Analysis of Multiple Factors Restricting The Maximum Height of Skydiving

Team 244, Problem A

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Abstract

This paper investigates the effects of velocity, acceleration and heat on the maximum height of skydiving. We consider variable factors such as external temperature, pressure, gas density, parachute resistance and heat exchange, and use the equation of state of human motion to model the speed, acceleration and height of the skydiver as functions of time. We compare our model with former world records and find that it is accurate and feasible. We concluded that the main factor that limits the maximum height of skydiving is not velocity or acceleration, but the heat generated by the friction with the air during the descent, which can raise the temperature inside the suit to levels beyond human tolerance. Finally, based on the temperature limit that the human body can withstand, we calculated the maximum height of skydive, which is 61,270 m. The modeling fits the reality quite well and based on the real data from skydivers, the model fits quite well with the velocity and time.

Key Words: Skydive, 61270m, Velocity, Acceleration, Heat

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1 Introduction

1.1 Background

Skydiving from space is an extreme sport that involves jumping from a very high altitude, usually above the Earth's atmosphere, and falling back to the surface with the aid of a parachute. The idea of skydiving from space has fascinated many people for decades, and some have attempted to achieve this feat with varying degrees of success. The current record holder is Alan Eustace, a former Google executive who jumped from a balloon at 135,899 feet (41,419 meters) in 2014 [1] .

Skydiving from space involves many risks and challenges, such as heat, low air pressure, high speed, rapid acceleration and spinning. To survive these conditions, the jumper needs a special space suit and a parachute system that can adapt to different altitudes and speeds. The space suit should provide the jumper with oxygen, communication, and navigation, as well as protection from the cold and the low pressure. The parachute system should also be able to deploy at the right time and speed, and to resist the forces and temperatures of the descent.

1.2 Problem Restatement

The goal of this problem is to find the maximum altitude from which a person can safely skydive from space to the Earth's surface. The maximum altitude depends on several factors, such as the design of the space suit and the parachute, the weather conditions, the trajectory of the jump, and the skill and experience of the jumper. However, in our model, for clear demonstration of major factors, we only take major influence factor into consideration. It is assumed that the space suit can effectively protect the jumper from the effects of low pressure and we neglect the effect of spinning. We only focus on the effects of high speed, rapid acceleration and temperature.

2 Assumptions

Assumptions are acknowledged when they are made throughout the paper, but a list of key assumptions is also provided here.

- The maximum height will not pass the Karman line (height<=100000m)
- The speed of the human body cannot exceed 1 Mach in troposphere
- The maximum acceleration in a short period of time is not more than 15G.
- The maximum temperature that the human body can withstand is not more than 325K
- The top of troposphere is 10000m

3 Notations

Symbols	Description	Value
v	Velocity of the person	TBD
K	Drag coefficient	TBD
ρ	The density of the air	TBD
z	Height of the person	TBD
g	Gravity acceleration	$9.80665\mathrm{m/s}^2$
m	Mass of the person	190 kg
σ	Boltzmann coefficient	$1.380649 * 10^-23J/K$
m_c	Mass of the container	$27.2kg$
m_p	Mass of the PLSS	$38.1kg$
m_m	Mass of the man	$80kg$
c_c	Heat capacity of the container	1.3J/$^\circ C$
c_p	Heat capacity of the PLSS	$1.1J/^\circ C$
c_m	Heat capacity of people	$1.0J/^\circ C$
λ	heat transition coefficient	0.2
R	The specific gas constant	$-287.05287\mathrm{J} * K/\ \mathrm{kg}$
R_E	The radius of the earth	$6356766\ \mathrm{m}$
A_1	Surface area before parachute deployment	$0.4\mathrm{m}^2$
A_2	Surface area after parachute deployment	25m^2
C_d	Air drag coefficient	1.3

Here the main notations are defined while some of their specific values will be discussed and given later.

4 Model

4.1 Velocity and Acceleration in the whole process

4.1.1 Before opening the parachute

When the man leave the transport in the stratosphere, the time for the man to shift position from a head down falling to a horizontal position(this is called arch position) is neglected as it could be done within 1s. In arch position, which has the largest friction force and is commonly used in sky diving, he can reduce his acceleration and speed to the highest extent, thus have the ability to be tolerant to the extreme condition.

The force analysis of the human body shows that people are subjected to the downward gravity and the resistance of the air to the human body. Assuming that the human body is completely horizontal, so the direction of air resistance should be straight up like the following diagram.

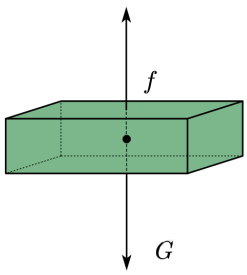

Figure 1: Force analysis of the person

Then according to Newton's second law, we get the following formula:

$$m\frac{dv}{dt} = -mg + Kv^2 \tag{1}$$

And the drag coefficient could be written as:

$$K = \frac{1}{2} * C_d * \rho * A_1 \tag{2}$$

Since we have $v = \frac{dz}{dt}$, Eq(1) could be written as:

$$\frac{mvdv}{dz} = -mg + Kv^2 \tag{3}$$

However, in order to calculate the drag coefficient K, we still need to find the density of the air ρ . According to the ideal gas law, we know that:

$$\rho = \frac{press * R}{temp} \tag{4}$$

And the exact value of pressure and temperature at certain height could be calculated by the following table and formulas:

Layer	Temperature gradient $\mathrm{K/m}$	Base temperature K	Base altitude m	Base pressure Pa
1	-0.0065	288. 15	0	101325
2	0	216.65	11000	22632.04
3	0.001	216.65	20000	5474.88
4	0.0028	228.65	32000	868.02
5	0	270.65	47000	110.91
6	-0.0028	270.65	51000	66.94
7	-0.002	214.65	71000	3.96
8	0	186. 95	84852	0.37

$$temp = T(i) + K(i) * (H_{geop} - H(i)) \tag{5}$$

$$press = P(i) * \exp \frac{-g_0 * (H_{\text{geop}} - H(i)}{T(i) * R} \tag{6}$$

in which $H_{geop} = \dfrac{(R_E * H_{in})}{R_E + H_{in}}$ and H_{in} is the height at the certain height.

Then we get the following figure about the relationship between air density and altitude

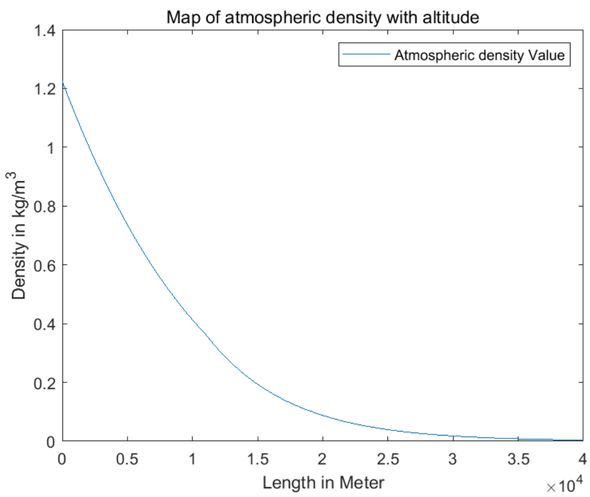

Figure 2: Map of atmosphere density with altitude

So we could solve Eq(3) by the values above and get the velocity before opening the parachute. And we know that $a = \frac{dv}{dt}$, then relationship between velocity and time, acceleration and time, altitude and time could be find.

Here I take an altitude of 50000m as an example to show the trend and value of the velocity, acceleration and altitude curves. The discussion of maximum height and feasibility analysis will be left in the Results section.

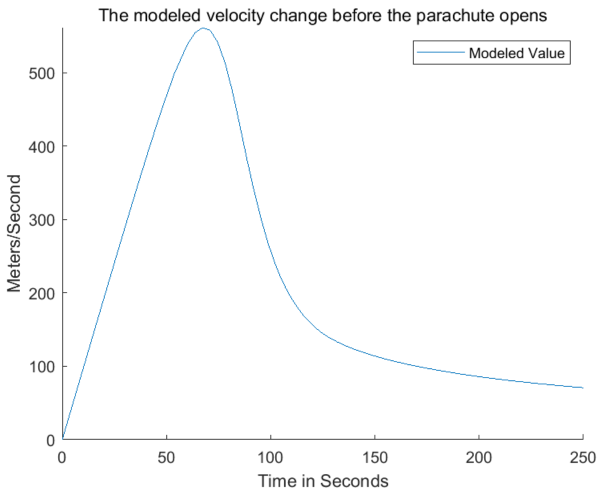

Figure 3: Modeled velocity before the parachute open

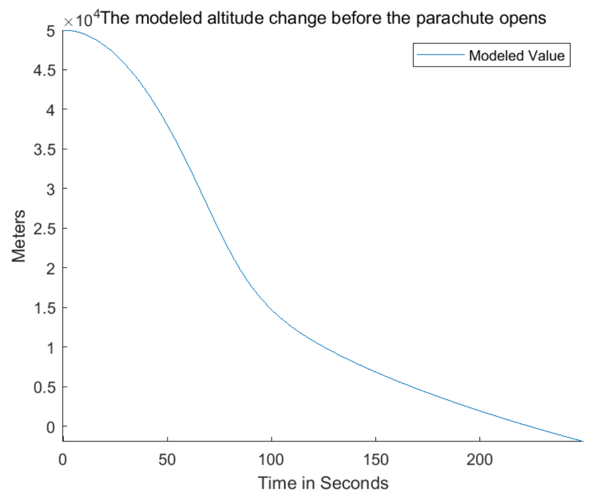

Figure 4: Modeled altitude before the parachute open

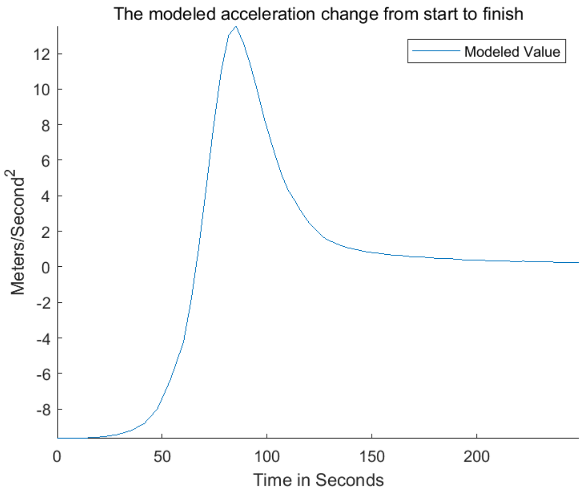

Figure 5: Modeled acceleration before the parachute open

4.1.2 After opening the parachute

The timing of parachute deployment is a matter of debate. However, to reduce the uncertainty of opening time, it is necessary to find the exact time. According to the data from Red Bull, which holds the record for jumping from 38,969 meters, the opening time is around 255s [2] . Since the difference between the maximum height in this study and this record is not too big, it is assumed that the parachute will always open at 255 s and fully deploy after about 4 s. As the parachute deploys, the cross-sectional area changes from $A_1 = 0.4m^2$ to $A_2 = 25m^2$

Applying A_2 to Eq(2) and Eq(3), we could get the following figures (Height at 50000m as an example. Detailed discussion will be showed in Results part):

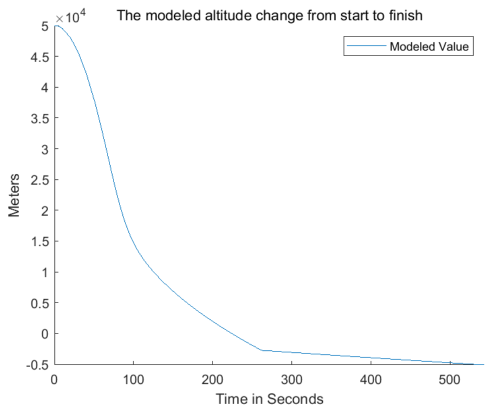

Figure 6: Modeled altitude of the whole process

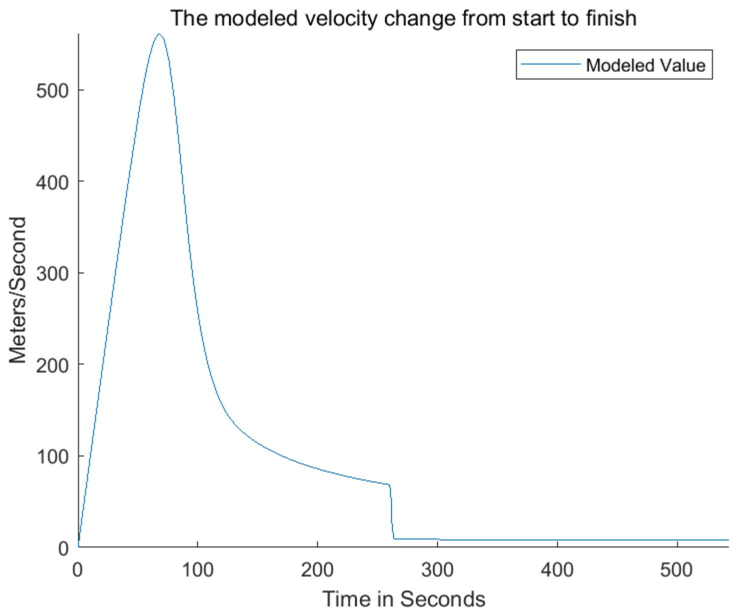

Figure 7: Modeled altitude of the whole process

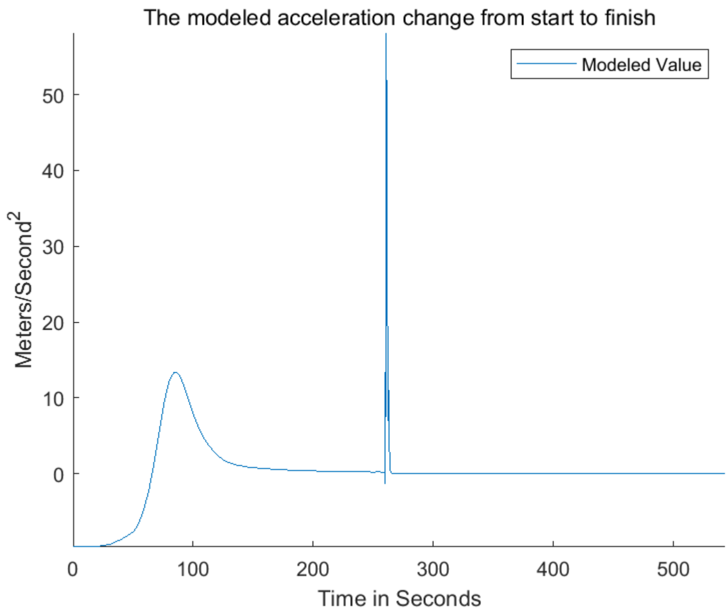

Figure 8: Modeled acceleration of the whole process

4.2 Heat transition in stratosphere

In the previous section, the effect of gravity acceleration posed on the man's body is under human's limit. Then the gravity acceleration is not a major factor to overcome for skydiver. For human himself, there is no speed or acceleration concern. However, in the whole process of falling similar to the motion of Satellite reentry module, human needs to overcome the extreme heat caused by friction without the protection of thermal-protective coating, only the space suit can provide heat protection to a limit extent. Then, the effect of friction and temperature limit should be taken into concern. According to Denys and Vadim's study [3], under the thermal analysis of a reentry vehicle, the surface temperature can reach 1200 $^{\circ}C$ [4]which is a big problem for the space suit.

In this section, the motion and temperature change in stratosphere is discussed, Mainly two kinds of effect caused by air is taken into concern: the friction force and the heat exchange between the suit and air.

The first effect can be expressed as the work down by air friction, which has similar formula as above:

$$\frac{Q_{friction}}{dt} = \frac{1}{2} K \rho v^3 A_3$$

where K is the drag coefficient, ρ is the density of air, A_1 is the area available for air friction.

The second effect can be capacity as the Stefan-Boltzmann equation:

$$\frac{Q_{exchange}}{dt} = \epsilon_{MIL} \sigma A_4 (T_1^4 - T_2^4)$$

where ϵ_{MIL} is the effective emissivity of MLI(Multi-Layer Insulation, the common material for space suit, has low heat transition constant), σ is the Stefan-Boltzmann constant, A_4 is the area available for heat transfer, and the T_1 and T_2 is the temperature of air and space suit. Due to the fact that MLI has low effective heat conductivity, the surface exposed of air has great temperature difference with other surface, meaning the valid area for air friction and heat exchange is the same, thus $A_3 = A_4 = A_1 = 0.4m^2$.

In reality, the biggest risk skydivers at high attitude(also called halo jump) are facing with is the unstable position of body, which can cause a death rotation. It makes no sense for the diver to carry a source to stablish the temperature in the suit, which can destroy the force balance when falling. So there is no external energy supply for the suit(which is the same in reality).

The process of heat exchange from the outside into the space suit can be described as a heat transition. This process is mainly affected by the property of suit material. MIL is a material formed by combination of different weak heat conductors. Based on the data provided by NASA[4], the suit can be divided into two materials: the PLSS(filler) and container. The material property of absorbing heat can be expressed as:

$$P_s = m_c * c_c + m_p * c_p$$

where P_s is the ability to absorb heat, with dimension of $J/°C$. m_c, m_p and c_c, c_p are the mass and heat capacity of the two materials respectively.Under the condition provided by NASA [5], the coefficients has values below: $m_c = 27.2kg$, $m_p = 38.1kg$,$c_c = 1.3J/°C$ and $c_p = 1.1J/°C$.

When in the whole process, the heat capacity for the man needs to be considered as well:

$$P_m = m_m * c_m$$

where P_m is the heat absorbing ability for human, which is expressed as the mass times the heat capacity. The whole process can be expressed as:

$$\lambda * (P_s + P_m) * \delta T = Q_{friction} + Q_{exchange}$$

where λ is the heat transition coefficient of space suit, which is determined by both the air condition and the material. In this model, the coefficient is chosen to be the same as NASA's one experiment [5], where $\lambda = 0.2$. Due to the fact that the partial differential equation of velocity and time is not solvable for a certain formula, in this model, we adapt polynomial approximation to determine the expression and calculate the heat.

By this approximation, the velocity is well expressed as a function of time:

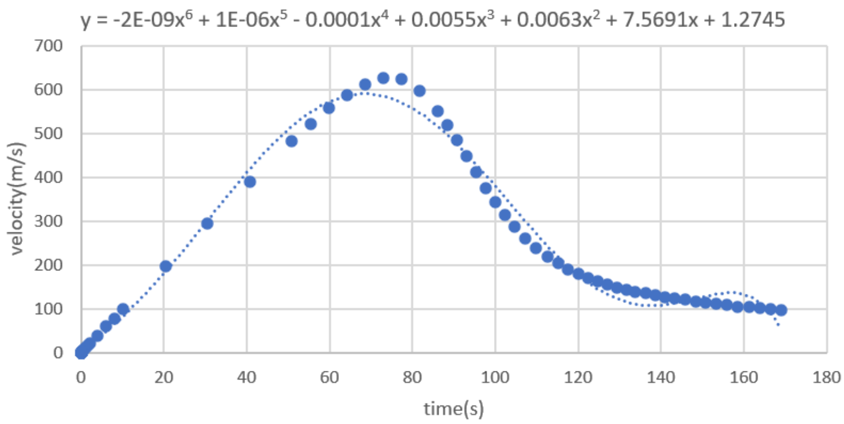

Figure 9: Approximation of velocity

$$v = -2 * 10^{-9}t^6 + 1 * 10^{-6}t^5 - 0.0001t^4 + 0.0055t^3 + 0.0063t^2 + 7.5691t + 1.2745$$

Then, the heat over the falling process can be expressed as:

$$Q_{friction} + Q_{exchange} = \int_0^{150} (\frac{1}{2}K\rho v(t)^3 A + \epsilon_{MIL}\sigma A(T_1^4 - T_2^4))dt$$

Thus the relationship among the temperature change, initial height and time can be calculated, the figure of them is as follows:

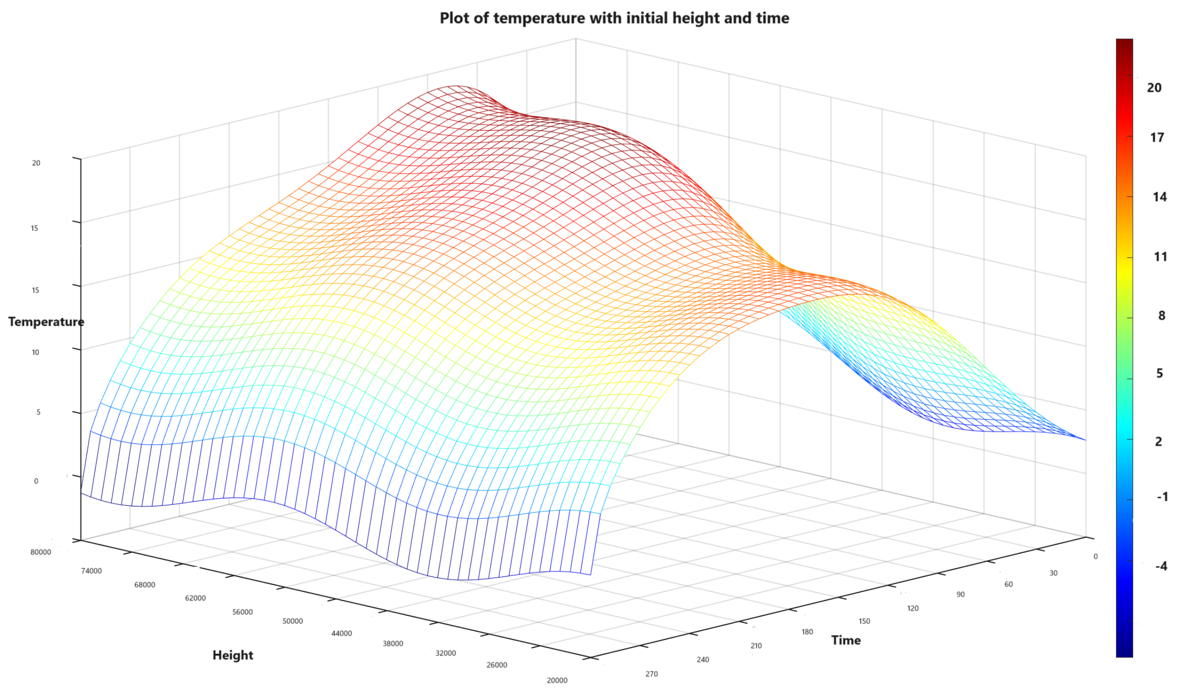

Figure 10: Plot of temperature with initial height and time

When starting from 61270m, the heat generated by air can be calculated to be $Q = 2.3425 * 10^9 J$ By plugging in all the values, the δT can be calculated:

$$\delta T = 14.98°C$$

The δT is used to demonstrate the temperature difference accumulated in the process of travelling in stratosphere. Suppose the starting temperature of the suit is $37°C$, then in the process of falling, the maximum temperature inside could reach $T = 51.98°C$, which reaches nearly the maximum temperature for a man to tolerant. And when the man enters leaves the stratosphere, the density of air increases, sharply, meaning the $Q_{exchange}$ and the velocity decreases sharply, and the temperature in the next stage is smaller than that in the stratosphere

In conclusion, in this model, we identified that the heat caused by falling is a key factor to determine the maximum height. When human falls at a height of 61270m, the maximum temperature in the space suit is nearly unbearable. When the height increases, the maximum temperature will increase sharply as the temperature is in direct proportion to the square of velocity. When the height decreases, the maximum temperature will decrease correspondingly.

5 Results Discussion

5.1 Velocity Discussion

We know that it is impossible for a person to travel faster than the speed of sound in the troposphere while wearing only a space suit (spacesuits are not metal, people get squashed). So we put in different altitudes and tried to find out if there was a troposphere moving at the speed of sound. According to our experimental data, the higher the altitude, the greater the speed of entry into the troposphere. As a result, we find that no matter how high we jump (within the Karman line), it is impossible for a person to enter the troposphere from the stratosphere at or near the speed of sound. By the time the person enter the troposphere, he is already traveling well below the speed of sound. Therefore, we can assume that speed will not be a factor in the whole process.

This is a jump very close to the Karman line (really high), and we can see that even under these extreme conditions, the speed of entry into the troposphere (around 150m/s) is still much less than the speed of sound (340m/s)

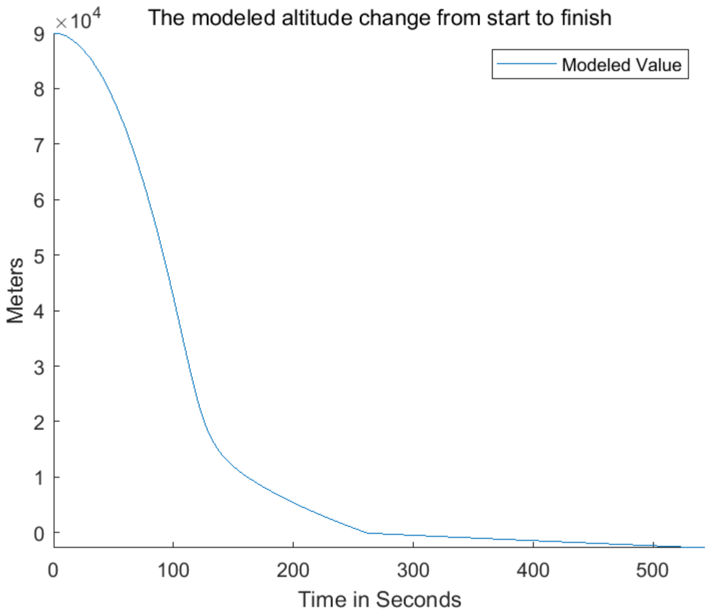

Figure 11: Relationship between altitude and time

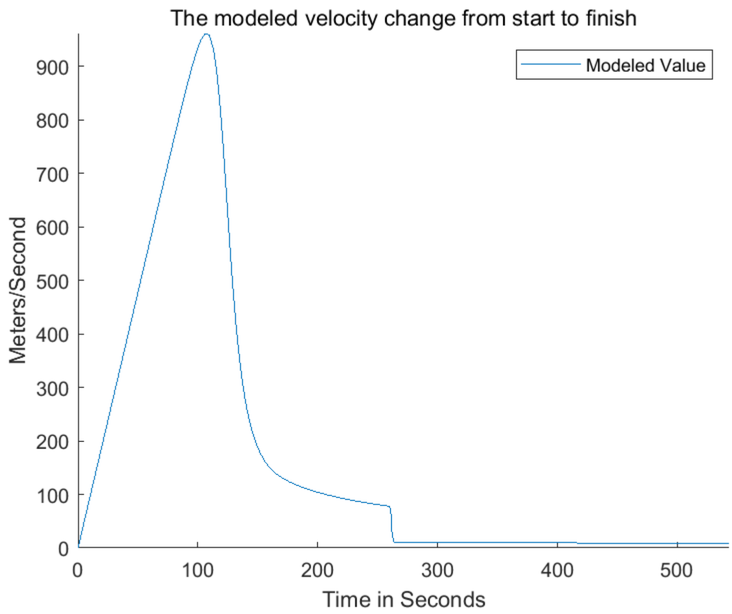

Figure 12: Relationship between velocity and time

5.2 Acceleration Discussion

By fitting the model, we found that the human body is not subjected to very large G values except at the moment of opening the parachute. By trying to input different altitudes, we found that the maximum acceleration value in the moment before opening the parachute increases gradually as the height of the jump increases. When we simulate the extreme case, the jump height is close to the Karman line, this acceleration reaches about 3g.

But such acceleration is still not a big problem for a trained athlete. We can also get from the velocity and time curve that the maximum velocity after entering the troposphere is only 150m/s even in the limit case. The speed is no different from that of a normal skydiving. Therefore, according to reference [4], we know that the acceleration experienced when the parachute is opened is about 10g. In this case, compared with ordinary skydiving, high-altitude skydiving does not bring significant challenges in terms of instantaneous acceleration, just a longer acceleration and deceleration process, but this is completely acceptable for a specially trained person

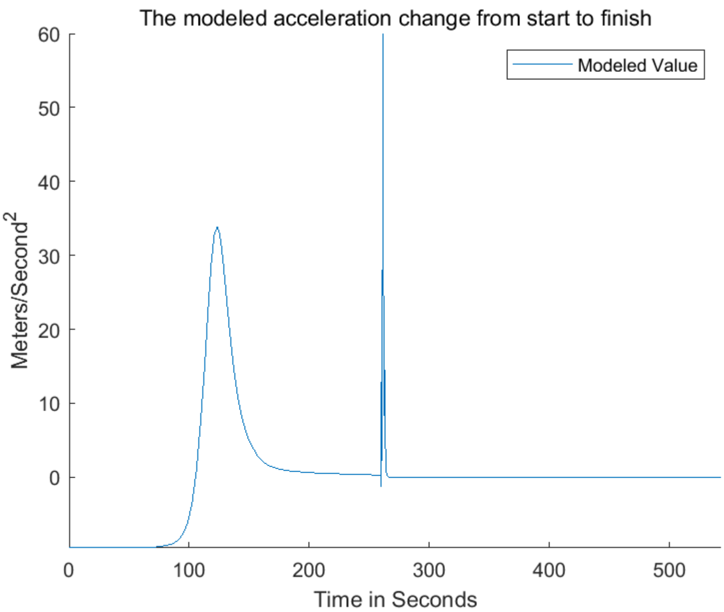

Figure 13: Relationship between Acceleration and time

5.3 Heat Discussion

Under calculation, we found that the temperature increase is the major factor for skydiving. According to our model, we determined that when the man is in the stratosphere, he reaches his maximum speed, thus is exposed to a long process of accumulating energy, which will cause the temperature of the suit to increase. Due to the limit of space suit and the need to keep balance, there is no possibility to carry extra battery to decrease the heat. Then the heat inside may be too high for a human to stay in. Under our modeling, when the height is 61270m, the maximum temperature increase is 14.98 degrees centigrade, then the temperature inside will be more than $51°C$, which is unbearable. When the starting height further increases, the maximum temperature inside will continously increase, thus the maximum height is limited under our discussion.

6 Model Evaluation

6.1 Strengths

1. High Accuracy: The precision and accuracy of our models are very high. We took real data from Red Bull during a high-altitude jump [6] and compared it to our model. (Set the same altitude 38969m)

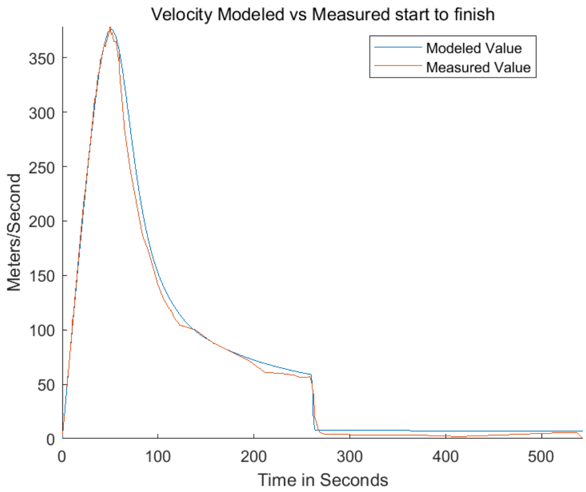

Figure 14: Modeled Velocity vs Measured Velocity

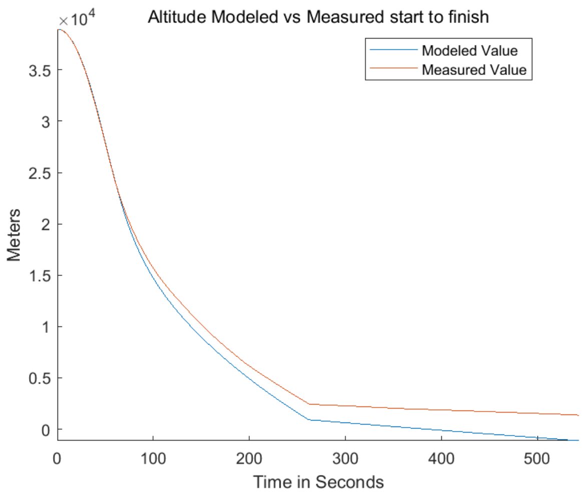

Figure 15: Modeled Altitude vs Measured Altitude

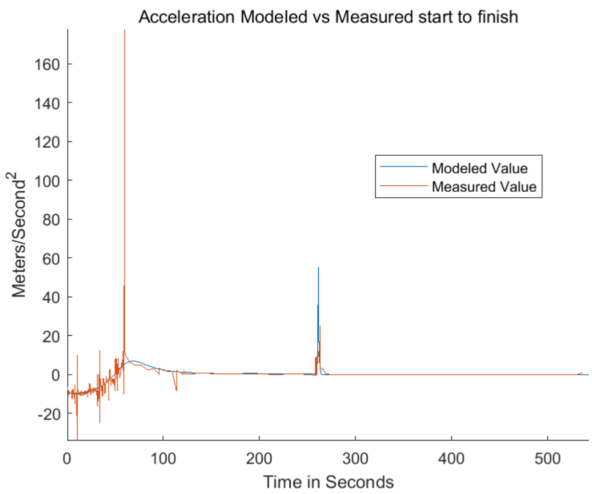

Figure 16: Modeled Acceleration vs Measured Acceleration

As you can see, our model does a pretty good job of describing velocity, acceleration and altitude versus time. At the same time, we are very close numerically, which proves that our model is very accurate and has high confidence and accuracy for other simulated heights.

2. Velocity, Acceleration and heat considered: Our model takes velocity, acceleration and heat into consideration. The process of skydiving itself is extremely complex, and it is unscientific to only consider the influence of a single factor on the final result. Therefore, we modeled the whole course of space skydiving from three perspectives. In this way, the physical model we build is more meaningful and can better simulate the actual situation.

6.2 Weaknesses

1. Polynomial approximation leads to error: When calculating heat, since the velocity curve is iterated by differential equation, there is no exact analytical solution. So we use polynomial approximation to represent the velocity curve. In this process, the actual curve and the polynomial curve can not be completely coincident, which will lead to a certain error.

2. Insufficient consideration of heat exchange: Our consideration of heat exchange is not sufficient. When we calculate that the heat generated by the friction between the suit and the air increases the temperature, we ignore the effects of heat radiation and heat conduction. Although we can assume that this process does not last long, and that the heat reduction is much less than the heat generated by

friction, this still causes certain effects that make the theoretical maximum we calculate smaller.

7 Conclusion

After precise physical modeling and accurate data calculation, we excluded the influence of speed and acceleration on the maximum altitude, and determined that the limiting factor of the maximum altitude is heat. From the model estimates, we know that even if the height of the jump is greatly changed, the speed of the human body entering the troposphere is not close to or at the speed of sound. In addition, the maximum acceleration of the human body occurs at the moment of opening the parachute, and does not exceed the limit that the human body can bear. However, as the height increases, the heat generated by the friction between the space suit and the air will increase sharply, and the temperature inside the space suit will exceed the limit that the human body can withstand, so the heat will limit the maximum height of the jump. We calculate this height by polynomial approximation and model prediction, which is 61,270 m.

References

[1] https://www.bbc.com/news/world-us-canada-29766189

[2] https://www.redbull.com/cn-zh/best-of-2012-red-bull-stratos

[3] B. Denys and R. Vadim, "Stratospheric sky-diving: parachute opening shock and impact forces analysis," Jan. 2015.

[4] Launius, Roger D., Jenkins Dennis R. (2011). Coming home: reentry and recovery from space. Washington, D.C: NASA/SP.

[5] https://ntrs.nasa.gov/api/citations/19850008599/downloads/19850008599.pdf

[6] https://www.youtube.com/watch?v=raiFrxbHxV0

A Filename.m

These are the essential matlab functions.

```
    % This is the matlab code to calculate the atmospheric correlation
     property
    function [rho,a,temp,press,kvisc,ZorH]=stdatmo(H_in,Toffset,Units
   ,GeomFlag)


if nargin == 0
    H_in = 0;
end

if nargin < 2 || isempty(Toffset)
    Toffset = 0;
end

% global u
U = false;
if nargin >= 3 && isstruct(Units)

%      u = Units;
end
if isa(H_in,'DimVar')
    U = true;
    H_in = H_in/u.m;
    Units = 'si';
end
if isa(Toffset,'DimVar')
    Toffset = Toffset/u.K;
end


% index   Lapse rate   Base Temp     Base Geopo Alt         Base
   Pressure
%   i       Ki (1/m)     Ti (K)         Hi (m)             P (Pa)
D =[1        -.0065       288.15         0                   101325
    2        0            216.65         11000
   22632.0400950078
    3        .001         216.65         20000
   5474.87742428105
    4        .0028        228.65         32000
   868.015776620216
    5        0            270.65         47000
   110.90577336731
    6        -.0028       270.65         51000
   66.9385281211797
    7        -.002        214.65         71000
```

```
    3.9563921603966
     8        0            186.94590831019 84852.0458449057
    0.373377173762337  ];

% Constants
R=287.05287;  %N-m/kg-K; value from ESDU 77022
% R=287.0531;   %N-m/kg-K; value used by MATLAB aerospace toolbox
    ATMOSISA
gamma=1.4;
g0=9.80665;     %m/sec^2
RE=6356766;     %Radius of the Earth, m
Bs = 1.458e-6;  %N-s/m2 K1/2
S = 110.4;      %K

K=D(:,2); %1/m
T=D(:,3); %1
H=D(:,4); %m
P=D(:,5); %Pa

temp=zeros(size(H_in));
press=temp;
hmax = 90000;

if nargin < 3 || isempty(Units)
    Uin = false;
    Uout = Uin;
elseif isnumeric(Units) || islogical(Units)
    Uin = Units;
    Uout = Uin;
else
    if ischar(Units) %input and output units the same
        Unitsin = Units; Unitsout = Unitsin;
    elseif iscell(Units) && length(Units) == 2
        Unitsin = Units{1}; Unitsout = Units{2};
    elseif iscell(Units) && length(Units) == 1
        Unitsin = Units{1}; Unitsout = Unitsin;
    else
        error('Incorrect Units definition. Units must be ''SI'', ''US
    '', or 2-element cell array')
    end

    if strcmpi(Unitsin,'si')
        Uin = false;
    elseif strcmpi(Unitsin,'us')
        Uin = true;
    else error('Units must be ''SI'' or ''US''')
    end

    if strcmpi(Unitsout,'si')
```

```
        Uout = false;
    elseif strcmpi(Unitsout,'us')
        Uout = true;
    else error('Units must be ''SI'' or ''US''')
    end
end

% Convert from imperial units, if necessary.
if Uin
    H_in = H_in * 0.3048;
    Toffset = Toffset * 5/9;
end



% Convert from geometric altitude to geopotental altitude, if necessary.
if nargin < 4
    GeomFlag = false;
end
if GeomFlag
    Hgeop=(RE*H_in)./(RE+H_in);
else
    Hgeop=H_in;
end

n1=(Hgeop<=H(2));
n2=(Hgeop<=H(3) & Hgeop>H(2));
n3=(Hgeop<=H(4) & Hgeop>H(3));
n4=(Hgeop<=H(5) & Hgeop>H(4));
n5=(Hgeop<=H(6) & Hgeop>H(5));
n6=(Hgeop<=H(7) & Hgeop>H(6));
n7=(Hgeop<=H(8) & Hgeop>H(7));
n8=(Hgeop<=hmax & Hgeop>H(8));
n9=(Hgeop>hmax);

% Troposphere
if any(n1(:))
    i=1;
    TonTi=1+K(i)*(Hgeop(n1)-H(i))/T(i);
    temp(n1)=TonTi*T(i);
    PonPi=TonTi.^(-g0/(K(i)*R));
    press(n1)=P(i)*PonPi;
end

% Tropopause
if any(n2(:))
    i=2;
    temp(n2)=T(i);
```

```
    PonPi=exp(-g0*(Hgeop(n2)-H(i))/(T(i)*R));
    press(n2)=P(i)*PonPi;
end

% Stratosphere 1
if any(n3(:))
    i=3;
    TonTi=1+K(i)*(Hgeop(n3)-H(i))/T(i);
    temp(n3)=TonTi*T(i);
    PonPi=TonTi.^(-g0/(K(i)*R));
    press(n3)=P(i)*PonPi;
end

% Stratosphere 2
if any(n4(:))
    i=4;
    TonTi=1+K(i)*(Hgeop(n4)-H(i))/T(i);
    temp(n4)=TonTi*T(i);
    PonPi=TonTi.^(-g0/(K(i)*R));
    press(n4)=P(i)*PonPi;
end

% Stratopause
if any(n5(:))
    i=5;
    temp(n5)=T(i);
    PonPi=exp(-g0*(Hgeop(n5)-H(i))/(T(i)*R));
    press(n5)=P(i)*PonPi;
end

% Mesosphere 1
if any(n6(:))
    i=6;
    TonTi=1+K(i)*(Hgeop(n6)-H(i))/T(i);
    temp(n6)=TonTi*T(i);
    PonPi=TonTi.^(-g0/(K(i)*R));
    press(n6)=P(i)*PonPi;
end

% Mesosphere 2
if any(n7(:))
    i=7;
    TonTi=1+K(i)*(Hgeop(n7)-H(i))/T(i);
    temp(n7)=TonTi*T(i);
    PonPi=TonTi.^(-g0/(K(i)*R));
    press(n7)=P(i)*PonPi;
end

% Mesopause
```

```
if any(n8(:))
    i=8;
    temp(n8)=T(i);
    PonPi=exp(-g0*(Hgeop(n8)-H(i))/(T(i)*R));
    press(n8)=P(i)*PonPi;
end

if any(n9(:))
    warning('One or more altitudes above upper limit.')
    temp(n9)=T(8); % Modified by Craig Scratchley, February 2017
    press(n9)=0; % Modified by Craig Scratchley, February 2017
end

temp = temp + Toffset;

rho = press./temp/R;

if nargout >= 2
    a = sqrt(gamma * R * temp);
    if nargout >= 5
        kvisc = (Bs * temp.^1.5 ./ (temp + S)) ./ rho; %m2/s
        if nargout == 6
            if GeomFlag % Geometric in, ZorH is geopotential altitude
     (H)
                ZorH = Hgeop;
            else % Geop in, find Z
                ZorH = RE*Hgeop./(RE-Hgeop);
            end
        end
    end
end

if Uout %convert to imperial units if output in imperial units
    rho = rho / 515.3788;
    if nargout >= 2
        a = a / 0.3048;
        temp = temp * 1.8;
        press = press / 47.88026;
        if nargout >= 5
            kvisc = kvisc / 0.09290304;
            if nargout == 6
                ZorH = ZorH / 0.3048;
            end
        end
    end
end

if U
    rho = rho*u.kg/(u.m^3);
```

```
    if nargout >= 2
        a = a*u.m/u.s;
        temp = temp*u.K;
        press = press*u.Pa;
        if nargout >= 5
            kvisc = kvisc*u.m^2/u.s;
            if nargout == 6
                ZorH = ZorH*u.m;
            end
        end
    end
end



% This is the function to calculate the drag coefficient
function res = drag(t, p, v, m)
% <This function calculates the drag force as the human falls>
airdata = stdatmo(p);
density = airdata(1);
if t <= 260
end
    A= 0.4;
    Cd=1.3;
a= [4,8,12,16,20,25];
CD= [0.3,0.6,0.9,1.2,1.5,1.75];
if t > 260
    A=a(1);
    Cd=CD(1);
end
if t > 260.5
    A=a(1);
    Cd=CD(1);
end
if t > 261
    A=a(2);
    Cd=CD(2);
end
if t > 261.5
    A=a(3);
    Cd=CD(3);
end
if t > 262
    A=a(3);
    Cd=CD(3);
end
if t > 262.5
    A=a(4);
    Cd=CD(4);
```

```
end
if t > 263
    A=a(5);
    Cd=CD(5);
end
if t > 263.5
    A=a(5);
    Cd=CD(5);
end
res=  0.5*Cd*stdatmo(p)*A ;
end

%This function calculate the mass of the human
function res = mass(t, v)
    % mass in kg of the human and all his equipment
    res = 190;
end

%This function calculates the acceleration of gravity at different altitudes
function a_grav = gravityEst(p)
    % estimate the acceleration due to gravity as a function of altitude, p
    A_GRAV_SEA = 9.807;  % acceleration of gravity at sea level in m/s^2
 a_grav = ((6.67*10^(-11))*(5.972*10^24))/((6371000+p)^2);
end

%This function solve the differential eqaution of the human's motion
function res = fall(t, X)
    %FALL <This function is used with ode45 to calculate altitude and acceleration>

    % do not modify this function unless required by you for some reason!

    p = X(1); % the first element is position
    v = X(2); % the second element is velocity

    dpdt = v; % velocity: the derivative of position w.r.t. time
    dvdt = acceleration(t, p, v); % acceleration: the derivative of velocity w.r.t. time

    res = [dpdt; dvdt]; % pack the results in a column vector
end


%This function calculates the acceleration of the human.
function res = acceleration(t, p, v)
```

```
    % t: time
    % p: position
    % v: velocity
    % output...
    % res: acceleration

    a_grav = gravityEst(p);
        m= mass(t, v);
        b = drag(t, p, v, m);

        f_drag = b * v^2;
        a_drag = f_drag / m;
        res = -a_grav + a_drag;
    end
end

%This function plot the relationship between two given physical
    quantities.
function res = plotComparisons(fignumber, graphtitle, x_label,
    y_label, T, M)
    figure(fignumber)
    hold on
    title(graphtitle)
    xlabel(x_label)
    ylabel(y_label)
    plot(T, M)
    %plot(modelx, modely)
    axis tight;
    legend('Modeled Value')
    hold off
end

%This is the main function of the matlab code

function main

clf
filename = 'RedBullStratosData180.xlsx';
altitudetotal = xlsread(filename,'Data','D4:D403');
airspeedtotal = (xlsread(filename,'Data','E4:E403'))/3.6;
timetotal = xlsread(filename,'Data','K4:K403');
acceltotal = diff((xlsread(filename,'Data','E4:E404'))/3.6)./diff(
    xlsread(filename,'Data','K4:K404'));

altitude1min = xlsread(filename,'Data','D4:D284');
airspeed1min = (xlsread(filename,'Data','E4:E284'))/3.6;
time1min = xlsread(filename,'Data','K4:K284');
accel1min = diff((xlsread(filename,'Data','E4:E285'))/3.6)./diff(
    xlsread(filename,'Data','K4:K285'));
```

```

altitude270sec = xlsread(filename,'Data','D4:D396');
airspeed270sec = (xlsread(filename,'Data','E4:E396'))/3.6;
time270sec = xlsread(filename,'Data','K4:K396');
accel270sec = diff((xlsread(filename,'Data','E4:E397'))/3.6)./diff(xlsread(filename,'Data','K4:K397'));
%read the real data from the excel document

[T, M]= ode45(@fall, [0, 543], [60000, 0]);
o = diff(M(:,2));
oo=[]; utt=[];
z = 1;j=1;
for (z=1:size(o))
    if (o(z)~=0)
        oo(j)=o(z)/(T(z+1)-T(z));
        utt(j)= T(z);
        j = j+1;
    end
end

plotComparisons(4,'The modeled altitude change from start to finish', 'Time in Seconds'...
    , 'Meters', T, M(:,1))
plotComparisons(5,'The modeled velocity change from start to finish', 'Time in Seconds'...
    , 'Meters/Second', T, abs(M(:,2)))

for u = 1:(size(T)-1)
    TT(u) = T(u);
end
plotComparisons(6,'The modeled acceleration change from start to finish', 'Time in Seconds'...
    , 'Meters/Second^2', utt, oo)
%Plot the modeled altitude, velocity and acceleration

[T, M]= ode45(@fall, [0, 543], [38969.4, 0]);
disp(T);
disp(M);
o = diff(M(:,2));
oo=[]; utt=[];
z = 1;j=1;
for (z=1:size(o))
    if (o(z)~=0)
        oo(j)=o(z)/(T(z+1)-T(z));
        utt(j)= T(z);
        j = j+1;
    end
end
plotComparisons(7,'Altitude Modeled vs Measured start to finish', '
```

```
    Time in Seconds'...
     , 'Meters', T, M(:,1), timetotal, altitudetotal)
plotComparisons(8,'Velocity Modeled vs Measured start to finish', '
    Time in Seconds'...
     , 'Meters/Second', T, abs(M(:,2)), timetotal, airspeedtotal)

%The following is a manual calculation of acceleration from the model
 for u = 1:(size(T)-1)
   TT(u) = T(u);
   end
 timetotalpart6accel = timetotal;
 timetotalpart6accel(end) = [];
plotComparisons(20,'Acceleration Modeled vs Measured start to finish'
    , 'Time in Seconds'...
     , 'Meters/Second^2', utt, oo, timetotalpart6accel, -acceltotal)
%Plot the figure of comparison between modeled value and real value.
end
```