Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
A		227

Maximum Altitude for Safe Space Diving

Summary

Space diving, an exhilarating extreme sport, has gained immense popularity worldwide. This paper aims to analyze the challenges and risks involved in a skydiver's process of descenting back to Earth in a space suit with a parachute. In addition, we also try to determine the maximum altitude at which a space dive can be safely performed.

In this study, we consider the skydiver in a space suit and the parachute as a cohesive "system". By making appropriate assumptions, we begin by analyzing the four forces acting on this system. Next, we examine how atmospheric density varies with altitude to derive the motion equations governing the descent process.

Then, we thoroughly evaluate the four primary challenges and dangers that the system encounters. Through the numerical solution of multiple differential equations, we establish the limitations imposed by each factor on the maximum altitude for parachute jumping. These factors include the fast landing velocity, large acceleration, death spiral and high temperature. While the limitation on maximum initial altitude imposed by the first factor is negligible, the second, third, fourth factors restrict the maximum initial altitude to $255km$,$510km$,$153km$,respectively.

Consequently, our findings indicate that the highest attainable altitude for parachute jumping in this context is $153km$.

Keywords: skydiving, motion, acceleration, temperature

Contents

1 Introduction

1.1 Background

Since the late 1400s, when the renowned Italian inventor and artist Leonardo da Vinci sketched a design of a pyramid-shaped parachute, there has been an enduring fascination with jumping from great heights.[1] In modern times, space diving has emerged as an exhilarating pursuit, involving the descent from high altitudes in space back to the Earth's surface using a parachute. This extreme sport offers an extraordinary adrenaline rush and provides breathtaking views, attracting a growing number of enthusiasts who continually seek higher and more challenging jumps.[2] However, as the altitude increases, space diving presents a unique set of challenges and dangers. For instance, among the almost 6.2 million jumps performed by 519,620 skydivers over 10 years between 2010 and 2019, 35 deaths and 3015 injuries were reported.[3] Hence, the safety of parachuting and the maximum altitude for skydiving must be thoroughly examined and analyzed to ensure a secure and enjoyable experience.

Figure 1: skydiving
[4]

Figure 2: skydiving
[5]

1.2 Problem Restatement

Now we consider a skydiver equipped with a space suit and parachute, with a total mass of $190kg$, descending back down to the Earth's surface from perhaps above the Earth's atmosphere. In this analysis, we treat the skydiver, his space suit and parachute as a 'system'. Our task is to examine the impact of various external physical factors on this system and explore the challenges and dangers involved in the descent. By carefully evaluating these factors, we can determine the maximum altitude achievable for a safe and controlled descent.

2 Assumption and Notation

2.1 Assumptions

- Ignore potential collisions with planes and other objects.

- Space suits can provide sufficient oxygen during landing periods.
- The skydiver jumps off at the equator, which is the most suitable place for a rocket launch. Therefore we set the ground gravitational acceleration to $9.78m/s^2$.
- Assume that the parachute is opened immediately after jumping to minimize the maximum acceleration and temperature experienced by the skydiver. And the opening time of the parachute is less than $0.5s$[6], thus we neglect it. Also, because we can use mechanical device to open the parachute, there is no need to worry about the parachute not opening when the atmospheric density is low.
- Ignore the effect of sound barrier. At altitudes where the velocity of the system exceeds the speed of sound, the air density in that region remains extremely low.
- Ignore the disturbance caused by winds in the troposphere.
- Assume that the rocket launches at night to mitigate the impact of solar radiation.
- Assume that the spacesuit and parachute can withstand temperatures up to 650 degrees, providing protection to the individual from high temperatures[7].

2.2 Notations

Table 1: Notations

Symbol	Definition	Numerical value
R	Earth radius	$6378km$
m	The total mass	$190kg$
g_0	Gravitational acceleration at the equator	$9.78m/s^2$
ρ_{para}	The density of the parachute material	$1.93g/m^3$
C	Coefficient factor	1.6
k	Boltzmann constant	$1.380649 \times 10^{-23}J/K$
ω_0	Speed of earth's rotation	$7.27 \times 10^{-5}rad/s$
μ	Average mass of air molecules	$4.82 \times 10^{-26}kg$
ρ_{air}	The density of air	

3 Physical Analysis of Descent Process

3.1 Force Analysis

First, let's consider the forces a skydiver will experience.

3.1.1 Gravity

Since the initial height of the skydiver could potentially reach the top of the atmosphere ($\sim 2000\,\mathrm{km}$) , the gravitational acceleration should be taken into account as a function of height. According to the law of universal gravitation,

$$g(h) = g_0 \left(\frac{R}{R+h}\right)^2 \tag{1}$$

where $g(h)$ is the acceleration at height h.

3.1.2 Air Buoyancy

According to Archimedes principle, air buoyancy is:

$$F_{\text{float}} = \rho(h) V g(h) \tag{2}$$

Where V represents the total volume. However, the maximum air density (approximately $1.29\,\mathrm{kg/m^3}$ at ground level) is only about one thousandth of the density of the human body (approximately $1000\,\mathrm{kg/m^3}$). Therefore, we can neglect the effect of air buoyancy.

3.1.3 Coriolis Force

Since the Earth is rotating at a constant angular velocity w_0, the consideration of the Coriolis force becomes necessary when selecting a reference frame at rest relative to the Earth:

$$F_{\text{cor}} = 2m\vec{v} \times \vec{\omega} \tag{3}$$

As we assume that the skydiver will jump near the equator (where many rocket launches occur), the formula can be simplified as follows:

$$F_{\text{cor}} = 2mv_{\perp}\omega \tag{4}$$

$v_{\perp}$ refers to the vertical velocity of the system, $v_{\parallel}$ refers to the horizontal velocity of the system. Since $v_{\parallel} \ll v_{\perp}$, the Coriolis force caused by $v_{\parallel}$ can be neglected.

3.1.4 Air Resistance

The relationship between air resistance and velocity can be inferred from Maxwell's speed distribution in a single direction:

$$f(v) = \sqrt{\frac{m}{2\pi kT}} e^{-\frac{mv^2}{2kT}} \tag{5}$$

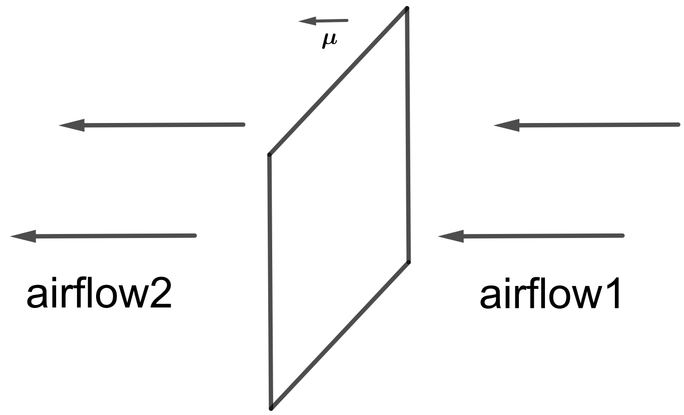

Figure 3: Schematic Diagram of Air resistance

The impetus from airflow1 is:

$$2mns\int_{\mu}^{+\infty}\sqrt{\frac{m}{2\pi kT}}e^{-\frac{mv^2}{2kT}}(v-\mu)^2\mathrm{d}v \tag{6}$$

Similarly, the resistance from airflow2 is:

$$2mns\int_{-\mu}^{+\infty}\sqrt{\frac{m}{2\pi kT}}e^{-\frac{mv^2}{2kT}}(v+\mu)^2\mathrm{d}v \tag{7}$$

where μ is the speed of the system, S is the area projected in the direction of velocity, m is the average mass of an air molecule, and n is the number density of air molecules. After approximate calculation, we determined the air resistance:

$$F_f = \frac{1}{2}nmSv^2 = \frac{1}{2}S\rho_{\text{air}}v^2 \tag{8}$$

The reality is more complicated because the parachute is not a flat surface, and the air strikes against the surface of the parachute. Therefore, we introduce a coefficient C to modify the equation, and according to relevant documents, C is approximately 1.6.
Finally, the air resistance can be considered as:

$$F_f = \frac{1}{2}CS\rho_{\text{air}}v^2 \tag{9}$$

However, it is important to note that air density ρ varies with height. To determine the forces acting on the skydiver, we first need to calculate the variation of air density with respect to height.

3.2 Variation of Air Density with Height

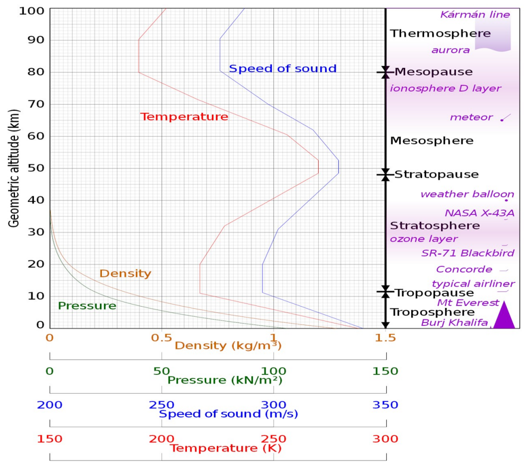

Figure 4: Diagram of Earth's Atmospheric Layering
[8]

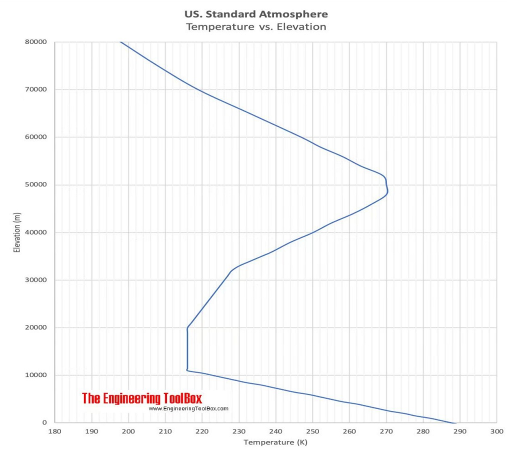

Figure 5: Illustration of the more accurate variation of atmospheric temperature with altitude
[9]

The density of air (ρ) depends on temperature (T) and varies with height (h). Based on the force balance in the atmosphere and the temperature variation with altitude, we can calculate the change in air density (ρ) with height (h) in the ranges of 0-160km:

$$\rho = \rho_0 \left(1 - \frac{k_1 h}{T_0}\right)^{\frac{\mu g}{k_1 R}-1}, \quad 0 \leq h < 10km, \tag{10}$$

where $\rho_0 = 1.225 kg/m^3, k_1 = 6.5 \times 10^{-3} K/m, T_0 = 298K,$

$$\rho = \rho_1 e^{-\frac{\mu g}{RT_1}(h-h_1)}, \quad 10km \leq h < 20km, \tag{11}$$

where $\rho_1 = 0.4334 kg/m^3, T_1 = 233K, h_1 = 10km,$

$$\rho = \rho_2 \left(1 - \frac{k_3(h-h_2)}{T_2}\right)^{\frac{\mu g}{k_3 R}-1}, \quad 20km \leq h < 50km, \tag{12}$$

where $\rho_2 = 0.1009 kg/m^3, k_3 = -1.8 \times 10^{-3} K/m, T_2 = 233K, h_2 = 20km,$

$$\rho = \rho_3 \left(1 - \frac{k_4(h-h_3)}{T_3}\right)^{\frac{\mu g}{k_4 R}-1}, \quad 50km \leq h < 160km, \tag{13}$$

where $\rho_3 = 0.0016kg/m^3, k_4 = 2.4 \times 10^{-3}K/m, T_3 = 287K, h_3 = 50km.$

derivation procedure:

The atmosphere must satisfy the force equilibrium:

$$\frac{\mathrm{d}p}{\mathrm{d}h} = -\rho g \tag{14}$$

According to the Clausius-Clapeyron equation:

$$\begin{aligned}
\rho &= \frac{p\mu}{RT} \\
T &= T_0 - kh \\
\frac{\mathrm{d}p}{p} &= \frac{\mu g}{R\left(T_0 - kh\right)}\mathrm{d}h \\
p &= p_0\left(1 - \frac{kh}{T_0}\right)^{\frac{\mu g}{kR}} \\
\rho &= \rho_0\left(1 - \frac{kh}{T_0}\right)^{\frac{\mu g}{kR}-1}
\end{aligned}$$

3.3 Dynamic Equation of the Skydiver

Now we've obtained all the forces the skydiver would experience(as shown below):

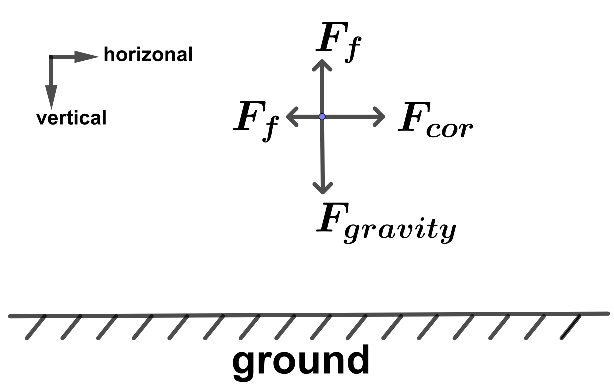

Figure 6: Total force analysis of the system

Thus, the dynamic equation of the skydiver can be obtained:

$$\begin{cases} m\frac{\mathrm{d}v_{\parallel}}{\mathrm{d}t} = 2m\omega v_{\perp} - \frac{1}{2}CS\rho v_{\parallel}^2 \\ -m\frac{\mathrm{d}^2h}{\mathrm{d}t^2} = mg_0\left(\frac{R}{R+h}\right)^2 - \frac{1}{2}CS\rho\left(\frac{\mathrm{d}h}{\mathrm{d}t}\right)^2 \\ v_{\perp} = -\frac{\mathrm{d}h}{\mathrm{d}t} \end{cases} \tag{15}$$

($v_{\parallel}$ is horizontal velocity,$v_{\perp}$ is vertical velocity)

The skydiver will experience such a process:

1. Exiting the rocket at a height much higher than $50km$
2. Accelerating at regions where the air density is extremely low while gaining horizontal velocity from the effect of Coriolis force
3. Decelerating at regions where the air density is high and the air resistance exceeds the gravity
4. Reaching the ground

4 Challenges and Dangers

Now let's consider the major challenges and dangers that the skydiver would encounter.

1. Firstly, let's address the issue of speed. If the speed upon reaching the ground (either horizontally or vertically) exceeds the human body's ultimate endurance, the skydiver would undoubtedly be in great danger.
2. Secondly, we cannot neglect the acceleration. If the acceleration is too rapid, it could potentially tear the parachute and exceed the limits of what the human body can withstand.
3. Thirdly, the potential rotation of the parachute and skydiver poses a probably and potentially catastrophic threat to the skydiver.
4. Lastly, we need to consider the potential threat of temperature due to the friction between the parachute and the air. Clearly, the initial height must be carefully chosen to ensure that none of these three dangers occur.

4.1 Final Velocity

Theoretically, the final velocity can be determined by solving the dynamic equation in (20), The dynamic equation is too complex to obtain an analytical solution, so we have written a Python program to obtain the numerical solution for the final velocity at different initial altitudes:

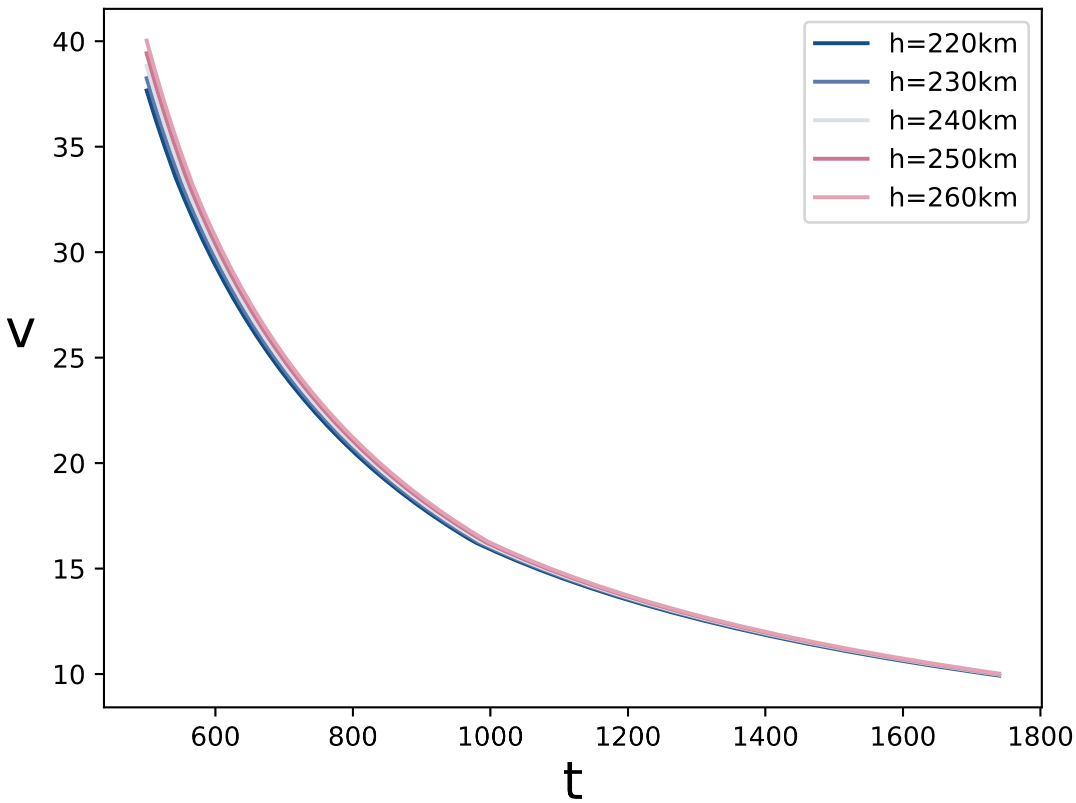

Figure 7: Final Velocity(m/s) versus Time(t) Graph at Different Initial Altitudes(km) (the skydiver lands at approximately $1800s$)

Based on our analysis, we can conclude that the final velocity is relatively low, posing no significant threat to the skydiver. Therefore, it can be neglected.

4.2 Acceleration

Based on the analysis, it has been determined that the maximum acceleration a human body can endure is approximately $10G$. However, to ensure the safety of the skydiver and prevent excessive harm, we have set the maximum acceleration to be no more than $8G$. To showcase the maximum acceleration at different initial altitudes, we have developed a Python program:

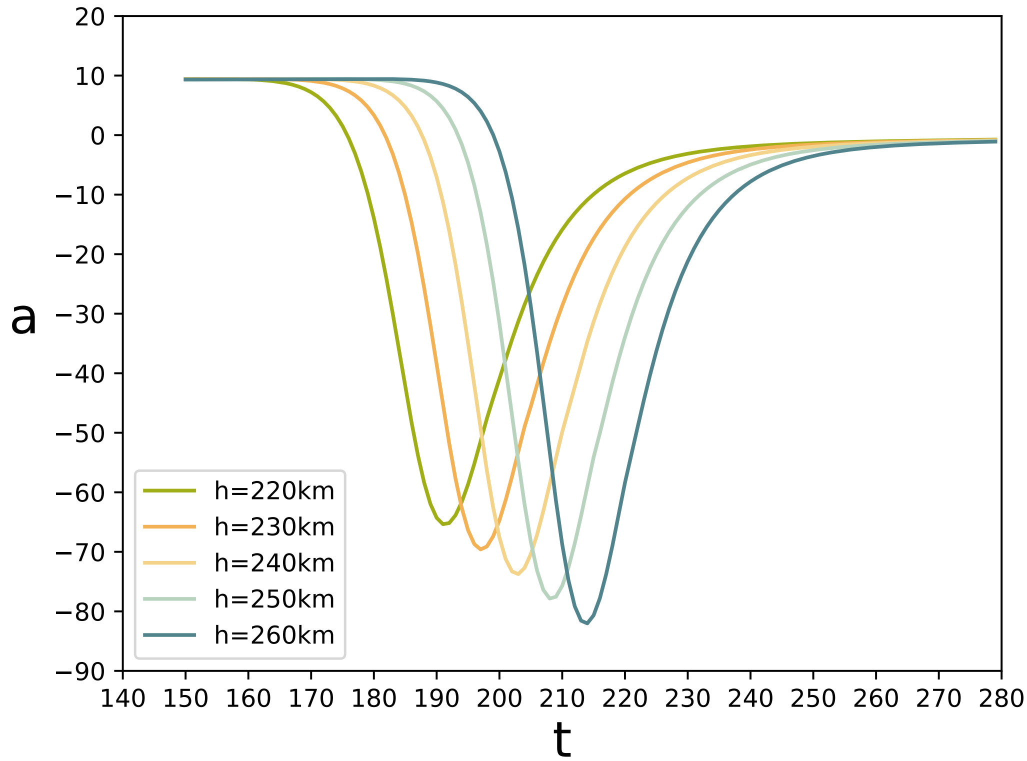

Figure 8: Acceleration(m/s^2) versus Time(s) Graph at Different initial altitude(km) (Acceleration is positive when directed downward.)

By comparing the maximum acceleration at various heights, we estimate that a maximum height of 255 kilometers is considered safe for the skydiver.

Also, it is crucial to prevent the parachute from becoming tangled. The following diagram illustrates a simplified force analysis of the parachute:

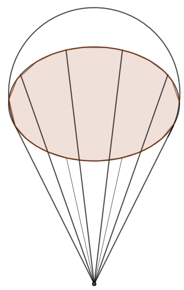

Figure 9: a Simple Parachute Model

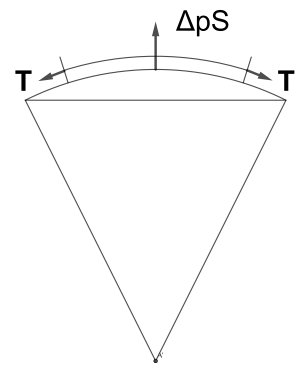

Figure 10: Force Analysis Diagram of the Parachute

According to force balance,

$$T \sin(\theta) = S\Delta p \tag{16}$$

$$p = \frac{T}{2\pi r d \sin(\theta)} = \frac{R}{2d}\Delta p \tag{17}$$

$$\Delta p = \frac{1}{2}C\rho v^2 \tag{18}$$

where
R is depicted in the picture as the radius of the parachute.
d represents the thickness of the parachute material.
T denotes the tangential tension on the parachute.
p represents the tangential pressure on the parachute.

To ensure the parachute does not fatigue, p_0 should not exceed the Young's modulus of the material. We consider the material to be $M60JB-6000$ carbon fiber, a highly durable material with a tensile strength of $3800MPa$. We further assume d to be 2mm and the radius, R, to be $5m$. Under these conditions, the pressure difference, Δp, should not exceed $1.51 \times 10^6 Pa$.

Utilizing a numerical solution implemented in Python, we obtained the relationship between Δp, and time under varying initial heights.

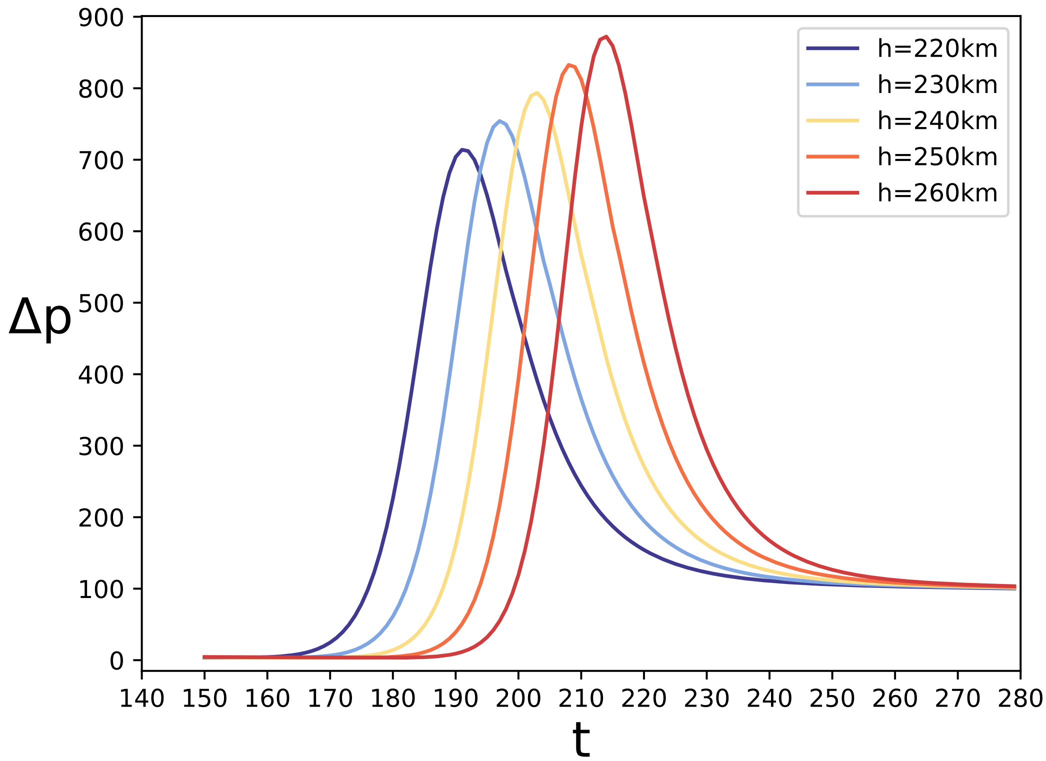

Figure 11: $\Delta p(pa)$ versus Time(t) Graph at Different Initial Altitudes(km)

Fortunately, the pressure difference $\Delta p \ll 1.5 \times 10^{-6} pa$, so the safety of the parachute is guaranteed.

4.3 Rotation

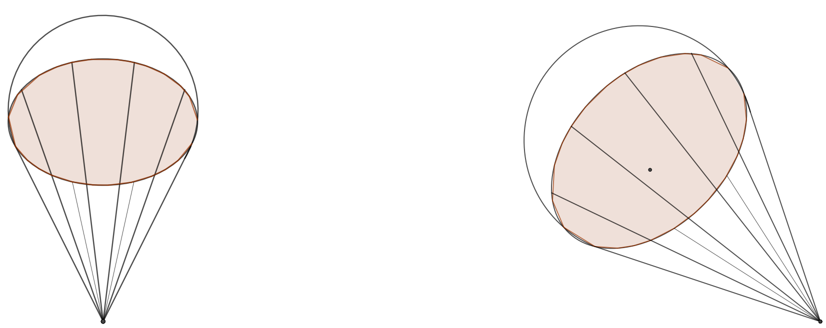

Figure : Schematic Diagram of Death Spiral

During a HAHO (High Altitude High Opening) jump, the most dangerous situation arises when the skydiver gets caught in uncontrollable spins, commonly referred to as a "parachute death spiral." Therefore, it is crucial to handle the potential rotation with utmost care to prevent tragic incidents from occurring.
To analyze this issue, we should first calculate the position of mass center of the system:

$$h = \frac{2\pi R^3 \rho d \int_0^\theta \sin(t)\cos(t)\mathrm{d}t}{M_{\text{total}}} = 1.99m \tag{19}$$

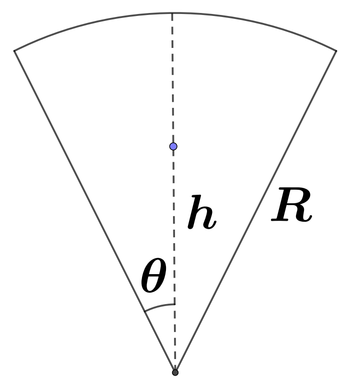

Figure 12: The Schematic Diagram of ρ, θ, h

Next, we will calculate the moment of inertia of the system about the center of mass:

$$\pi\rho d R^4 [\int_0^\theta (1 - \frac{1}{2}\sin^2(t))\sin(t)\mathrm{d}t] - M_{\text{total}} h^2 = 570 kg \cdot m^2 \tag{20}$$

where $\rho = 1.93kg/m^3$, where ρ is the density of the parachute.
The torque relative to the center of mass is entirely provided by the Coriolis force. Therefore, the torque relative to the center of mass can be calculated as:

$$M = [\int_0^\theta 4\pi\omega R^2 \rho d \sin(t)(R\cos(t) - h)v_\perp \mathrm{d}t] - 2m_{\text{people}} v_\perp \omega = 9.75 \times 10^{-5} v_\perp \tag{21}$$

Since the rotation is very slow, we neglect the resistance of the system's rotation.
According to rotation theorem:

$$\begin{aligned}\int M\mathrm{d}t &= 9.75 \times 10^{-5} \int v_\perp \mathrm{d}t \\ &= 9.75 \times 10^{-5} h \\ &= I \cdot \varphi\end{aligned}$$

where φ is the deflected angle of the system.
We assume φ cannot be more than $5°$, then h is approximately $510km > 255km$.

4.4 Temperature

In pursuit of exploring the universe, one significant challenge humans face is the intense heat generated when vehicles travel at high speeds upon exiting the atmosphere. This obstacle applies equally to skydivers. Therefore, it is crucial to prevent the high temperatures from damaging the parachute and endangering the skydiver.

To minimize the impact of temperature, it is assumed that the skydive takes place at night to avoid the effects of solar radiation.

When the parachute encounters the air at a velocity of v_0, the resulting increase in air temperature (ΔT) can be calculated:

$$\Delta T = \frac{v_\perp^2}{2C_p}[10] \tag{22}$$

where C_p is the heat capacity of air at constant temperature.

$$P = \frac{\alpha}{2} S\rho v_\perp^3 \tag{23}$$

where P is the thermal power caused by air compression. α is the ratio of the heat transferred to the parachute to the total heat generated.

An approximation can be used to calculate it:

$$\alpha = \frac{k_{\text{air}}}{k_{\text{para}} + k_{\text{air}}} \tag{24}$$

$$(k_{\text{air}} \sim 0.025W/(m \cdot k),\ k_{\text{para}} \sim 5W/(m \cdot k))$$

where k_{air} is the thermal conductivity of air, and k_{para} is approximately 5, representing the thermal conductivity of the parachute. Thus we can assume that $\alpha = 1$, meaning that almost all heat is transferred to the parachute.

Thus, the equation describing the temperature as a function of time can be formulated as follows:

$$CM_{\text{para}} \frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\alpha}{2} S\rho v_\perp^3 - \sigma S \left(2T^4 - T_{\text{ground}}^4\right) \tag{25}$$

$$(M = 81.14kg, T_{\text{ground}} = 298K, T_{\text{initial}} = 298K, t = 0, \sigma = 5.67 \times 10^{-8} W \cdot m^2 \cdot K^{-4}, S = 20m^2, C = 150J/K, g = 9.8m/s^2)$$

where C_0 represents the heat capacity of the parachute, and M_{para} denotes the mass of the parachute.

We computed the numerical solution of the equation for various initial heights:

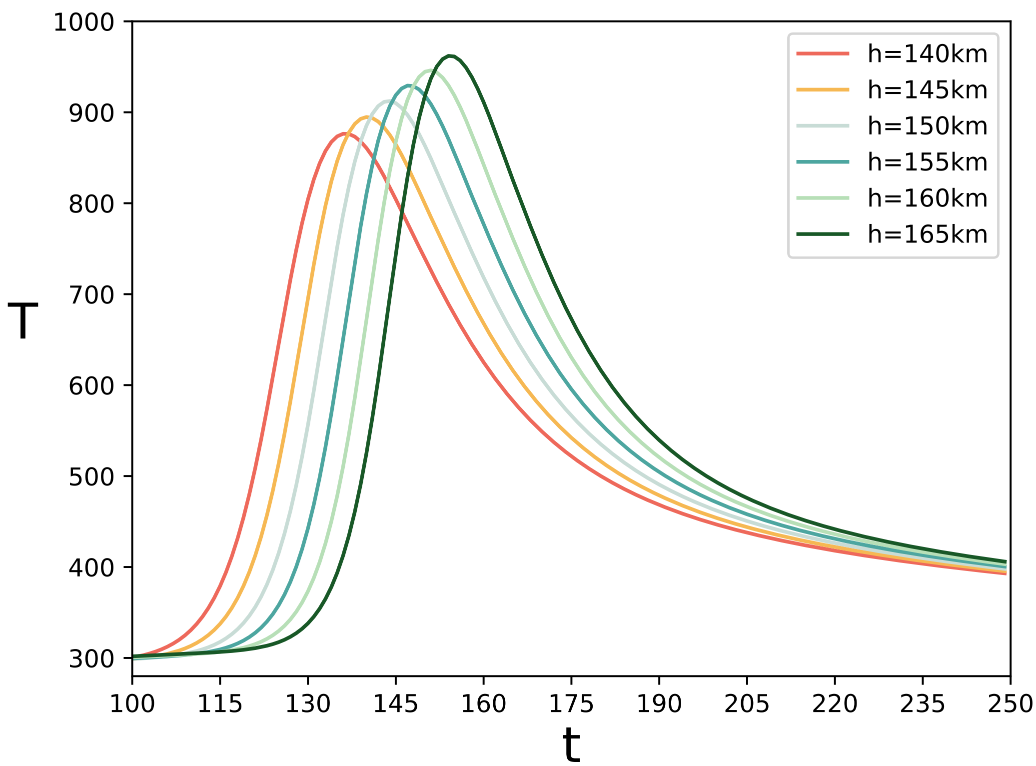

Figure 13: $T(K)$ versus Time(t) Graph at Different Initial Altitudes(km)

According to the relevant documentation, it has been determined that the melting point of carbon fiber is between $800° \sim 900°$. However, considering the material's softening and allowing for a margin of safety, we assume that the maximum temperature the parachute can withstand should not exceed $650°$.

By comparing this temperature limit with the data presented in Figure 13, we can conclude that the initial height for the space dive is restricted to $153km$.

5 Conclusion

Through the analysis and comparison of different maximum altitudes, we have determined that the maximum altitude for the skydiver's descent in our physical model is $153km$.

To determine the outcome, we begin by making several reasonable assumptions. Then, we analyze the four primary forces acting on the system by presenting their respective formulas and providing the overall dynamic equations alongside a diagram. Specifically, we thoroughly examine the variations in air density at different altitudes. Subsequently, by computing numerical solutions of relevant differential equation systems with a Python code, we find the maximum altitude under the limitations of different factors, so that the skydiver could stay safe in the face of each factor. As a result of our solutions, the challenges of landing velocity on the skydiver can be neglected, and the maximum altitude is $255km$ under acceptable acceleration, $510km$ under acceptable rotation angle, and $153km$ under acceptable temperature.

Overall, we successfully analyze the challenges and dangers in such a descent and find the maximum altitude. Based on our models and results, we believe that a skydiver can maximize his own safety by jumping from an altitude below $153km$.

6 Strengths and Weaknesses

6.1 Strengths

- **Our assumptions are reasonable and well-founded**
  We have taken into account as many realistic factors as possible and made reasonable assumptions based on the literature to simplify the model.
- **Our physical model is comprehensive and universally applicable**
  We have factored in a range of variables, from acceptable acceleration to safe temperature, to determine the final maximum altitude.
- **Our analysis of the forces is thorough and meticulous**
  We have thoroughly analyzed a variety of forces, including gravity and air resistance, and have also taken into account some less obvious forces like the Coriolis force.
- **Our solutions have a high level of accuracy**
  By listing all the relevant differential equations, we develop a Python code that computes the numerical solutions of these equations with a large number of data points.
- **Our graphics are exquisite and highly illustrative**
  During the analysis, we create visually appealing diagrams and function images, using carefully chosen colors and patterns to enhance the understanding of the results.

6.2 Weakness

- **Some factors have been neglected**
  For instance, we have disregarded the influence of wind due to its complexity. Also, We have neglected the potential electromagnetic effects caused by the ionosphere on the system. I hope that in the future, we will have more time to take these two factors into consideration and further modify the model to better align with reality.

References

[1] C. Mooney, *Skydiving*. The Rosen Publishing Group, Inc, 2015.

[2] T. H. Barrows, T. J. Mills, and S. D. Kassing, "The epidemiology of skydiving injuries: World freefall convention, 2000–2001," *The Journal of emergency medicine*, vol. 28, no. 1, pp. 63–68, 2005.

[3] C. Fer, M. Guiavarch, and P. Edouard, "Epidemiology of skydiving-related deaths and injuries: A 10-years prospective study of 6.2 million jumps between 2010 and 2019 in france," *Journal of science and medicine in sport*, vol. 24, no. 5, pp. 448–453, 2021.

[4] "Photography-man-parachute-sea-wallpaper," *warpaperflare*, 2016.

[5] "Get your head in the clouds: Skydiving is therapeutic," *skydiveorange*, 2018.

[6] Z. Q. Peng Yong Song Xumin, "Calculation method for parachute inflation time," *Aerospace Return and Remote Sensing*, vol. 25, no. 1, pp. 17–20, 2004.

[7] "What is zirconium – properties of zirconium element – symbol zr." (Aug. 10, 2018), (visited on 11/05/2023).

[8] "Comparison-us-standard-atmosphere-1962," *Wikimedia Commons*,

[9] "U.s. standard atmosphere," *Wikimedia Commons*,

[10] H. W. Sun Xiwan Guo Zhenyun, "Research of calculation methods of aerodynamic heating for hypersonic re-entry vehicles," *Aerospace Return and Remote Sensing*, vol. 36, no. 1, 2016.

Appendices

Python source code:

```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

def func(hv,t): #differential equation system of h,v,T,t
    m=190
    g=9.8
    R=6378000
    C=1.6
    s=20
    w=7.27e-5
    M=81.14
    c=150
    T0=298
    o=5.67e-8 #some constants
    if hv[0]<10000:
        rho=1.225*(1-0.0065*hv[0]/298)**4.215
```

```
    elif 10000<=hv[0]<20000:
        rho=0.4334*np.exp(-1.455e-4*(hv[0]-10000))
    elif 20000<=hv[0]<50000:
        rho=0.1009*(1+1.8e-3*(hv[0]-20000)/233)**(-19.833)
    elif 50000<=hv[0]<160000:
        rho=0.0016*(1-2.4e-3*(hv[0]-50000)/287)**(13.125)
    else:
        rho=0 #rho in different h
    dhdt1=hv[1]
    dhdt2=-g*(R/(R+hv[0]))**2+C*s*rho/(2*m)*hv[1]**2
    dv2dt=2*w*(-dhdt1)-C*s*rho/(2*m)*hv[2]**2
    dTdt=(g+dhdt2)*(-dhdt1)/(2*c)-o*s*(2*hv[3]**4-T0**4)/(c*M)
    #differential equation system
    return [dhdt1,dhdt2,dv2dt,dTdt]

def init(h0): #numerical solutions of h,v,a,T
    step=1
    t=np.arange(0,1741,step)
    hv0=[h0,0,0,298] #initial values
    sol=odeint(func,hv0,t) #solving equations
    v1=[] #vertical velocity
    v2=[] #horizontal velocity
    v3=[] #resultant velocity
    h=[] #height
    a=[0] #acceleration
    T=[] #Kelvin temperature
    for i in range(len(sol)): #store numerical solutions
        v1.append(-sol[i][1])
        v2.append(sol[i][2])
        v3.append(np.sqrt(sol[i][1]**2+sol[i][2]**2))
        h.append(sol[i][0])
        T.append(sol[i][3])
        if i==0:
            continue
        else:
            a.append((v3[len(v3)-1]-v3[len(v3)-2])/step)
    a[0]=a[1]
    return t,v3,a,T

def findmaxa():
    maxh=240000
    for i in range(240000,260001,1000):
        [t,v,a,T]=init(i)
        if max(abs(aa) for aa in a)<80 and maxh<i:
            maxh=i
    return maxh

def findmaxT():
    maxh=150000
    for i in range(150000,160001,1000):
        [t,v,a,T]=init(i)
        if max(T)<=923 and maxh<i:
            maxh=i
    return maxh
```

```
def ga(a): #calculate the pressure difference
    p=[]
    for i in range(len(a)):
        p.append(9.5*(9.8-a[i]))
    return p

ss=input("which maximum(a or T):")
if ss=='a':
    print(findmaxa()) #print the maximum of jumping height considering the maximum of a
elif ss=='T':
    print(findmaxT()) #print the maximum of jumping height considering the maximum of T

#sketch
s=input("which figure(a or v or p or T):")
if s=='a': #a-t image (at different heights)
    plt.figure()

    [t,v,a,T]=init(220000)
    l1,=plt.plot(t[150:280],a[150:280],color='#9FAE16')
    [t,v,a,T]=init(230000)
    l2,=plt.plot(t[150:280],a[150:280],color='#F1B154')
    [t,v,a,T]=init(240000)
    l3,=plt.plot(t[150:280],a[150:280],color='#F3D389')
    [t,v,a,T]=init(250000)
    l4,=plt.plot(t[150:280],a[150:280],color='#B7D3BE')
    [t,v,a,T]=init(260000)
    l5,=plt.plot(t[150:280],a[150:280],color='#51838C')
    plt.legend(handles=[l1,l2,l3,l4,l5]
               ,labels=['h=220km','h=230km','h=240km','h=250km','h=260km'],loc='best')

    xt=np.arange(140,281,10)
    yt=np.arange(-90,21,10)
    plt.xlim(140,280)
    plt.ylim(-90,20)
    plt.xticks(xt)
    plt.yticks(yt)

    plt.xlabel("t",fontsize=20,labelpad=3)
    plt.ylabel("a",fontsize=20,rotation=0,labelpad=12)
    plt.savefig('a_t.pdf',format='pdf')
    plt.show()

elif s=='v': #v-t image (at different heights)
    plt.figure()
    [t,v,a,T]=init(220000)
    l1,=plt.plot(t[500:],v[500:],color='#134F85')
    [t,v,a,T]=init(230000)
    l2,=plt.plot(t[500:],v[500:],color='#5C7AAC')
    [t,v,a,T]=init(240000)
    l3,=plt.plot(t[500:],v[500:],color='#D9E0E6')
    [t,v,a,T]=init(250000)
    l4,=plt.plot(t[500:],v[500:],color='#CC7892')
    [t,v,a,T]=init(260000)
```

```
        l5,=plt.plot(t[500:],v[500:],color='#E2A2B3')
        plt.legend(handles=[l1,l2,l3,l4,l5]
                   ,labels=['h=220km','h=230km','h=240km','h=250km','h=260km'],loc='best')

        plt.xlabel("t",fontsize=20,labelpad=3)
        plt.ylabel("v",fontsize=20,rotation=0,labelpad=12)
        plt.savefig("v_t.pdf",format="pdf")
        plt.show()

    elif s=='p': #p-t image (at different heights)
        plt.figure()

        [t,v,a,T]=init(220000)
        p=ga(a)
        l1,=plt.plot(t[150:280],p[150:280],color='#403990')
        [t,v,a,T]=init(230000)
        p=ga(a)
        l2,=plt.plot(t[150:280],p[150:280],color='#80A6E2')
        [t,v,a,T]=init(240000)
        p=ga(a)
        l3,=plt.plot(t[150:280],p[150:280],color='#FBDD85')
        [t,v,a,T]=init(250000)
        p=ga(a)
        l4,=plt.plot(t[150:280],p[150:280],color='#F46F43')
        [t,v,a,T]=init(260000)
        p=ga(a)
        l5,=plt.plot(t[150:280],p[150:280],color='#CF3D3E')
        plt.legend(handles=[l1,l2,l3,l4,l5]
                   ,labels=['h=220km','h=230km','h=240km','h=250km','h=260km'],loc='best')

        xt=np.arange(140,281,10)
        yt=np.arange(0,901,100)
        plt.xlim(140,280)
        plt.ylim(-15,901)
        plt.xticks(xt)
        plt.yticks(yt)

        plt.xlabel("t",fontsize=20,labelpad=3)
        plt.ylabel("\u0394p",fontsize=20,rotation=0,labelpad=15)
        plt.savefig('p_t.pdf',format='pdf')
        plt.show()

    elif s=='T': #T-t image (at different heights)
        plt.figure()

        [t,v,a,T]=init(140000)
        l1,=plt.plot(t[100:250],T[100:250],color='#EE695B')
        [t,v,a,T]=init(145000)
        l2,=plt.plot(t[100:250],T[100:250],color='#F6B853')
        [t,v,a,T]=init(150000)
        l3,=plt.plot(t[100:250],T[100:250],color='#C8DCD6')
        [t,v,a,T]=init(155000)
        l4,=plt.plot(t[100:250],T[100:250],color='#4DA6A0')
        [t,v,a,T]=init(160000)
```

```
l5,=plt.plot(t[100:250],T[100:250],color='#B7DFB7')
[t,v,a,T]=init(165000)
l6,=plt.plot(t[100:250],T[100:250],color='#185827')
plt.legend(handles=[l1,l2,l3,l4,l5,l6]
           ,labels=['h=140km','h=145km','h=150km'
                    ,'h=155km','h=160km','h=165km'],loc='best')

xt=np.arange(100,251,15)
yt=np.arange(300,1001,100)
plt.xlim(100,250)
plt.ylim(280,1000)
plt.xticks(xt)
plt.yticks(yt)

plt.xlabel("t",fontsize=20,labelpad=3)
plt.ylabel("T",fontsize=20,rotation=0,labelpad=12)
plt.savefig('T_t.pdf',format='pdf')
plt.show()
```