Space Parachute Model Considering Rotation, Superheat, and Large Acceleration

Problem A

Team：447

Abstract：

The landing process of a skydiver can be divided into three stages: falling without a parachute, falling with the secondary parachute and falling with the main parachute. Based on these three stages, the possible dangers to skydivers are fast rotation, high temperatures, and high acceleration. To address these three questions, the atmospheric density distribution is calculated first (Figure 3). On this basis, **firstly**, we considered one of the biggest challenges: the skydiver's spin (which can occur at any altitude). According to the data and torque balance equation (equation 5), the relationship between velocity loss and angular velocity increase is calculated. It is concluded that compared with the free fall speed, the actual angular velocity of the skydiver increases by **2.2 rad/s** for every **1 m/s** lost, and a method to deal with the spin is proposed. **Secondly**, just considering the dangers posed by acceleration, the skydiver’s vertical motion equation is obtained (equation 14) and solved numerically (Figure 16). In order to avoid the harm caused by the human body acceleration exceeding 8g in the instant of opening the parachute, we adopted the way of opening the auxiliary parachute first and then the main parachute and analyzed the opening process of the main parachute in detail. The result shows that the maximum height that can be reached is **162.7km**. **Thirdly**, considering the effects of high temperatures on skydivers, the concepts of stagnation temperature and stagnation point are introduced. Its temperature was calculated from the basic adiabatic flow equation (equation 13). The surface temperature of the spacesuit can be expressed through its temperature. Combined with the maximum temperature that a spacesuit can withstand, this gives a maximum drop height of **63.78km**.

Contents

1 Introduction

1.1 Problem Background

Eleven years ago, Austrian skydiver Felix Baumgartner reached an altitude of 37 kilometers in a specially designed hot air balloon. When the weather was right and all systems were go, he jumped off the air balloon in a pressurized suit and free-fall to earth.

The Austrian skydiver reached a speed of Mach 1.2(1,110 kilometers per hour) and fell freely for four and a half minutes before opening his parachute at an altitude of 5,000 feet (1,524 meters) to float him to the ground. It is undeniable that he successfully challenged the limits of mankind.

Figure 1 Felix Baumgartner[1]

Space skydiving involves a rocket ride into space, so it can face problems such as overheating, overweight, and difficulty in determining the landing point. Not only that, skydivers may spin around as they cross the speed of sound, which is also a major source of danger. How to get this right and make space jumping possible is what scientists are currently trying to achieve.

1.2 Analysis of the problem

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problem:

What is the maximum altitude at which a skydiver can successfully descend to the ground, given that the total mass of the jumper, space suit, and parachute is 190 kilograms.

Skydiver will encounter the threats below during his descent:

- **Horizontal rotation in fall**
  Since the descending altitude has reached tens of kilometers, the skydiver will reach very fast speeds. Therefore, the huge air resistance can cause the skydiver to imbalance and produce a strong self-rotation.
- **Great deceleration when opening a parachute**
  When opening the parachute, a great acceleration may be generated due to the great speed.
- **Small horizontal and vertical velocities required for landing**
  When landing, there is a limit to how much weight a person's legs can carry.
- **Elevation of heat in spacesuits**
  Relative to the skydiver's incoming air is compressed instantaneously, the temperature at the stagnation point rises sharply to the stagnation temperature, and the heat of the suit gradually increases.

1.3 Our Work

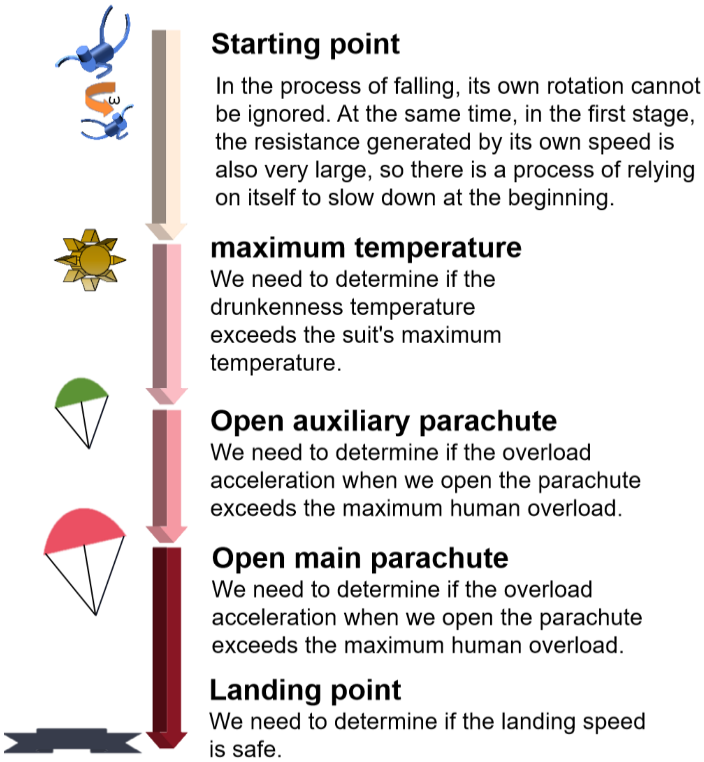

Figure 2 The process of landing

2 Assumptions and Justifications

✧ **The parachute is round**

By far the most used parachute is still the round parachute. Compared to other parachute shapes, a round parachute has the most significant deceleration effect and a wider range of application conditions, in addition to its simple structure and low cost.

✧ **The process of opening the parachute**

Due to the small area of the auxiliary parachute and the high speed of the parachutist when opened, it can be considered to be an instant opening. However, the main parachute area is large, and the speed of the skydiver is small when it is opened, so the closing process and inflation process need to be considered when it is opened.

✧ **Performance of the space suit**

The suit we have chosen is specific, it has a maximum temperature of 121 ℃ and a minimum temperature of -156℃. There is a perfect and good cooling and heating system inside the suit, so we just need to verify that the temperature of the suit is within this ran[2]

✧ **The maximum load the skydiver can carry is 8G[3]**

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	***Description***	***Unit***
Z	*geometric height*	*m*
H	*gravitational potential height*	*m*
R_0	*the radius of the Earth*	*m*
ρ	*Atmospheric density at local altitude*	Kg/m^3
ρ_{sl}	*Sea level atmospheric density*	Kg/m^3
$(CA)_{sk}$	*The area of the parachute in the closing stage*	m^2
$(CA)_s$	*The area of the parachute in the inflation stage*	m^2
λ_{m1}	*Parameters of the parachute itself*	*1*
λ_{m3}	*Parameters of the parachute itself*	*1*
t_{m1}	*The time of the closing stage*	*s*
t_{m2}	*The duration of maintenance after the closing stage*	*s*
t_{m3}	*The time of the inflation stage*	*s*
v_l	*Parachute's initial closing velocity*	*m/s*
v_{DR}	*The velocity after the closing stage*	*m/s*
V_x	*Velocity along the X-axis*	*m/s*
V_y	*Velocity along the Y-axis*	*m/s*
V_z	*Velocity along the Z-axis*	*m/s*
ω_z	*Angular velocity along the Z-axis*	*rad/s*
F_z	*Air resistance along the Z-axis*	*N*
M_z	*Moment along the Z-axis*	$N \cdot m$
Θ_x	*Deflection angle along the X-axis*	*rad*
Θ_y	*Deflection angle along the X-axis*	*rad*
V_{lost}	*Temperature loss due to air resistance at the same altitude*	*m/s*
Δt	*The minimum time for the main parachute to achieve the target deceleration effect during the closing stage*	*s*
a	*The local speed of sound*	m/s^2
γ	*The ratio of isobaric heat capacity to isovolumetric heat capacity*	*1*
T	*The temperature of the air at rest*	*K*
$T0$	*The temperature we are looking for*	*K*

4 Model preparation

4.1 atmospheric density

According to the formula of the standard atmospheric parameters of "Yang Bingwei", it can be obtained[4]:

Conversion between geometric height Z and gravitational potential height H:

$$H = Z/(1 + Z/R_0) \tag{1}$$

R_0=6.356766*10^3km is the radius of the Earth.

$$\begin{cases} W = 1 - \dfrac{H}{44.3308} \\ T = 288.15W(K) & 11.0191 < Z \le 20.0631\,\text{km} \\ \dfrac{\rho}{\rho_{SL}} = W^{4.2559} \end{cases} \tag{2}$$

Due to the excessive number of formulas, this paper only lists the density formulas for heights of 11.0191-20.0631km.

The image obtained is as follows:

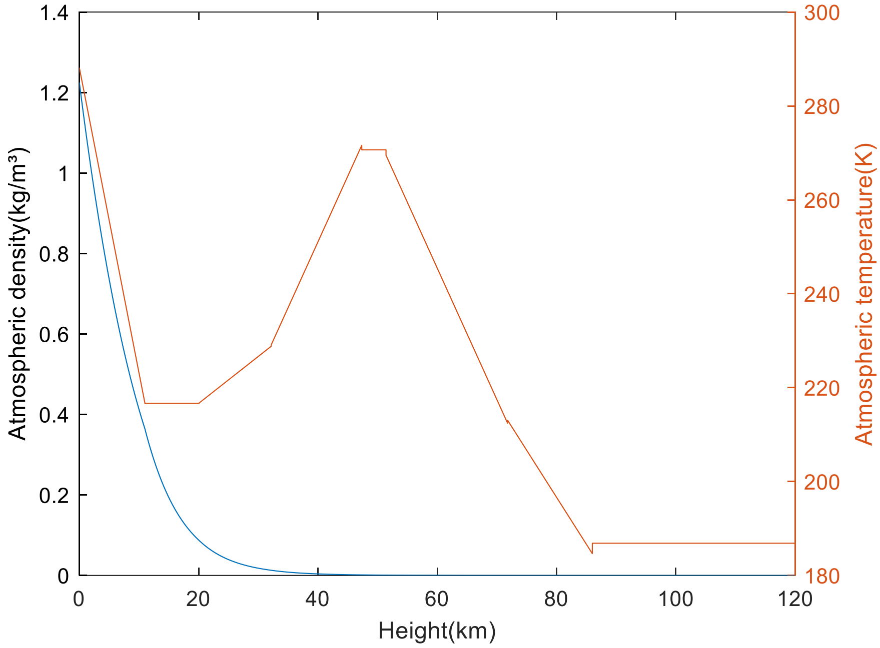

Figure 3 Atmospheric density and temperature fitting image

In the subsequent modeling process, density and temperature of atmosphere is distributed according to this image.

4.2 parachute opening process

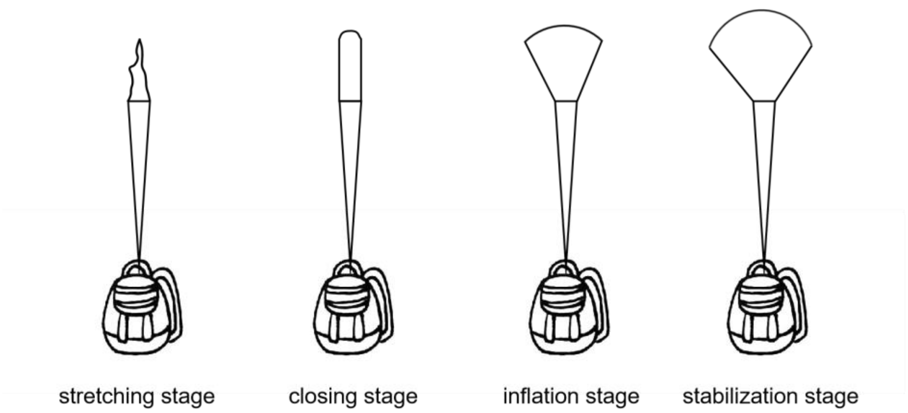

Figure 4 The four stages of opening a parachute

The parachute opening process can be divided into four stages, namely: stretching stage, closing stage, inflation stage, and stabilization stage. The change of parachute area can be seen in the following figure:

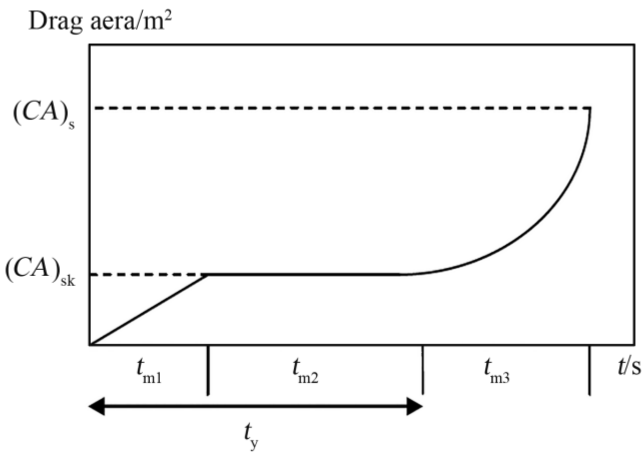

Figure 5 Area of resistance change during opening process[5]

(CA)$_{sk}$ is the area of resistance in the closing state, and *(CA)*$_s$ is the area of resistance in the fully tensioned state.

t_{m1} is the time of the closing stage [5]:

$$t_{m1} = \frac{\lambda_{m1}(CA)_{sk}^{0.5}}{v_L} \tag{3}$$

Where, λ_{m1} is the setting parameter, v_L is the parachute's initial closing velocity.

$$t_{m3} = \frac{\lambda_{m3}\left[(CA)_s^{0.5} - (CA)_{sk}^{0.5}\right]}{v_{DR}} \tag{4}$$

λ_{m3} is the setting parameter and v_{DR} is the velocity after the closing stage. The parachute is selected with λ_{m1} of 37.3 and λ_{m3} of 4.42[5].

5 Analysis of rotation dangers

5.1 The Establishment of Model

During landing, the skydiver controls the fluid forces acting on the body and the resulting direction and speed of descent by changing the position of the body. However, changes in body position also cause changes in stability. If stability is lost and the diver is caught in an unstable state such as spinning, a fatal accident may occur. Centrifugal force will throw the blood from the human brain, causing the eyes to be black, rotate the foot as the center, the blood will quickly rush to the brain, will cause the field of vision to become red, the formation of red vision, two cases will make people fall into a coma, heavy blood backfill-ing, bursting the skull.[6]

In this model, we relate rotational motion to vertical motion, describing the hazards that can result from rotation during descent.

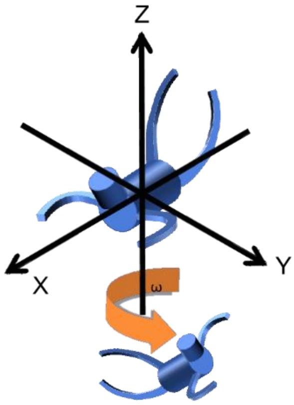

Figure 6 Coordinate system for the landing process

As shown in the figure, we set the mass of the astronaut as *m*, the velocities and angular velocities in the three directions of *x y z* as V_x, V_y, V_z, ω_x, ω_y, ω_z. The forces in the three directions are set as F_x, F_y, F_z. By expressing the components of the inertia tensor and the moments generated by the forces around the three axes in terms of I, M_x, M_y and M_z, respectively. C_n is the air resistance coefficient of the human body, taken as 0.865[7].The resulting equation of motion[8] is：

$$\begin{aligned} M_z = & \frac{1}{2}\rho|V_x|V_x C_n \int_l \cos^3\Theta_y \cdot y dA_{nx} \\ & + \frac{1}{2}\rho|V_y|V_y C_n \int_L \cos^3(\Theta_x) \cdot x dA_{ny} \\ & + \frac{1}{2}\rho|V_z|_n V_z C_n \int_L (\cos^2(\Theta_x)\sin(\Theta_x)\cdot x + \cos^2(\Theta_y)\sin(\Theta_y)\cdot y) dA_{nz} \end{aligned} \tag{5}$$

Since V_z is much larger than V_x, V_y, we can simplify the equation for M_z as：

$$M_z = \frac{1}{2}\rho|V_z|_n V_z C_n \int_L (\cos^2(\Theta_x)\sin(\Theta_x)\cdot x + \cos^2(\Theta_y)\sin(\Theta_y)\cdot y) dA_{nz} \tag{6}$$

Because Θ_x and Θ_y are both small variable, we assume that it tends to approach 0.The above formula can be simplified to：

$$M_z = \frac{1}{2}\rho|V_z|_n V_z C_n \left(\Theta_x \int_L x \cdot dA_{nz} + \Theta_y \int_L y \cdot dA_{nz}\right) \tag{7}$$

For the same person, the static moment: $\int_L x \cdot dA_{nz}$ and $\int_L y \cdot dA_{nz}$ are fixed.

$$M_z \propto \frac{1}{2}\rho|V_z|_n V_z C_n S \cdot (\Theta_x + \Theta_y) \tag{8}$$

Suppose $\Theta_x + \Theta_y$ stay the same, then：

$$M \propto F_z \tag{9}$$

Further changes obtain:

$$\omega_z \propto V_{lost} \tag{10}$$

V_{lost} is the difference between the velocity of a free-falling body and its actual velocity in the presence of air resistance. According to the query information, when jumping from the height of 39km at 0.8 times the speed of sound, the body rotates 180 times per minute, namely ω_z is 6π rad/s[9]. According to the kinetic aerodynamic model, the V_{lost} is 8.5 m/s. Take into the equation (6) to calculate:

$$\omega_z \approx 2.2 V_{lost} \tag{11}$$

Analysis this formula, we can draw the final conclusion: the loss of 1 m/s landing speed will increase the angular speed of 2.2 rad/s. Therefore, skydiver need to keep themselves level at all times and be able to make adjustments in time in order to minimize their rotation.

To ensure safety, we can refer to Baumgartner's team, using supersonic missiles to design an auxiliary stabilization device, a special drag parachute installed on the body of the skydiver, will be opened when the accelerometer reading on the jumpsuit exceeds 3.5G for six consecutive seconds, playing a stabilizing role. But overcoming the spin is more about the parachutist's ability to control the air[10].

6 Analysis of high-temperature danger

6.1 The concept of stagnation point

The stationary point is the point at which the relative velocity of the air to the object is 0. Points A and B in the diagram are the stagnation points.

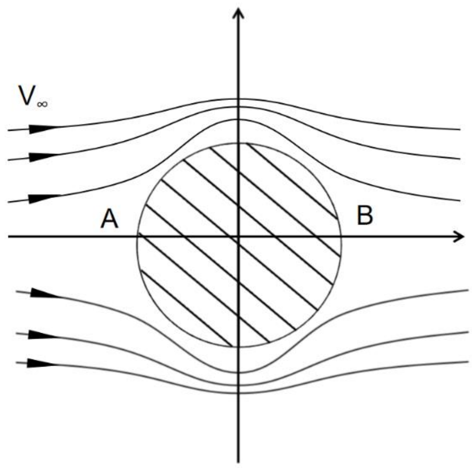

Figure 7 stagnation point diagram

The temperature at the stagnation point can be viewed as a conversion of all the kinetic energy of the air into internal energy. Theoretically, this is the highest temperature that can be reached on the outer surface of a spacesuit. But the heat transfer capacity of the thin atmosphere is limited, and the suit must absorb enough heat to approach the temperature.

Assuming that half of loss energy turns into the heat that is absorbed by the spacesuit, the outer surface of the spacesuit is made of polyethylene, the outer layer of the mass of 10Kg, the specific heat capacity of 1.5J/(g • k), if the skydiver were in 40km, 60km, 80km at the time of jumping, the heat absorbed on the outer surface of the spacesuit are 4764kJ, 5340kJ and 5857kJ respectively, the outer surface can be warmed up 317.6K, 356K, 390K, it can be considered that the outer surface of the suit is close to the stationary point temperature, due to the existence of convective heat transfer between the suit and the air, and the outer surface temperature is not exactly equal to the stationary point temperature. When designing the thermal protection capability of the capsule, engineers often take 80% of the stationary temperature as the maximum temperature of the capsule's surface. It is impossible for a spacesuit to reach the same high temperature as a space capsule, and it cannot dissipate heat on a large scale by radiation like a space capsule, but basically only by convection heat transfer with the air, so the ratio of its outer surface temperature to the stagnation temperature is generally higher than 80%. In this model we approximate the outer surface temperature of the suit to be 85% of the stationary point temperature.

6.2 Calculation of stagnation point temperature

To calculate the temperature at the stagnation point, we introduce the following equation. Firstly, we need to calculate the local speed of sound:

$$a = \sqrt{\gamma R T} \tag{12}$$

a is the local speed of sound, γ is the ratio of isobaric heat capacity to isovolumetric heat capacity, R is the gas constant.

Thus, the velocity of the gas can be converted to Mach number. From the speed of sound we can find the temperature of the stagnation point:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} Ma^2 \tag{13}$$

T is the temperature of the air at rest and T_0 is the stagnation temperature. The following figure illustrates the maximum surface temperature skydivers will encounter when jumping at different height.

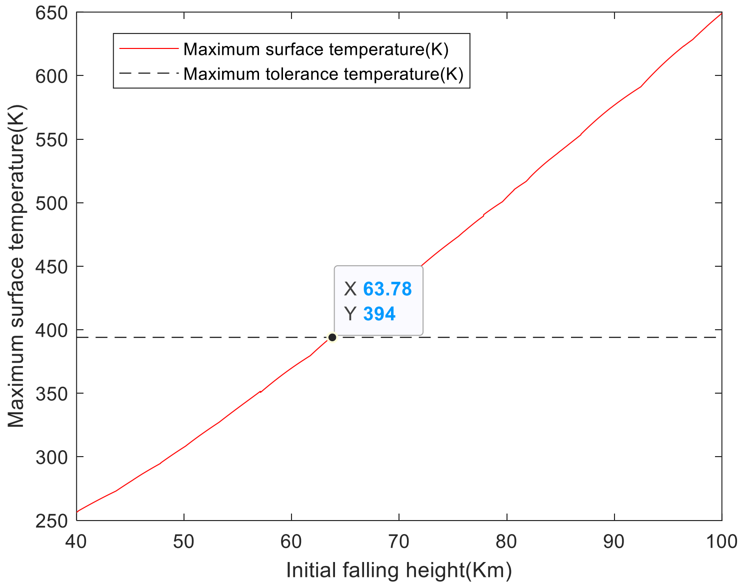

Figure 8 Variation of maximum surface temperature with initial height

From this it can be concluded that astronauts will exceed the maximum thermal load(121℃[11]) when jumping at altitudes above 63.78 Km.

7 Analysis of overload and speed danger

7.1 Establishment of the process

In this model, the main problem we solve is the problem of overload (acceleration) at the moment of opening the parachute and speed when landing on the ground.

After reviewing Felix Baumgartner's skydiving case[13], we have the following statistics：

Table 2: Felix Baumgartner's skydiving statistics

Time(s)	Altitude(m)	Velocity(km/h)	Accident
0	38969	0	
39.4	31676	1238	reach the speed of sound
49.5	27993	1356	Reach maximum speed
60.0	24144	1201	Maximum resistance
100.3	15505	502	
255.2	2838	202	
260.6	2533	181	Open the parachute.

Referring to this case, we design the movement process of the skydiver as the following three steps (Figure 2):

1. Non-parachute drop
2. Open the secondary parachute
3. Open the main parachute

The rationale for this process is argued below:

Since the total mass stated in the question is 190kg, and our assumed mass of the man and spacesuit is 160kg, the parachute can only have a weight of 30kg. Since the total density of the parachute is 0.2 kg/m^2[12], the total area of the parachute is 150m^2. After analysis, the area of the main parachute is 146m^2 and the area of the secondary parachute is 4m^2.

The skydiver’s velocity change is an acceleration-deceleration process. If only consider vertical acceleration does not exceed 8g, parachute can be opened before the speed rises. But in reality, it can't be analyzed in this way, due to the thin air, low temperatures and risk of rotation at high altitudes. Direct opening of the parachute at high altitude is very risky, so the first procedure is non-parachute drops.

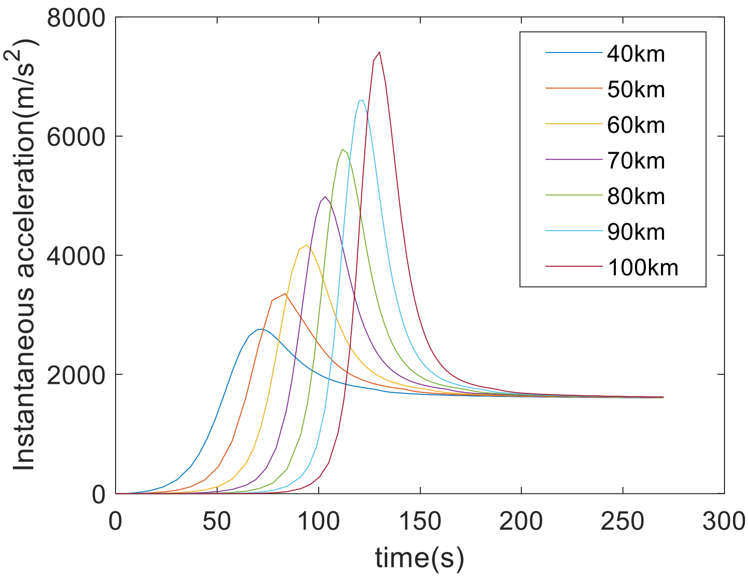

Figure 9 Changes in acceleration during descencending at different altitudes

Fig9 is the load on the skydiver if the main parachute is opened directly without opening the secondary parachute. The load is at least 100G, so it is not reasonable to open the main parachute directly. So, we need to open the secondary parachute before the main parachute. The explanation of this figure is described below

7.2 Solution of the process

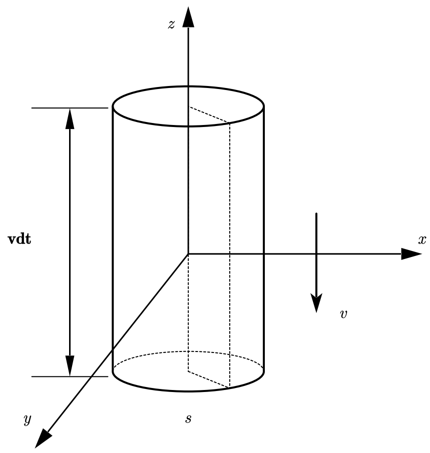

Figure 10 Air expelled in descending

For the descending process, the air expelled is seen as a cylinder and the equation of motion can be obtained as follows:

$$\frac{dv}{dt} = mg - \frac{1}{2} C\rho s v^2 \tag{14}$$

s is the contact area of the spacesuit (cylinder) with the air, ρ is the density of the air, *v* is the speed of descent without opening the parachute. C is the air drag coefficient of the parachute[14]. These are the equations of motion without the parachute. Since ρ is related to h and the equations of motion are nonlinear second order differential equations. So, this equation is solved numerically. The state of motion during landing without parachute can be obtained:

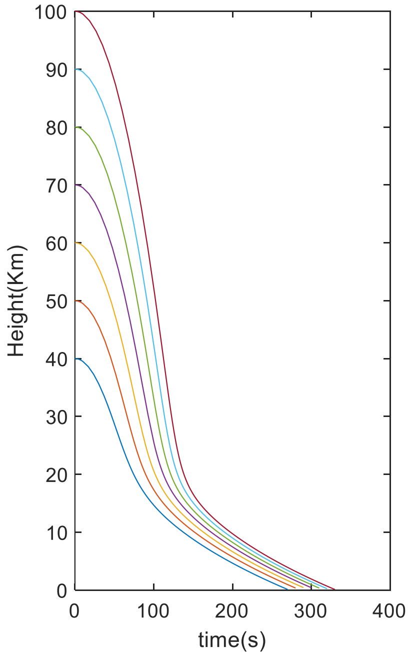

Figure 11 Movement of the skydiver before opening of the secondary parachute

By means of the state of motion, skydiver's acceleration can be obtained at the moment when the secondary parachute or main parachute is opened (Figure 9 Changes in acceleration during descencending at different altitudes)

$$a = \frac{\frac{1}{2}\rho c s v^2 - G}{m} \tag{15}$$

The secondary parachute is opened as soon as the acceleration resulting from the opening of the parachute reaches 8g. Find the speed and altitude of the skydiver when the secondary parachute is opened:

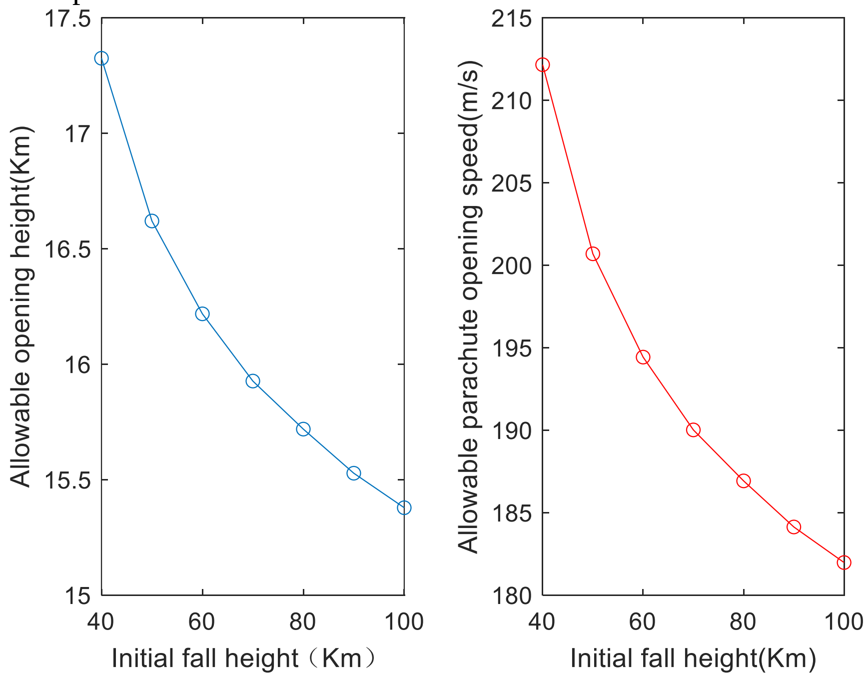

Figure 12 Height and speed of skydivers when open the secondary parachute

The next thing we'll discuss is the opening time of the main parachute.

Taking the point where the secondary parachute is opened as the initial point, the s of the above equation becomes the area of the parachute, which is solved numerically again to obtain the motion state as follows:

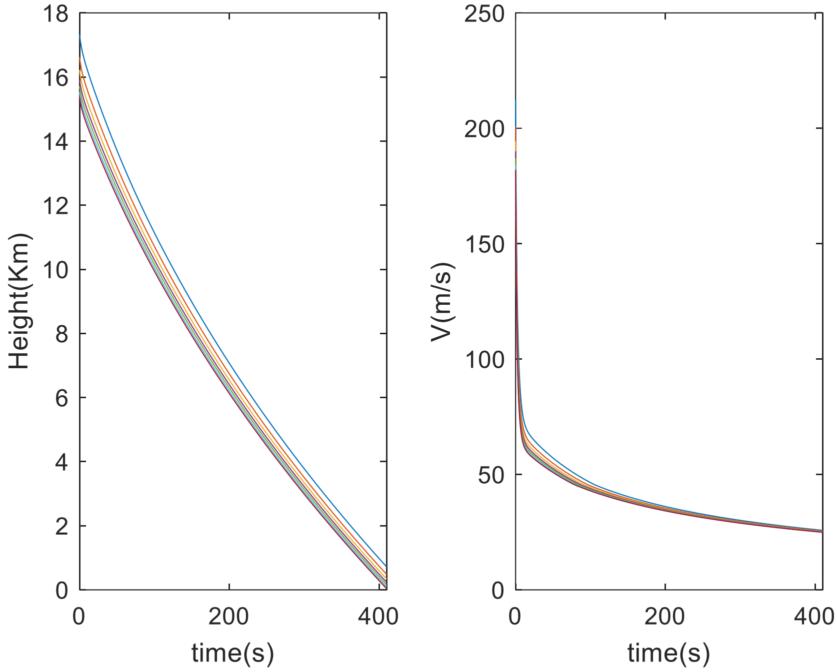

Figure 13 Movement of the skydiver after opening of the secondary parachute

The opening of the main parachute cannot simply be treated as an instantaneous process. Due to the action of the secondary parachute, the skydiver has descended to a lower speed. Because the speed is low and the size of the main parachute is large, the process of opening the parachute needs to be considered as an non-instantaneous process. To simplify the calculation, the opening process of the main parachute is divided into the instant opening of two parachutes, whose area is s_{sk} and s_c. s_{sk} =30m^2, s_c=116m^2. The area of the first umbrella s_{sk} is used to equate to the case where the main umbrella is partially opened during the t_y time period in Fig14, the second umbrella is used to equate to the case where the remaining part of the main umbrella is fully opened during the t_{m3} time period in Fig14.

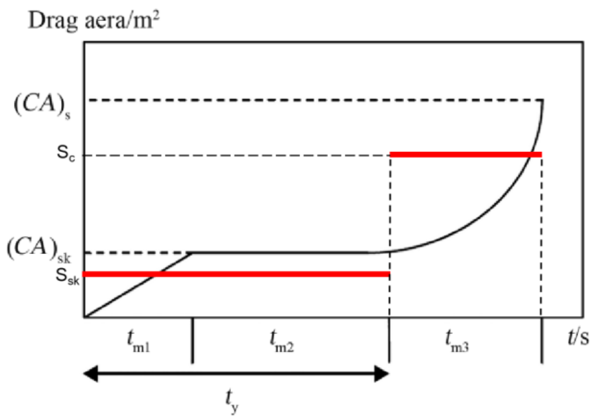

Figure 14Analysis of the main parachute opening process[5]

The state of motion is known. Combined with equation (15) The acceleration of main parachute opening can be solved. As above, we open the parachute at a predicted acceleration of 8g. But it is possible that the skydiver may have decelerated too slowly in t_y time, resulting in an acceleration of more than 8g when the area reaches the s_c.

To rule this out, we calculated the time required as Δt, which means after Δt, if the area of parachute reaches the s_c, acceleration is less than 8g.

Here is a comparison table of Δt and t_{m1}:

Table 3: Main parachute opening time

H	40km	50km	60km	70km	80km	90km	100km
t_{m1}	6.2	6.5	6.7	6.8	6.9	6.98	7.05
Δt	1.5	1.43	1.4	1.37	1.35	1.33	1.32

$$t_y > t_{m1} > \Delta t$$

So, if the acceleration is lower than 8g, we can open the main parachute.

The motion image after opening the main parachute is:

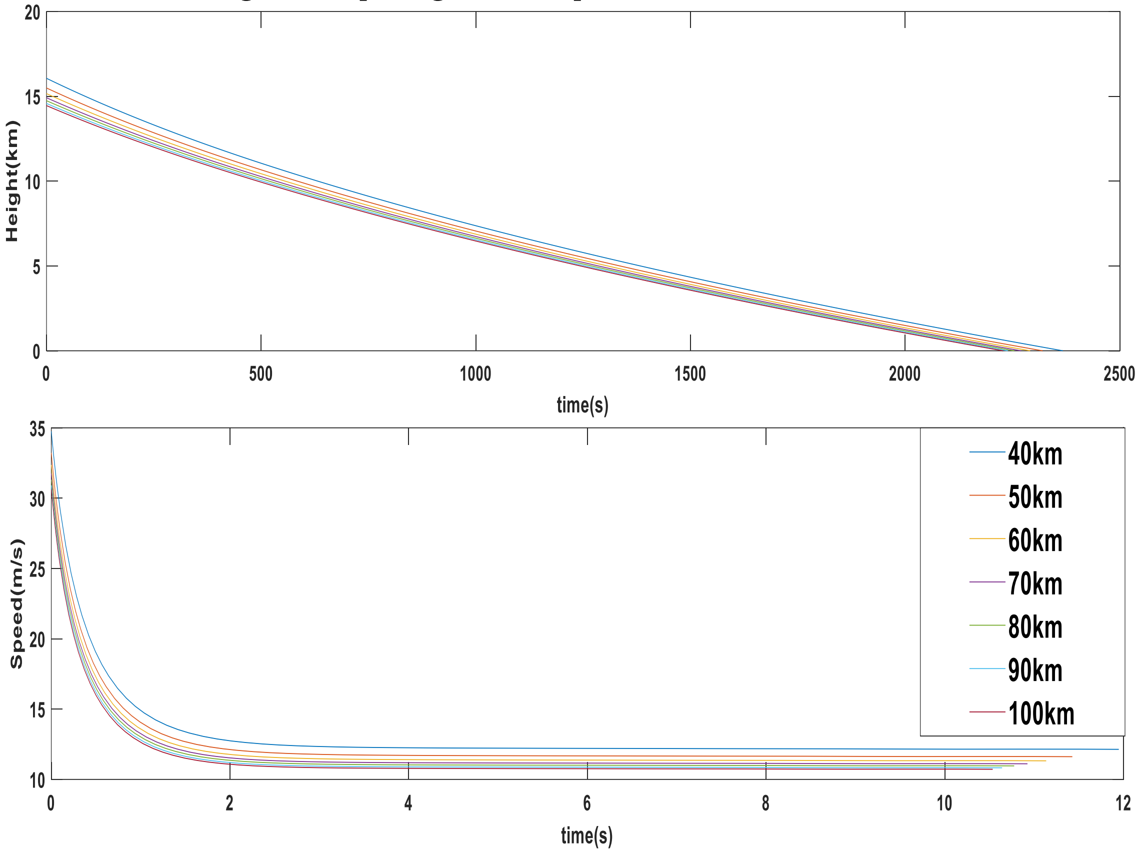

Figure 15 Movement of the skydiver after opening of the main parachute

It can be seen that after two seconds, the speed had been greatly reduced, from 35 metres per second to around 10 meters per second.

7.3 Conclusion

In the end, we get the motion curves for different starting heights of descent：

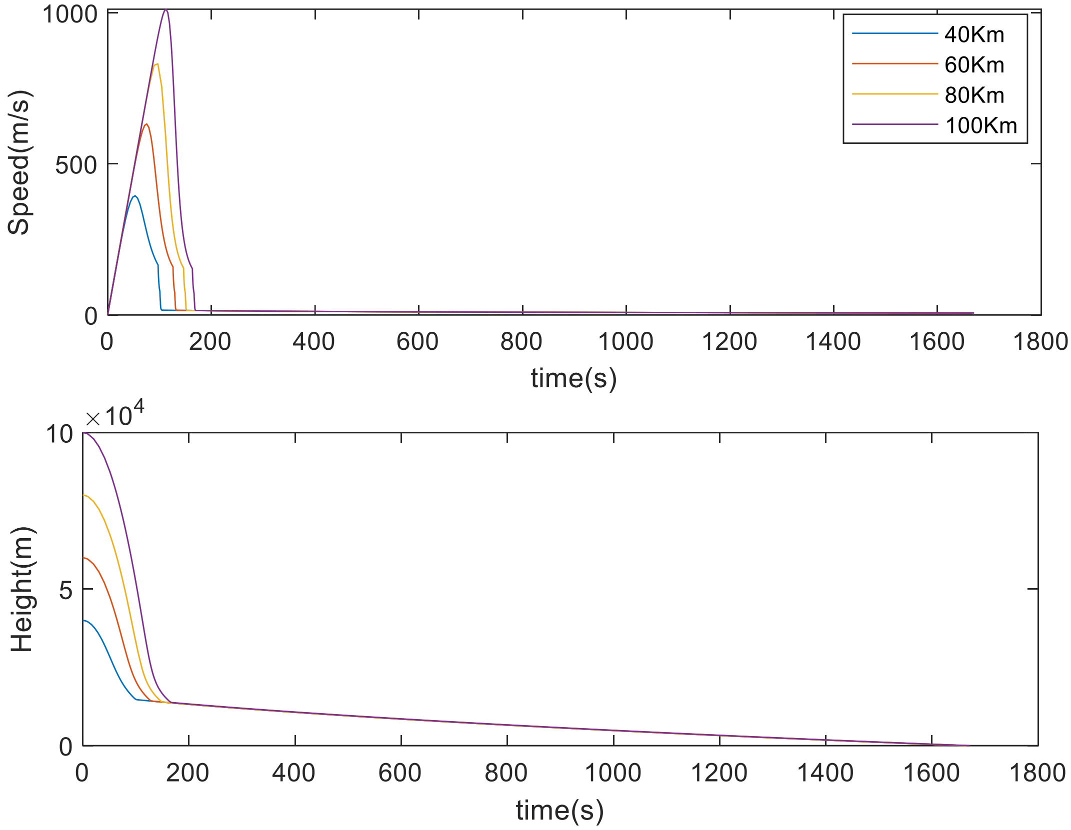

Figure 16 Movement of the skydiver

It can be seen from the figure that different jump heights in the opening of the parachute will soon converge to the same state of motion, this is because the time to open the parachute is always when the acceleration on the skydiver has just dropped to 8g.At the same time, the total area of the parachute is unchanged, which leads to the equilibrium speed is unchanged. It is worth mentioning that the astronauts finally landed on the ground at a speed of 6.5m/s, which means that the parachute to the end of the parachute has not been reduced to equilibrium state of the speed.

So, if only the effects of high acceleration and landing speed are taken into account, it will land safely at 100km and below. Risks at this point arise mainly from temperature and rotation.

However, our calculations show that the skydiver will not be able to reach the ground safely at 162.7km, due to the inability to open the secondary parachute within the permissible acceleration.

8 Model Evaluation and Further Discussion

8.1 Strengths

1. We discuss the potential hazards that may occur to a skydiver in different stages of descent, perform qualitative calculations and propose solutions accordingly.
2. we model the next stage by calculating the motion state of the skydiver in the previous stage and make reasonable simplifications.
3. we optimize the skydiving process of the skydiver according to the current level of

science and technology, and find out the maximum height of his/her permissible skydiving on this basis.

8.2 Weaknesses

1. We use approximate treatment in solving the heat generation model that determines the maximum height of skydiving, and do not establish the convective heat transfer model between the spacesuit and the air.
2. We have optimized the whole skydiving process more, such as opening the parachute at a state where the instantaneous acceleration of opening is 8g, which in fact will make the whole skydiving task more complicated and increase the uncertainty.

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Appendices

Appendix 1
Introduce: Yang Bing wei atmospheric density temperature fitting curve code

```
g=9.80665;rousl=1.225;%kg/m³
R0=6.356766*1e6;
h1=0:0.01:11.0191;H1=h1./(1+h1/R0);
h2=11.0191:0.01:20.0631;H2=h2./(1+h2/R0);
h3=20.0631:0.01:32.1619;H3=h3./(1+h3/R0);
h4=32.1619:0.01:47.3501;H4=h4./(1+h4/R0);
h5=47.3501:0.01:51.4125;H5=h5./(1+h5/R0);
h6=51.4125:0.01:71.8020;H6=h6./(1+h6/R0);
h7=71.8020:0.01:85.9999;H7=h7./(1+h7/R0);
h8=85.9999:0.01:120;H8=h8./(1+h8/R0);
W1=1-H1/44.3308;rou1=W1.^4.2559*rousl;T1=288.15*W1;
W2=exp((14.9647-H2)/6.3416);rou2=0.15898*rousl*W2;T2=216.65*ones(1,size(h2,2));
W3=1+(H3-24.9021)/221.552;rou3=0.032722*rousl*W3.^(-35.1629);T3=221.552*W3;
W4=1+(H4-39.7499)/89.4107;rou4=3.2618*(1e-3)*W4.^(-
13.2011)*rousl;T4=250.35*W4;
W5=exp((48.6252-H5)/7.9223);rou5=9.492*(1e-
4)*W5*rousl;T5=270.65*ones(1,size(h5,2));
W6=1-(H6-59.439)/88.2218;rou6=2.528*(1e-4)*W6.^11.2011*rousl;T6=247.021*W6;
W7=1-(H7-78.0303)/100.295;rou7=1.7632*(1e-5)*W7.^16.0816*rousl;T7=200.59*W7;
W8=exp((87.2848-H8)/5.47);rou8=3.6411*(1e-6)*W8;T8=186.87*ones(1,size(h8,2));
rou=[rou1,rou2,rou3,rou4,rou5,rou6,rou7,rou8];
h=[h1,h2,h3,h4,h5,h6,h7,h8];
T=[T1,T2,T3,T4,T5,T6,T7,T8];
plot(h,rou)
ylabel('Atmospheric density(kg/m³)')
yyaxis('right');
plot(h,T)
ylabel('Atmospheric temperature(K)')
xlabel('Height(km)');
```

Appendix 2

Introduce: Code for calculating differential equations of motion

```
clear;clc
for h0=60000
    m=190;
    G=6.67*1e-11;
    M=5.965*1e24;
    R0=6.356766*1e6;
    global S1;global S2;global S3;global C1;
    S1=4;S2=30;S3=116;C1=1;
    y0=[h0,0];
    opts1=odeset('Events',@Myevents);
    [t,y,ti,yi,ki]=ode45(@(t,y) funluo(t,y),[0,400],y0,opts1);
    if ki==2 || ki==3
        break
    end
    y10=[yi(1),yi(2)];
    opts2=odeset('Events',@Myevents1);
    [t1,y1,ti1,yi1,ki1]=ode45(@(t1,y1) funluo1(t1,y1),[ti,ti+500],y10,opts2);
    if ki1==2
        break
    end
    opts3=odeset('Events',@Myevents2);
    y20=[yi1(1),yi1(2)];
    [t2,y2,ti2,yi2,ki2]=ode45(@(t2,y2) funluo2(t2,y2),[ti1,ti1+1400],y20,opts3);
    if ki2==2
        break
    end
    opts4=odeset('Events',@Myevents3);
    y30=[yi2(1),yi2(2)];
    [t3,y3,ti3,yi3,ki3]=ode45(@(t3,y3) funluo3(t3,y3),[ti2,ti2+3000],y30,opts4);
end
```