height makes the diver experience much greater acceleration in the process.

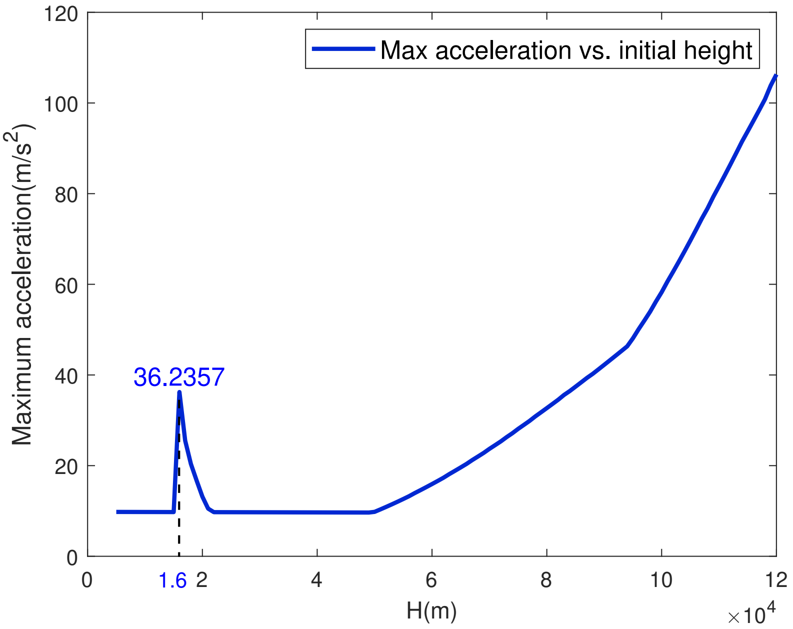
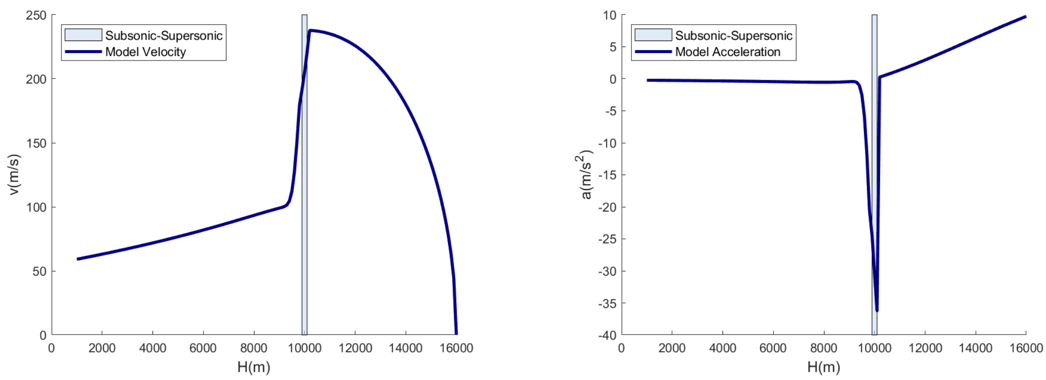


Figure 8: The relationship between density of air and altitude.



(a) Model velocity (v-x plot)

(b) Model acceleration (a-x plot)

Figure 9: Supersonic model with initial height 16km

From the graphs of acceleration of falling from 16km high (Figure 8 a), we observe an impulse at 10km above the ground. This is the phenomenon of entering supersonic wave and quickly exiting it. In addition, the discontinuity of the velocity graph may be due to a sharp increase of air density [**?**]. This analysis further supports the validity of our model.

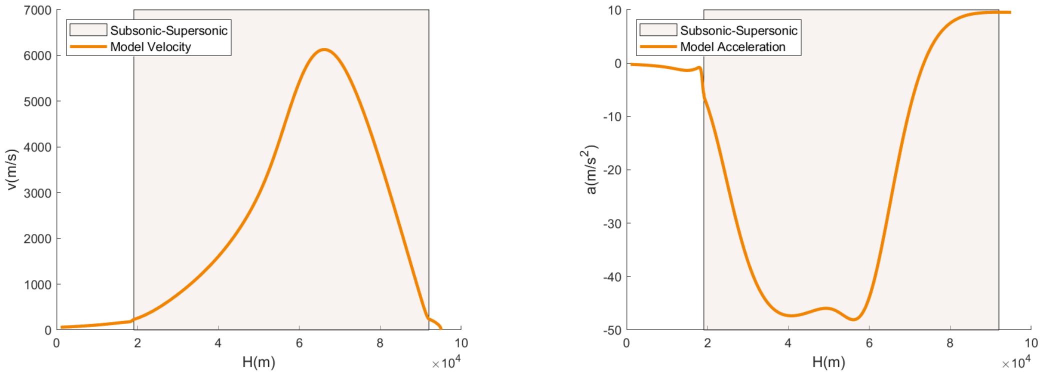
8 Results

8.1 Acceleration restriction

After the model verification for the correctness, we can apply our model regarding space-diving velocity and acceleration to the determination of the highest safety height for releasing the diver. Inputting the program with initial height increasing with an interval of one kilometer, we terminate the program processing when finding the absolute value of acceleration greater than five times of gravity acceleration, 5G, according to the statement in assumption section. Simultaneously, we let the program report the corresponding height above the ground, the velocity and the acceleration at this instant, where the height should be the highest safety height that space diver shall not pass.

Based on the statistics gained from program, we claim that the highest safety height should be 94 kilometers, while height of 95 kilometers leads to the danger of space diver. When we have initial height of 95 kilometers, the acceleration can reach to the value of $-47.6549\mathrm{m/s}^2$ at the height above the ground as 57.4 kilometers, where the acceleration is greater than five times of gravity acceleration at the same height for the first time. Furthermore, by monitoring we find that the statement of acceleration passing 5G maintains throughout 3.1 kilometers, until 54.4 kilometers.

The following are the graphs concerning velocity and acceleration with initial releasing height of 95 kilometers respectively. We can then conclude that velocity has its maximum value as $6.13 \times 10^3\mathrm{m/s}$, and the diver has its velocity increased till this point relatively rapidly and then slows down to roughly 60m/s when it is one kilometer above the earth surface. For its acceleration, we find that it decreases rapidly to roughly smallest value, throughout which the acceleration causes space diver in danger, and maintains a steady value about $-46\mathrm{m/s}^2$ then increases rapidly till its velocity back to the range of subsonic region. Eventually, its acceleration gradually converges to 0. As a matter of fact, the maximum velocity has reached to around 10 Mach, which is actually impossible for human body to load. Detailed information related to human body structure, functions and so forth should be taken into account in real circumstances, as well as more accurate value of physical quantities including air drag coefficient, air density and air pressure. More concrete and precise value of highest safety height for releasing can be gained by applying model of flat spinning. In general, with the previous assumptions, we have the highest safety height for releasing restricted by space diver's acceleration as 94 kilometers.



(a) Model velocity (v-x plot)

(b) Model acceleration (a-x plot)

Figure 10: Supersonic model with initial height 95km

8.2 Temperature restriction

In order to explore the temperature restriction on initial height, We build a concept model of the earth's atmosphere. We then use *NRLMSIS (2.0) Atmosphere Model* to simulate the atmospheric environment and get atmospheric temperature data at different height.

By comparison and analyze, we find that restricted by the heat tolerance of a space suit the the maximum altitude of a space dive should not exceed 120 km.

8.3 Flat spin restriction

Even though flat spin effect is hard to analyze and model numerically, through a series of assumptions, we finally assess the threat of flat spin during a high jump.

We first model the shock wave during the supersonic fall. Though a vital formula, we can approximate the pressure difference in front and behind a shock wave at different Mach. Then we assume that a little interference can leads to uncontrollable flat spin. This spin can be caused by shock wave pressure difference acting on human body model. Only when Mach is large enough, the pressure difference can cause deadly spin in a short time.

Though assuming and modelling, we estimate that if a space diver falls from 78 km above sea level, pressure difference cause by supersonic effect will leads to dangerous flat spin.

9 Conclusion

In this paper, various dangers that space diver may encounter throughout the process of jumping are analyzed and discussed, including large acceleration, high temperature, shock wave and flat spin. Among them, we mainly focus on the restriction due to acceleration, and assume that five times of gravity acceleration can bring danger to space diver. With data modeling and equation deducing, we obtained equations or sets of statistics including temperature, air density and speed of sound with respect to distant to the ground. With these basic information, we form the model of space diver's velocity and acceleration, consisting of three states known as first subsonic stage, supersonic stage and second subsonic stage, affected by both the gravity and the drag force produced by air. Solving the model, we have the statistics of values and accelerations with interval of 100 meters starting from the releasing position to one kilometer above the earth surface, corresponding to different initial height. Comparing the values of acceleration with gravity acceleration at corresponding height, we conclude that 94 kilometers will be the highest safe height for releasing for space diver, with the assumption that space diver can have perfect control of his body postures throughout the entire process. Since this model focus on an ideal circumstance, where the assumption of diver's control of body postures is nearly impossible in real situation based on our discussion, we have the rough modelling and deducing to discuss the effect of shock wave and flat spin. Under shock wave affecting, 5 Mach of velocity will result in pressure ratio of 9.52 P_t which is dangerous for space diver, while flat spin discussion state that 78 km will roughly be the highest initial height for a safe dive.

Considering main factors together, the maximum altitude from which a person could successfully descend to the surface is approximately 80 km.

References

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A Supersonic Fall.m

Code of the physical model of the entire space diving process.

```
    clearvars,clc;
T1 = [280.5 274.6 269.7 265.0 259.8 253.5 246.1 238.3 230.9 224.6 ...
220.0 217.4 216.2 215.9 216.0 216.0 215.5 214.5 213.2 212.0 211.0 ...
210.3 210.0 209.9 210.0 210.3 210.7 211.3 212.0 213.0 214.4 216.2 ...
218.4 220.9 223.6 226.4 229.1 231.8 234.6 237.4 240.2 243.2 246.2 ...
249.0 251.6 253.8 255.3 256.2 256.6 256.6 256.2 255.5 254.6 253.6 ...
252.4 251.2 249.8 248.5 247.0 245.4 243.7 241.9 239.9 237.8 235.6 ...
233.2 230.8 228.2 225.6 222.9 220.3 217.7 215.1 212.7 210.3 208.0 ...
205.9 203.9 202.2 201.1 200.7 201.3 202.8 205.1 207.8 210.5 212.8 ...
214.2 214.7 214.4 213.3 211.6 209.4 206.7 203.7 200.4 197.2 194.2 ...
191.5 189.2 187.6 186.7 186.6 187.6 189.8 193.4 198.5 205.0 212.9 ...
222.4 233.3 245.6 259.4 274.4 290.4 306.7 323.2 339.6 355.8 371.7 ...
387.2 402.2 416.6 430.5 443.8 456.7 469.3 481.6 493.6 505.2 516.6 ...
527.6 538.4 548.9 559.2 569.1 578.9 588.3 597.6 606.6 615.4 623.9 ...
632.3 640.4 648.3 656.0 663.5 670.9 678.0 685.0];
T = transpose(T1);
Alt = [0];
Alt0 = 0;
for i=1:149
    Alt0 = Alt0+2;
    Alt = [Alt,Alt0];
end
%xspan = [0,1000];
xspan = [0,1,2].*100;
%V0 = 0;
U0=0;
Cd = 1;
A = 0.18;
Index = 0;
NIndex = 0;
N2Index = 0;
iexit1 = 1000000;
iexit2 = 1000000;
flag = 0;
U = [U0];
U10 = U(end);
%h = 39000;
Maxa = [];
Maa = [];
Mia = [];
MaxMach = [];
for j=70:1:90
    h = j*1000;
    Cs = [sqrt(1.4*8.314*T(h/1000))];
for i=2:(h/100-10)
```

```
 Cs0 = sqrt(1.4*8.314*T(h/1000-floor((i-1)/10))/(29*10^(-3)));
 Cs = [Cs,Cs0];
 if Index==0 && sqrt(U10)<=0.8*sqrt(1.4*8.314*T(h/1000-floor((i-1)
/10))/(29*10^(-3)))
     [x,U] = ode45(@(x,U) 2.*(3.979*10^14./(6.371*10^6 + h - x).^2
 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U),xspan,U0);
     U10 = U(end);
     a = 3.979*10^14./(6.371*10^6 + h - x).^2 - ((1/380).*Cd.*A
*5.236*10^13).*exp(-805.525.*(h - x)./(6.371*10^6 + h - x))
./((6.371*10^6 + h - x).^2).*U;
     xspan0 = xspan(end);
     xspan = [xspan,xspan0+100];
     U1 = U;
     a1 = a;
     Num = length(U1);
     Nxspan = [Num-1,Num,Num+1].*100;
 elseif sqrt(U10)>0.8*sqrt(1.4*8.314*T(h/1000-floor((i-1)/10))
/(29*10^(-3)))
     iexit1 = i;
     if Index~=2 && Index~=1
         [x,U13] = ode45(@(x,U13) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U13),Nxspan,
U10);
         U10 = U13(2);
         Index = 1;
         U = [];
         a = [];
         Cd = 5;
         A = 0.2;
         continue
     end
     if Index == 1
         U10 = U1(end);
         [x,U11] = ode45(@(x,U11) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U11),Nxspan,
U10);
         U12 = U11(2:end);
         x12 = x(2:end);
         a12 = 3.979*10^14./(6.371*10^6 + h - x12).^2 - ((1/380).*
Cd.*A*5.236*10^13).*exp(-805.525.*(h - x12)./(6.371*10^6 + h - x12
))./((6.371*10^6 + h - x12).^2).*U12;
         U1 = [U1;U12];
         a1 = [a1;a12];
         U10 = U1(end);
         Nxspan0 = Nxspan(end);
         Nxspan = [Nxspan,Nxspan0+100];
```

```
            Index=2;
        else
            [x,U11] = ode45(@(x,U11) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U11),Nxspan,
U10);
            U12 = U11(end);
            x12 = x(end);
            a12 = 3.979*10^14./(6.371*10^6 + h - x12).^2 - ((1/380).*
Cd.*A*5.236*10^13).*exp(-805.525.*(h - x12)./(6.371*10^6 + h - x12
))./((6.371*10^6 + h - x12).^2).*U12;
            U1 = [U1;U12];
            a1 = [a1;a12];
            U10 = U1(end);
            Nxspan0 = Nxspan(end);
            Nxspan = [Nxspan,Nxspan0+100];
        end
        U2 = U1;
        a2 = a1;
        NNum = length(U1);
        NNxspan = [];
        NNxspan = [NNum-1,NNum,NNum+1].*100;
    elseif Index~=0 && sqrt(U10)<=0.8*sqrt(1.4*8.314*T(h/1000-floor((
i-1)/10))/(29*10^(-3))) && h-(i*100)>25000
        iexit2 = i;
        if NIndex~=2 && NIndex~=1
            [x,U23] = ode45(@(x,U23) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U23),Nxspan,
U10);
            U10 = U23(2);
            Cd = 1;
            A = 0.18;
            NIndex = 1;
            continue
        end
        if NIndex==1
            U10 = U2(end);
            [x,U21] = ode45(@(x,U21) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U21),NNxspan,
U10);
            U22 = U21(2:end);
            x22 = x(2:end);
            a22 = 3.979*10^14./(6.371*10^6 + h - x22).^2 - ((1/380).*
Cd.*A*5.236*10^13).*exp(-805.525.*(h - x22)./(6.371*10^6 + h - x22
))./((6.371*10^6 + h - x22).^2).*U22;
            U2 = [U2;U22];
            a2 = [a2;a22];
```

```
            U10 = U2(end);
            NNxspan0 = NNxspan(end);
            NNxspan = [NNxspan,NNxspan0+100];
            NIndex=2;
        else
            [x,U21] = ode45(@(x,U21) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U21),NNxspan,
U10);
            U22 = U21(end);
            x22 = x(end);
            U2 = [U2;U22];
            a22 = 3.979*10^14./(6.371*10^6 + h - x22).^2 - ((1/380).*
Cd.*A*5.236*10^13).*exp(-805.525.*(h - x22)./(6.371*10^6 + h - x22
))./((6.371*10^6 + h - x22).^2).*U22;
            a2 = [a2;a22];
            U10 = U2(end);
            NNxspan0 = NNxspan(end);
            NNxspan = [NNxspan,NNxspan0+100];
        end
        U1 = U2;
        a1 = a2;
        Num = length(U1);
        Nxspan = [];
        Nxspan = [Num-1,Num,Num+1].*100;
    elseif Index~=0 && sqrt(U10)<=0.8*sqrt(1.4*8.314*T(h/1000-floor((
i-1)/10))/(29*10^(-3))) && h-(i*100)<=25000
        if N2Index~=2 && N2Index~=1
            if iexit1<iexit2
                N3xspan = NNxspan;
            else
                N3xspan = Nxspan;
            end
            [x,U33] = ode45(@(x,U33) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U33),Nxspan,
U10);
            U10 = U33(2);
            Cd = 1.2;
            A = 0.8;
            N2Index = 1;
            continue
        end
        if N2Index==1
            U10 = U1(end);
            [x,U31] = ode45(@(x,U31) 2.*(3.979*10^14./(6.371*10^6 + h
 - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U31),N3xspan,
U10);
```

```
                U32 = U31(2:end);
                x32 = x(2:end);
                a32 = 3.979*10^14./(6.371*10^6 + h - x32).^2 - ((1/380).*
    Cd.*A*5.236*10^13).*exp(-805.525.*(h - x32)./(6.371*10^6 + h - x32
    ))./((6.371*10^6 + h - x32).^2).*U32;
                a2 = [a2;a32];
                U2 = [U2;U32];
                U10 = U2(end);
                N3xspan0 = N3xspan(end);
                N3xspan = [N3xspan,N3xspan0+100];
                N2Index=2;
            else
                [x,U31] = ode45(@(x,U31) 2.*(3.979*10^14./(6.371*10^6 + h
     - x).^2 - ((1/380).*Cd.*A*5.236*10^13).*exp(-805.525.*(h - x)
    ./(6.371*10^6 + h - x))./((6.371*10^6 + h - x).^2).*U31),N3xspan,
    U10);
                U32 = U31(end);
                x32 = x(end);
                U2 = [U2;U32];
                a32 = 3.979*10^14./(6.371*10^6 + h - x32).^2 - ((1/380).*
    Cd.*A*5.236*10^13).*exp(-805.525.*(h - x32)./(6.371*10^6 + h - x32
    ))./((6.371*10^6 + h - x32).^2).*U32;
                a2 = [a2;a32];
                U10 = U2(end);
                N3xspan0 = N3xspan(end);
                N3xspan = [N3xspan,N3xspan0+100];
            end
            U1 = U2;
            a1 = a2;
            Num = length(U1);
        end
end
Csfinal = sqrt(1.4*8.314*T(h/1000-floor((h/100-10)/10))/(29*10^(-3)))
    ;
Cs = [Cs,Csfinal];
Cs = transpose(Cs);
V = sqrt(U1);
Mach = V./Cs;
MaxMach = [MaxMach,max(Mach)];
%S = [h:-100:1000];
%plot(S,V);
%plot(S,a1);
disp(h);
aNum = length(a1);
aAbs = abs(a1);
Maxa = [Maxa,max(aAbs)];
Maa = [Maa,max(a1)];
Mia = [Mia,min(a1)];
Times = 0;
```

```
for k = 1:aNum
if a1(k)<=-5*(3.979*10^14./(6.371*10^6 + h - (k-1)*10)^2)
    Times = Times+1;
    disp(h);
    disp((k-1)*100);
    disp(h-(k-1)*100);
    disp(sqrt(U2(k)));
    disp(a1(k));
    flag = 1;
    break
elseif a1(k)>-5*(3.979*10^14./(6.371*10^6 + h - (k-1)*10)^2) && Times
    ~=0
    break
end
end
xspan = [0,1,2].*100;
a1 = [];
a2 = [];
V = [];
U1 = [];
U2 = [];
U0=0;
Cd = 1;
A = 0.18;
Index = 0;
NIndex = 0;
N2Index = 0;
iexit1 = 1000000;
iexit2 = 1000000;
U = [U0];
U10 = U(end);
end
```

B Mach.m

Code of Mach calculating and flat spinning model.

```
 clearvars, clc;
M = [7.40489543367327 7.76218304495451 8.11772458138331
   8.53847045648394 ...
    8.9201542450429 9.37072425154140 9.78510649681305
   10.2177871605247 ...
    10.7052900916285 11.1628561506380 11.6329635935257
   12.1190359235425 ...
    12.6142263294677 13.1812581750441 13.6459671421466
   14.1934478412229 ...
    14.7322269761828 15.3062212491345 15.8215793325490
   16.4094197087820 ...
    17.0156325965005]; %
Ratio = zeros(21,1); %upstream p and downstream p ratio
F = zeros(21,1);
X = 70000:1000:90000;
Y = 171.9*ones(21,1);

for i = 1:21 %-
    Ratio(i) = 1/(((2.4*M(i)*M(i)*0.75)/(0.4*M(i)*M(i)*0.75+2))^(3.5)
     * ...
        ((2.4)/(2*1.4*M(i)*M(i)*0.75 - 0.4))^(2.5));
end

for p = 1:21
    F(p) = 0.01* (Ratio(p) - 1) * 89.51488;
end

figure
B = bar(X,M,0.5);
for f = 1:3:21
    text(X(f),M(f),num2str(M(f)),...
        'Fontsize',10,'HorizontalAlignment','center',...
        'VerticalAlignment','bottom');
end

%
B.FaceColor = [109 109 210]./255;
B.LineWidth = 0.5;
xlabel( 'H(m)','Fontsize',12,'Interpreter', 'none' );
legend({'Max Mach'},'Fontsize',12,'Location', 'NorthWest', '
   Interpreter', 'none');
ylabel( 'Mach','Fontsize',12,'Interpreter', 'none' );
grid on

figure
```

```
Bb = bar(X,F,0.5);
for f = 1:3:21
    text(X(f),F(f),num2str(F(f)),...
        'Fontsize',10,'HorizontalAlignment','center',...
        'VerticalAlignment','bottom');
end

%
Bb.FaceColor = [252 202 12]./255;
Bb.LineWidth = 0.5;
hold on
plot(X,Y,'LineWidth',1.5,'Color',[0 0 0.75])
xlabel( 'H(m)','Fontsize',12,'Interpreter', 'none' );
legend({'Estimate force','Dangerous line'})
set(legend,'Fontsize',12,'Location', 'NorthWest', 'Interpreter', '
    none');
ylabel( 'Force(N)','Fontsize',12,'Interpreter', 'none' );


grid on


```