Trampoline Jumping Analysis

Team 102

Problem B

abstract

Trampoline is a competitive sport that uses the bouncing power of a trampoline to perform acrobatic tricks in the air. It can also be a popular recreation for children.

To begin with, we built a model of a trampoline and calculated its expressions for elastic force, air resistance, friction force, and damping force.

In order to calculate the maximum height that each child can achieve, we first established an ideal model, regarded each child as a Particle, did not do external work, used Hooke's law to represent the elasticity of the trampoline, ignored various resistance factors, simulated with MATLAB, and by analyzing its image, we can get the energy transfer relationship in the process of this movement. When each child compresses the lowest point of the trampoline and is bounced, the next child just arrives and continues to bring the trampoline down, then the third child repeats the process, in this way, the energy of the first two children is transferred to the third child through the trampoline, so that the third child will reaches the highest height.

Then we improved the model, in this new model we introduced the previous trampoline model, and estimate each child's jumping ability by their maximum height when jumping individually. Again, we listed the corresponding differential equations and calculated them in MATLAB, but instead of directly calculating the theoretical maximum value that each child could reach, we controlled for different initial conditions and found the highest height reached by each child through exhaustive methods. The end result was the 25kg child could jump to 3.67 meters, the 40kg child could jump to 2.54 meters, and the 50kg child could jump to 2.09 meters.

Contents

1 Introduction

The trampoline is a professional and recreational facility not only can bring happiness to players and also plays an important role in balance resilience exercise. However, there are risks when engaging in physical activity especially when there is more than one person. For single players, the risk of it is lower because they can control their height, but for shared players, the risk of it is hard to prevent. In shared trampolines, one can transfer his energy to others. In detail, the potential energy of one player can be transferred to trampoline when landing earlier than another player. And the energy stored by trampoline will transfer to another person which make him bounce higher. We consider the case when there are three kids share the same trampoline and aim to find the highest each one can reach.

2 Problem analysis

The main task is to find the maximum height each one can reach. For this problem three kids A, B, C respectively weighs 25kg, 40kg, 50kg. The diameter of trampoline is 5m. A alone can bounce to a maximum height of 0.5m, while B, C can alone bounce to a maximum height of 0.8m, 1.2m. Firstly, we want to analysis the dynamic characteristics of trampoline, then we can know the interaction between kids and trampoline.

To make the problem clearly, we first want to simplify the forces between kids and trampoline, just consider it following Hook's low. Knowing how the simplified cases works, we want to adjust the model and consider the more reality cases.

Finally, we can solve the maximum height in complex cases.

3 Theoretical model

3.1 Trampoline Static and Dynamic Analysis

3.1.1 Static Analysis of Trampoline

[1]The Trampoline is consisted of many springs around the central mat. We take a specific and common trampoline for analysis. Below is the parameter of the trampoline:

Table 1: Parameter of the trampoline

Parameter	**Meaing**	**Value**	**unit**
R	Trampoline radius	2.5	m
M_m	Mat mass	1	kg
N_s	Number of spring	55	1
K_s	Stiffness of spring	3000	N/m

If we make a small distance x from the horizontal, the train of one spring is :

$$\triangle L = \sqrt{R^2 + x^2} - R \approx \frac{x^2}{2R} \tag{1}$$

Actually, we predict the largest of x to be 0.5m, and $\frac{x}{2R} = 0.1$. The spring force is:

$$F_s = K_s \triangle L = K_s(\sqrt{R^2 + x^2} - R) \tag{2}$$

The total force can be divided into the horizontal part and the vertical part, let the vertical part to be F_v:

$$F_v = N_s K_s (\sqrt{R^2 + x^2} - R) \frac{R}{\sqrt{R^2 + x^2}} \tag{3}$$

The graph of F_v is demonstrated below:

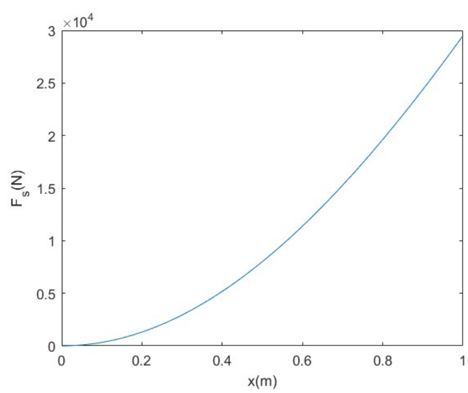

Figure 1: $F_v(x)$

3.1.2 Dynamic Analysis of Trampoline

When someone is jumping in the trampoline there will be some energy loss. There are three factors, the air resistance on the mat F_a, the damping resistance F_d and frictions f. We use calculus to calculate the F_a, divide the mat into many pieces, and let the velocity in the center to be v, the piece is r from the center, then velocity of each piece is:

$$v_r = \frac{R-r}{R} v \tag{4}$$

We can get the air resistance to this piece:

$$dF_a = C_a \frac{R}{\sqrt{R^2+x^2}} (\frac{R-r}{R} v)^2 2\pi r dr \tag{5}$$

We project the total resistance to the vertical direction, so the total air resistance is:

$$F_a = \int_0^R C_a \frac{R}{\sqrt{R^2+x^2}} (\frac{R-r}{R} v)^2 2\pi r dr = \frac{\pi C_a R^3}{6\sqrt{(R^2+x^2)}} v^2 \tag{6}$$

The damping resistance is proportional to central velocity:

$$F_d = C_d v \tag{7}$$

To put it easily, we use the statistics below which are proved to be consistent to reality to calculate:

Table 2: Resistance parameter

Parameter	**Meaing**	**Value**	**unit**
C_a	Air resistance constant	2.3	Ns^2/m^4
C_d	Damping resistance constant	16	Nsm
f	Stiffness of spring	2.5	N

3.2 The most ideal model

To understand the energy transfer of trampoline problem, let's start with the simplest model:

1.Treat each child as a particle.

2.Treat the highest potential energy children can get to be the initial energy of each one.

3.The elastic force provided by the trampoline satisfies Hooke's law $F_v = -kx$. We predict x less than $0.5m$, reference to figure 1, we let $k = 8000N/m$.

4.Regardless of other drag factors.

In this way, when a child touches the trampoline and compresses it, he is subjected to his own gravity $G = mg$ and trampoline elastic force $F_v = -kx$. Only the child who is at the bottom can get the trampoline force. After building the model according to this assumption, for each of the children we can list the differential equation below:

$$m\frac{d^2x_i}{dx_i{}^2} = \begin{cases} -mg & \text{if } x_i \neq a \\ -mg - kx_i & \text{if } x_i = a \end{cases}$$

$$a = min\{x_i, 0\}$$

We can solve the differential equations in MATLAB.

3.3 Simulation and analysis to ideal model

To get the central physical ideas in the process, Let's start by analyzing the energy transfer process when there are two children jumping. Suppose them fall from different height at the same time. Intentionally set the initial conditions, we can get figure 2.

At the beginning, they do a freefall movement, firstly B touches the trampoline and compress it. The trampoline exerts a force that hinders B's movement, while A continues to fall freely, so A falls faster than B and will catch up with B. Here is what interesting happens. If A catches B when B slightly bounce up. Then, A compresses the trampoline downwards and is subjected to the resistance provided by the trampoline, while B begins to bounce up freely. The majority of energy of B is stored in trampoline and will transfer to A when A bounces up. The result is A get the majority energy from B and B remains relatively low energy. A will bounce up significant high.

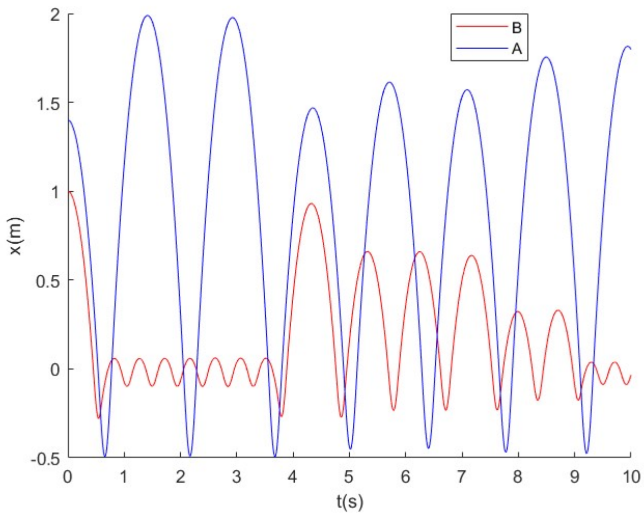

Figure 2: double damping

The problem is more complicated when there are three children jumping, but the physical idea is the same. Below is the initial statistics. We get it from the maximum they can reach. We set initial condition in this way, initially all of children are above the trampoline and their kinetic energy and potential energy are equal to their initial energy, so they can release from different height with accordingly velocity.

Table 3: Initial condition

Children	**Mass(kg)**	**Initial Energy(J)**
A	25	122.5
B	40	313.6
C	50	588.0

What we want is that when someone slightly bounces up, another one catches him and push the trampoline done. Then when this one slightly bounces up, the last one catches and eventually can get the majority energy and bounce up higher than what he can alone.

We use MATLAB to stimulate and we get the following graph: Figure 3 to Figure 5.

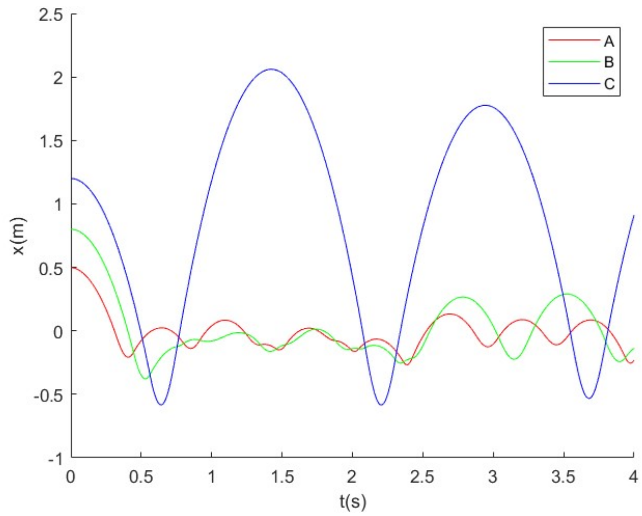

Figure 3: C reaches highest

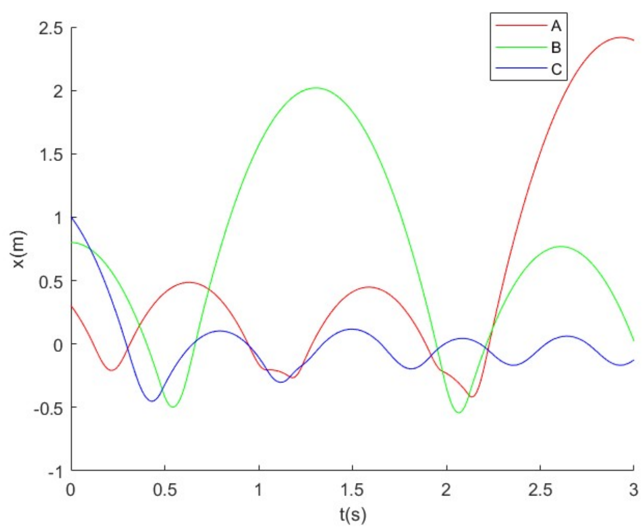

Figure 4: B reaches highest

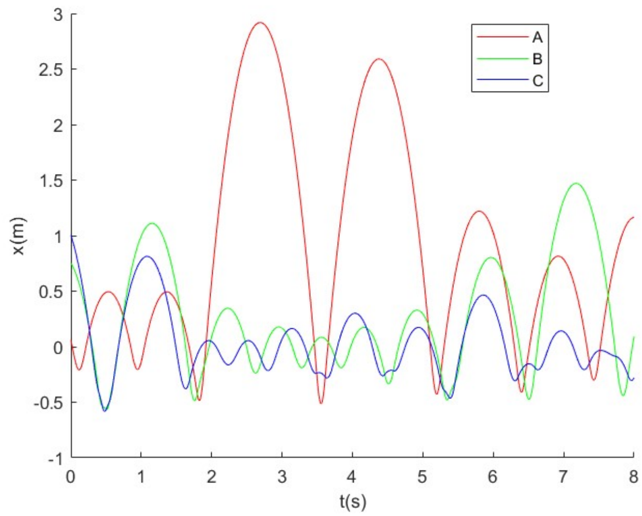

Figure 5: A reaches highest

As we can see, if they jump together, every one of them can get the height they cannot do alone.

Table 4: Add caption

Children	Alone maximum height(m)	Together maximum height(m)
A	0.5	2.9
B	0.8	2.2
C	1.2	2.1

Through trampoline, work can be stimulated in one person. Table 4 is the maximum height calculated by MATLAB. Actually, these are not the only conditions can lead to the highest.

3.4 Adjustment to the model

Next, we want to adjust our model.

Elastic Force: The force is not linearly to x as Hook's low, equation(3) gives the accurate formula. Considering distance x is a lot smaller than radius R, we can use more accurate simplification:

$$F_v \approx \frac{N_s K_s}{2R} x^2 \tag{8}$$

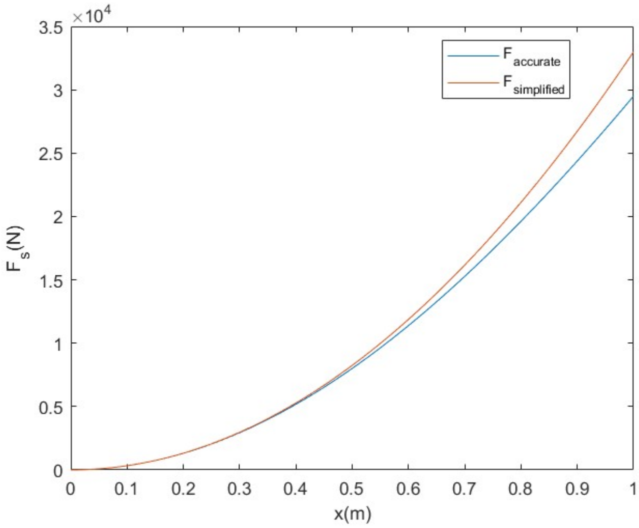

Figure 6: Comparison between accurate and simplified force

This simplification is quite well when $x < 0.5$.

Energy Loss: The energy loss is due to damping resistance, air resistance and fraction. All of them have been discussed above.

Energy gain: Actually, if the child don't use his strength, he will stop quickly as stated in figure 7. We predict that when one kid gets to his maximum, the energy gain equals to the energy loss. So, we consider his strength to be the energy loss divided by the distance from bottom to $x = 0$, which compensates the loss. We take the first bounce in figure 7 to calculate energy loss and distance. We calculate the force and demonstrate below:

Table 5: Strength of each child

Children	**Strength**	**Unit**
A	122.7	N/m
B	187.6	N/m
C	239.5	N/m

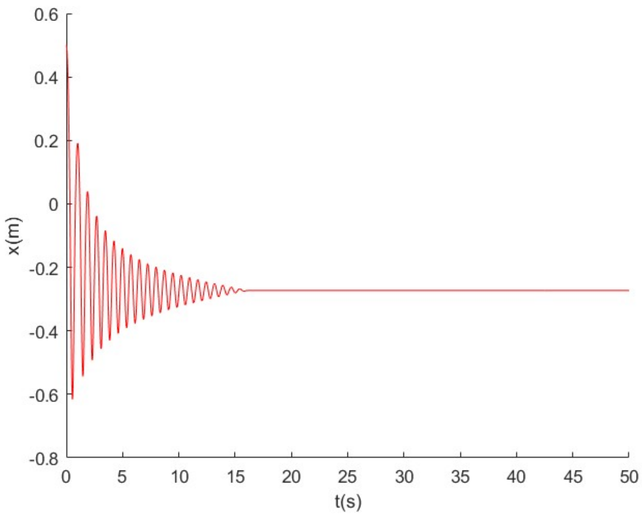

Figure 7: Energy Loss

Using the more accurate model, we get figure 8 to 10 to be the maximum height, Table 6 demonstrates statistics:

Table 6: Adjust modle

Children	**Alone maximum height(m)**	**Together maximum height(m)**
A	0.5	3.67
B	0.8	2.54
C	1.2	2.09

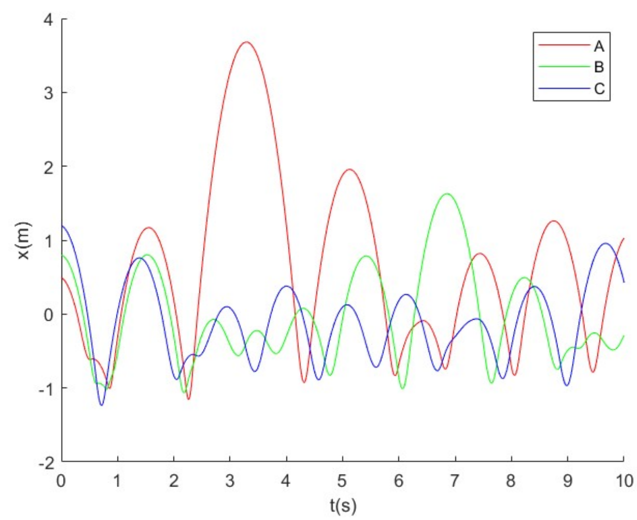

Figure 8: A reaches highest

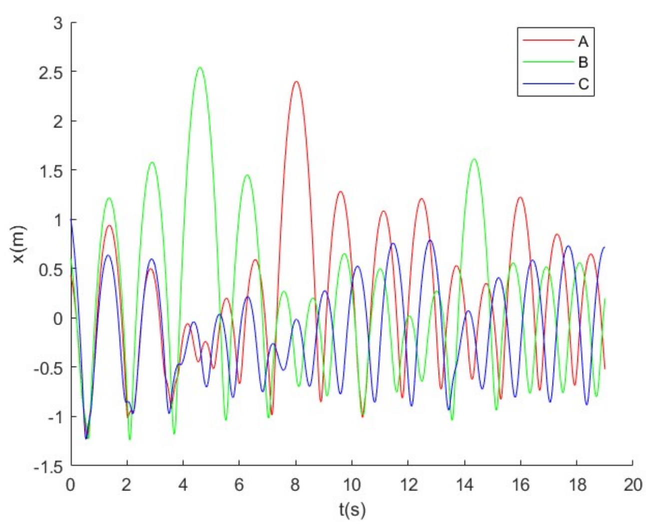

Figure 9: B reaches highest

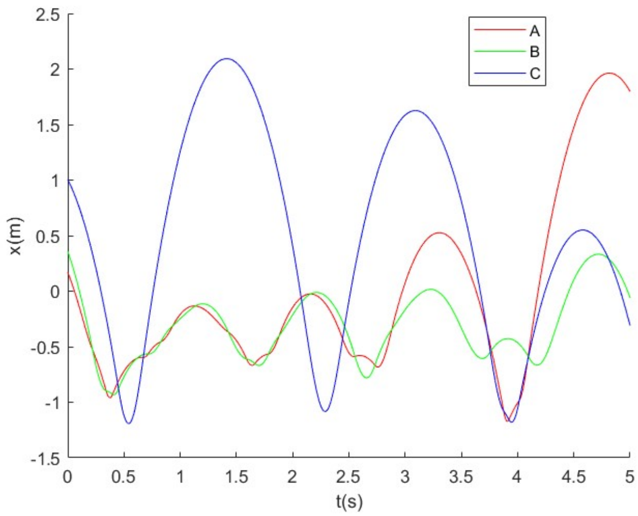

Figure 10: C reaches highest

4 Discussion and conclusion

We first use many simplifications, which can give a clear nature of the problem. To make it more close to reality, we consider many factors which may affect. But because of the complexity of reality, there must be some factors we don't fully consider such as the specific process of one kid using his strength, hoping we can do experiment to determine them.

At the same time, we also thought that for these children, judging by their weight, they may still be very young, especially the 25kg child, the height of the jump alone is only 0.5m, but when the three children jump together, they can reach a height of more than 3 meters, which is very dangerous for them. It can be seen that trampoline sports may produce some potential safety hazards, and we must pay attention to safety when using trampoline sports and games.

References

[1] D. Eager, S. Zhou, K. Ishac, I. Hossain, A. Richards, and L.N. Sharwood. Investigation into the trampoline dynamic characteristics and analysis of double bounce vibrations. *Sensors*, 22(8):2916, 2022.

Appendix

```
clear
clf
for i=0:0.01:0.3
for j=0:0.01:0.4
for m=0:0.01:0.5

[t,y]=ode45(@(t,x)func(t,x),0:0.01:30,
[0.5-4.9*i^2,-9.8*i,0.8-4.9*j^2,-9.8*j,1.2-4.9*m*m,-9.8*m]);
figure
hold on
plot(t,y(:,1),"r")
% plot(t,y(:,2))
plot(t,y(:,3),"g")
hold on
% plot(t,y(:,4))
plot(t,y(:,5),"b")
hold off
xlabel('t(s)')
ylabel('x(m)')
end
end
end
```

Figure 11: Ideal model

```
function dxdt=func(~,x)
g=9.8;
k=8000;
alpha=25;
beta=40;
gamma=50;
A=[x(1),x(3),x(5),0];
a=min(A);
dxdt=[x(2);
      -g-k*x(1)/alpha*(x(1)==a);
      x(4);
      -g-k*x(3)/beta*(x(3)==a);
      x(6);
      -g-k*x(5)/gamma*(x(5)==a)];
end
```

Figure 12: Ideal model

```
clear
clf
for i=0:0.01:0.5
for j=0:0.01:0.5
for m=0:0.01:0.5
[t,y]=ode45(@(t,x)func(t,x),0:0.01:5,
[0.5-4.9*i^2,-9.8*i,0.8-4.9*j^2,-9.8*j,1.2-4.9*m*m,-9.8*m]);
figure
hold on
plot(t,y(:,1),"r")
%plot(t,y(:,2),"y")
plot(t,y(:,3),"g")
hold on
%plot(t,y(:,4),"b")
plot(t,y(:,5),"b")
%plot(t,y(:,6))
hold off
%plot(y(:,1),y(:,2))
xlabel('t(s)')
ylabel('x(m)')
end
end
end
```

Figure 13: Adjusted model

```
function dxdt=func(~,x)
g=9.8;
k=6600;
c=2.3;
d=16;
R=2.5;
gamma=50;
alpha=25;
beta=40;
phi=2.5;
A=[x(1),x(3),x(5),0];
a=min(A);
dxdt=[x(2);
      -g*(x(1)>a)
+(-alpha*g+1/2*k*x(1)^2-((pi*c*R^2/6-pi*c*x(1)^2/12)*x(2)^2+d*x(2)+phi)*sign(x(2)))/
alpha*(x(1)==a)+4.90946*(x(2)>0&&x(1)==a);
      x(4);
      -g*(x(3)>a)
+(-beta*g+1/2*k*x(3)^2-((pi*c*R^2/6-pi*c*x(3)^2/12)*x(4)^2+d*x(4)+phi)*sign(x(4)))/
beta*(x(3)==a)+4.8695*(x(4)>0&&x(3)==a);
      x(6);
      -g*(x(5)>a)
+(-gamma*g+1/2*k*x(5)^2-((pi*c*R^2/6-pi*c*x(5)^2/12)*x(6)^2+d*x(6)+phi)*sign(x(6)))/
gamma*(x(5)==a)+5.8700*(x(6)>0&&x(5)==a)
      ];
end
```

Figure 14: Adjusted model