A Method for Quickly Estimating the Trajectory of Spherical Projectiles

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November 9, 2025

Abstract

This study addresses the precise strike problem of a 5-kg spherical projectile with an 11-cm diameter, focusing on the hit requirement for a 1200-meter horizontal distance when both the artillery and the target are located at an altitude of 500 meters. A ballistic calculation model incorporating air resistance, gravity, and environmental interference is established to provide practical technical solutions for artillery operations under varying environmental conditions within the 1000-1500 meter range.

The research first clarifies the model assumptions and parameter system. Based on engineering practice, a piecewise empirical formula for the drag coefficient is adopted. Combined with Newton's second law and fluid mechanics principles, the motion equations of the projectile in a three-dimensional coordinate system are derived. An exponential model for air density varying with altitude and a calculation formula for the frontal area are introduced, and the Euler method with a time step of 0.01 seconds is used to achieve discrete numerical solution of the ballistics.

In the baseline scenario without wind influence, a trajectory search algorithm with a step size of 0.1° for launch angle and 1 m/s for initial velocity is employed to screen multiple sets of optimal launch parameters, all of which can achieve precise hits on the 1200-meter target and obtain key indicators such as projectile flight time. To address environmental variable interference, the research is further extended to a three-dimensional scenario. The effects of wind speed and direction along the x/y/z axes, as well as changes in the target's horizontal distance and altitude on the ballistics, are quantitatively analyzed to form clear gun barrel angle correction rules.

Model evaluation shows that this method balances computational efficiency and reliability in medium-range ballistic estimation under standard atmospheric conditions, and can provide decision support for rapid artillery deployment. However, it has limitations such as the static wind assumption, neglect of the Coriolis force, and cumulative errors of the Euler method, making it suitable for non-extreme environments and scenarios that do not require ultra-high-precision correction. Through the combination of theoretical derivation and numerical simulation, the research results provide a complete technical framework for trajectory prediction and parameter optimization of artillery spherical projectiles, and have important reference value for military operations, engineering simulation, and projectile design fields.

Keywords: Trajectory Estimation Spherical Projectile Drag Force Euler Method Launch Parameter Optimization

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1 Problem Analysis

1.1 Introduction

The shooting problem involves complex and diverse ballistic challenges, where the hit accuracy of artillery shells is not solely determined by technical parameters but largely depends on the precise calibration of various launch parameters. These parameters include muzzle velocity, launch angle, projectile spin rate, and propellant charge quality—each interacting dynamically to govern the projectile's flight trajectory from the barrel to the target. In real-world combat scenarios, achieving reliable and precise target engagement requires rigorous consideration of numerous environmental and battlefield variables, which introduce additional uncertainties into the ballistic model. Atmospheric conditions stand out as a critical influencing factor: wind speed and direction (both horizontal and vertical) exert continuous aerodynamic forces on the projectile, altering its flight path and terminal velocity. Meanwhile, air density—varying with altitude, temperature, and humidity—directly affects the projectile's air drag coefficient. This effect is particularly pronounced for supersonic projectiles, where air resistance can drastically reshape trajectory characteristics. Against this backdrop, this study proposes a comprehensive and adaptive method for optimizing and determining shooting plans, which integrates theoretical ballistic principles with the practical constraints of modern battlefields. It incorporates empirical findings from ballistic research—including the influence of altitude on trajectory correction and rationalized empirical formulas for supersonic air resistance—to enhance model accuracy.

1.2 Problem Analysis

The artillery system fires a 5-kg spherical projectile with 11-cm diameter, achieving muzzle velocities up to 450 m/s. The primary challenge involves hitting a target at 1200-meter horizontal distance while both cannon and target are positioned at 500-meter altitude. The solution must account for spherical projectile ballistics and develop practical methods for engaging targets at 1000-1500 meter ranges under varying environmental conditions.

2 Model

2.1 Model Assumptions

- 1. Resistance is proportional to the square of speed
- 2. Use the Euler method for ballistic numerical calculations, with a time step Δ t = 0.01s
- 3. The surface of the shell is smooth, and the shell does not rotate after being fired, disregarding the Magnus effect[1][3]
- 4. The temperature is standard atmospheric temperature
- 5. The air is dry, and humidity has no effect
- 6. The observer can accurately report wind speed and direction
- 7. The Coriolis force (effect of Earth's rotation) can be neglected within the range
- 8. The combustion process of gunpowder is instantaneous, and the shell reaches the designated initial velocity immediately after leaving the muzzle
- 9. The wind speed and direction will not change during the injection process
- 10. Air density varies with altitude
- 11. The buoyancy of air is much smaller than gravity, and its impact can be ignored[5]

2.2 Parameter Setting

Table 1: Symbols and Physical Meanings in Projectile Simulation

Symbol	Physical Meaning	Units
T	Total simulation time	s
Δt	Simulation time step	s
M	Projectile mass	kg
Ma	Speed	mach
D	Projectile diameter	m
A	Frontal area	m^2
ρ	Air density	kg/m^3
θ	Launch angle	°
V	Velocity vector	m/s
v	Speed magnitude	m/s
C_d	Drag coefficient	Dimensionless
k	Drag acceleration coefficient	1/m
$\vec{a}_f$	Air resistance acceleration	m/s^2
$\vec{a}_g$	Gravitational acceleration	m/s^2
a	Total acceleration	m/s^2
v_r	real velocity	m/s

2.3 Formula Derivation

The empirical formula for air resistance segmentation:[4]

$$C_d(Ma) = \begin{cases} 0.47 & Ma \leq 0.3 \\ 0.47 - 0.20(Ma - 0.3) & 0.3 < Ma \leq 0.8 \\ \frac{1.22}{1+0.35(Ma-1)^2} & 0.8 < Ma \leq 2.5 \\ 0.95 & Ma > 2.5 \end{cases} \tag{1}$$

It is a simplified fit for the variation of air resistance coefficient of objects (especially blunt objects, such as spheres and non-streamlined objects) in the transonic and supersonic ranges in engineering
Air resistance arises from the transfer of momentum from the projectile to the air during its flight:[6]

$$F_d = \frac{1}{2} \times \rho \times v^2 \times C_d \times A \tag{2}$$

Air density:

$$\rho = 1.225 \cdot e^{-h/8500} \tag{3}$$

Frontal area:

$$A = \pi \times \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \tag{4}$$

2.3.1 Question 1's Formula Derivation

According to Newton's second law, force equals the product of an object's mass and its acceleration. Therefore, the acceleration due to air resistance is equal to the ratio of air resistance to the mass of the iron ball
To simplify the equation, the constant terms unrelated to velocity are combined into a single parameter, denoted as k. The k coefficient encompasses all inherent properties of the projectile (mass, size, shape) and environmental factors (air density), determining the sensitivity of drag to velocity.
Drag coefficient formula:

$$k = \frac{1}{2} \cdot C_d \cdot \rho \cdot \frac{S}{M} \tag{5}$$

Gravitational acceleration:

$$\vec{a}_g = [0, -9.8]\mathrm{m/s}^2 \tag{6}$$

The direction of the acceleration due to drag is opposite to that of the velocity vector. In a two-dimensional coordinate system, we decompose the velocity and drag into the x and y directions, and then obtain the air resistance acceleration
Air resistance acceleration:[2]

$$\vec{a}_f = -k \cdot [v_x^2, v_y^2] \tag{7}$$

Total acceleration:

$$\vec{a} = \vec{a}_g + \vec{a}_f \tag{8}$$

We employ Euler's Method for numerical integration, using discrete time steps use Δt to approximate continuous time
Core motion equation (discrete form):
Speed update equation:

$$\vec{v}(t + \Delta t) = \vec{v}(t) + \vec{a}(t) \cdot \Delta t \tag{9}$$

Position update equation:

$$\vec{r}(t + \Delta t) = \vec{r}(t) + \vec{v}(t) \cdot \Delta t \tag{10}$$

Termination and target conditions:
Landing conditions:

$$y(t) < 0 \tag{11}$$

Target hit condition:

$$|x(t) - 1200| < 0.5\mathrm{m} \quad \text{and} \quad y(t) < 0 \tag{12}$$

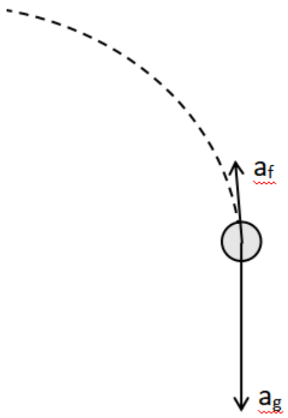

Figure 1: Analysis diagram of object acceleration

2.3.2 Question 2's Formula Derivation

We intend to calculate the necessary angular adjustments to counteract the impacts of distance variation, altitude difference variation, and wind effects along the x, y, and z axes, respectively. Subsequently, we will integrate these adjustments to determine the overall angular modification required under the combined influence of wind and distance variation.
Most of the formulas are the same as those in the first question, with the following adjustments:
X-axis acceleration:
Establish a Cartesian coordinate system with the muzzle as the origin, the horizontal direction of the target as the positive direction of the x-axis, the vertical upward direction as the positive direction of the y-axis, and the right direction of the muzzle as the positive direction of the z-axis

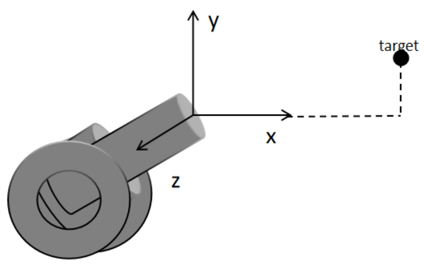

Figure 2: Cartesian coordinate system

$$a_x = \frac{dv_x}{dt} = -\frac{C_d \rho A}{2M} v_r \cdot (v_x - \omega_x) \tag{13}$$

Y-axis acceleration:

$$a_y = \frac{dv_y}{dt} = -g - \frac{C_d \rho A}{2m} v_r (v_y - \omega_y) \tag{14}$$

Z-axis acceleration:

$$a_z = \frac{dv_z}{dt} = -\frac{C_d \rho A}{2M} v_r (v_z - \omega_z) \tag{15}$$

Real velocity:

$$v_r = \sqrt{(v_x - \omega_x)^2 + (v_y - \omega_y)^2 + (v_z - \omega_z)^2} \tag{16}$$

Total acceleration:

$$\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z \tag{17}$$

To eliminate the influence of the aforementioned wind, we can make the following adjustments to the shell's muzzle angle:

1 wind in the X-axis direction:
By adjusting (increasing or decreasing) the angle of the cannonball's velocity along the x-axis, the influence of the wind component in the x-direction can be eliminated

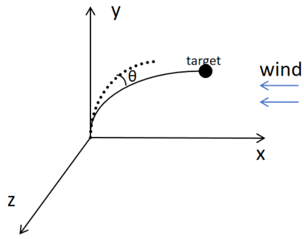

Figure 3: the elimination of X-axis wind

2 wind in the Y-axis direction:
By adjusting (increasing or decreasing) the angle of the shell's velocity on the y-axis, the influence of the y-axis component of wind can be eliminated

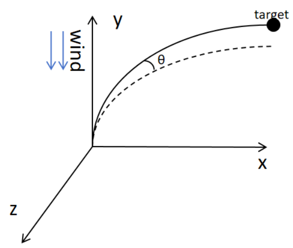

Figure 4: the elimination of Y-axis wind

3 wind in the Z-axis direction:
By adjusting (increasing or decreasing) the angle of the shell's velocity along the z-axis, the influence of the wind component in the z-axis direction can be eliminated

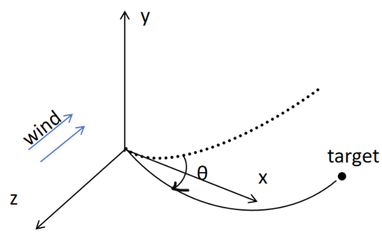

Figure 5: the elimination of Y-axis wind

4 Change in altitude difference:
By adjusting (increasing or decreasing) the angle of the shell velocity on the y-axis, the impact of altitude changes can be eliminated

5 Change in horizontal distance:
By adjusting (increasing or decreasing) the angle of the shell velocity on the y-axis, the impact of changes in horizontal distance can be eliminated

3 Results

3.1 Task 1: Solution for precise artillery shelling under no wind influence

Based on the aforementioned formula, we have summarized the artillery trajectory search algorithm. By traversing all possible combinations of initial velocity and angle with a step size of 0.1° and 1m/s, we ultimately identify the parameter solution that can accurately hit a target at a distance of 1200 meters.

Illustration of the solution obtained by traversing with a step size of 0.1° and 1m/s:

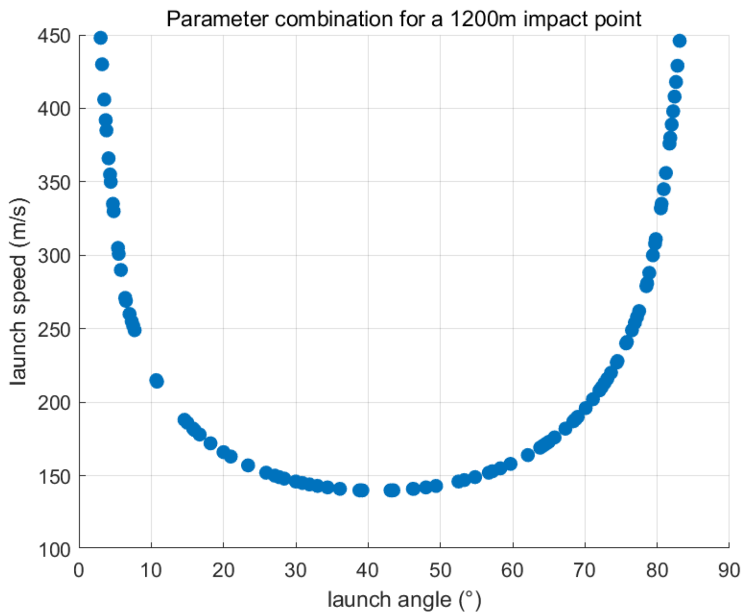

Figure 6: Diagram of relationship between launch angle and speed

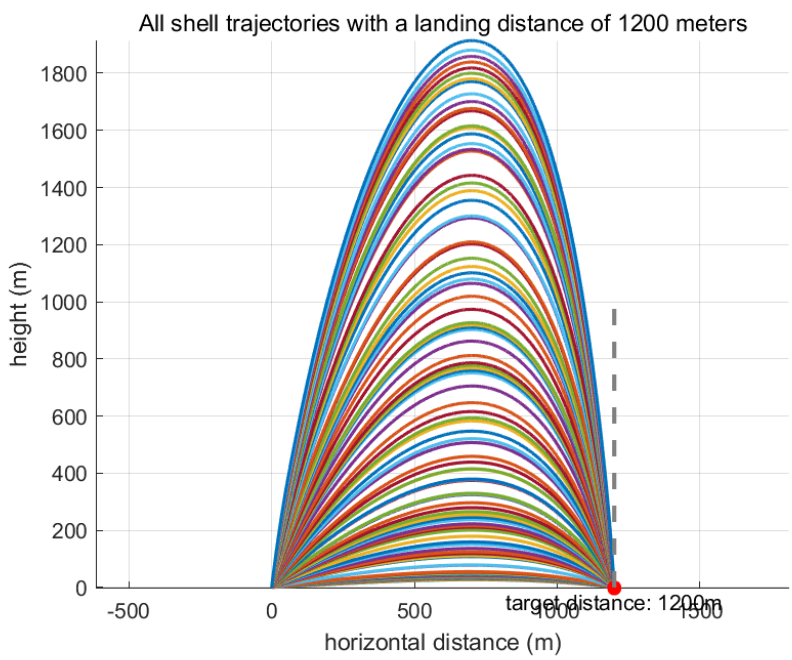

Figure 7: All trajectory images with landing points at a distance of 1200m

Due to the inability to display all results obtained with a step size of 0.1°, the results obtained with a step size of 1° are presented in the table below:

Table 2: Ballistic parameters

Launch angle(°)	initial velocity(m/s)	landing distance(m)	arrival time(s)
3	448	1200.28	4.69
7	260	1199.91	6.36
15	186	1200.47	9.43
16	181	1199.73	9.75
20	166	1200.31	10.95
21	163	1199.72	11.23
30	146	1200.50	13.61
33	143	1199.73	14.35
39	140	1199.99	15.84
48	142	1199.82	18.14
65	173	1199.81	23.50
69	190	1199.67	25.18
72	208	1200.39	26.64
82	389	1200.38	32.34

3.2 Task 2:Solution for precise artillery shelling under complex environment

Due to the inability to present all calculation results, we have taken a single speed value(v=300m/s) for illustrative purposes. In reality, calculations need to be performed for each speed, ultimately resulting in a comparison table that includes wind direction, wind speed, shooting speed, shooting angle, distance, and altitude difference. This table is used to quickly calculate the conditions required to hit a specific point.

During the solving process, we found that for a given point, there would be a solution with a high emission angle and a solution with a low emission angle. Therefore, in the following process, we divided it into two cases: high emission angle and low emission angle.

As the X-axis wind increases or decreases, in order to hit the designated target, the elevation angle of the gun barrel (i.e., the angle at which the shell is fired) needs to be adjusted as shown in the figure: For every 1 m/s increase in the x-axis component of the wind, the gun barrel is raised by 0.319° at high elevation angles and lowered by 0.0121° at low elevation angles

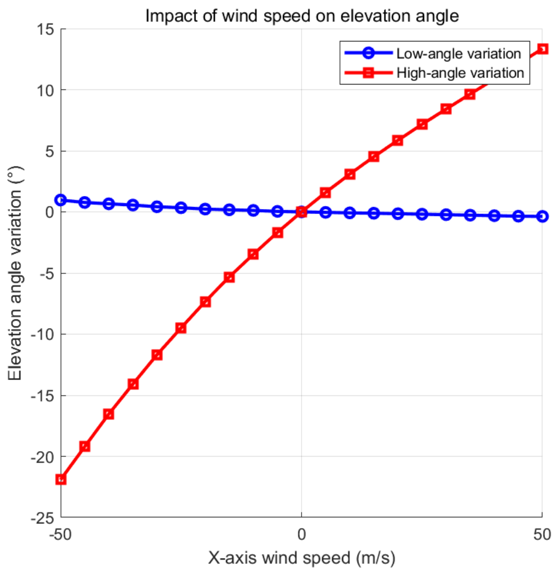

Figure 8: Impact of X-asis wind speed on elevation angle

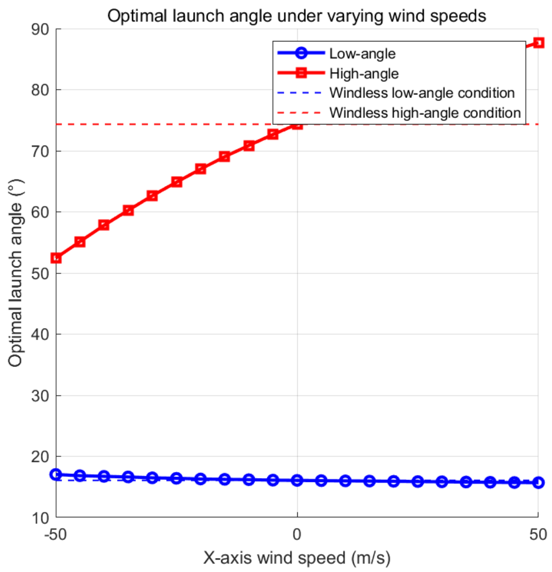

Figure 9: Optimal launch angle under varying X-axis wind speeds

As the Y-axis wind increases or decreases, in order to hit the designated target, the elevation angle of the gun barrel (i.e., the angle at which the shell is fired) needs to be adjusted as shown in the figure: For every 1 m/s increase in the y-axis component of the wind, the gun barrel is lowered by 0.095° at high elevation angles and raised by 0.0854° at low elevation angles

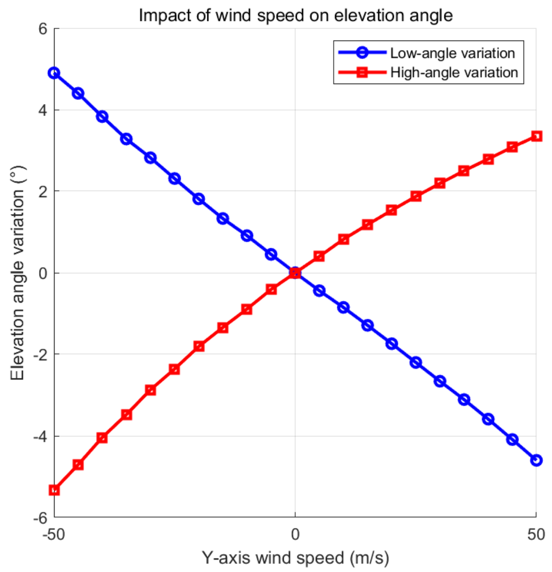

Figure 10: Impact of Y-axis wind speed on elevation angle

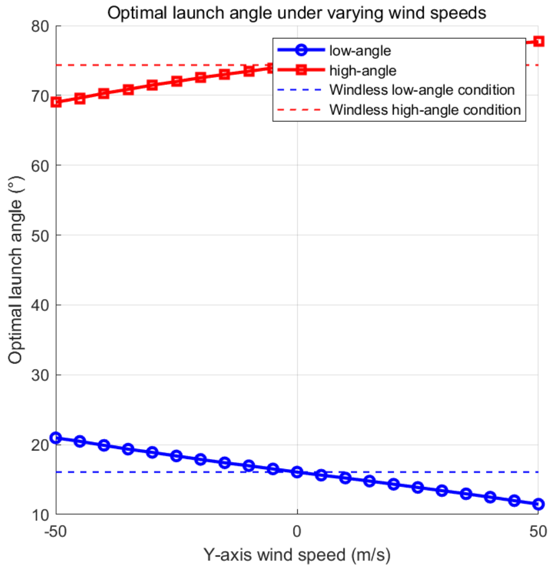

Figure 11: Optimal launch angle under varying Y-axis wind speeds

As the Z-axis wind increases or decreases, in order to hit the designated target, the elevation angle of the gun barrel (i.e., the angle at which the shell is fired) needs to be adjusted as shown in the figure: For every 1 m/s increase in the z-axis component of the wind, the gun barrel is raised by 0.101° at high elevation angles and lowered by 0.7928° at low elevation angles

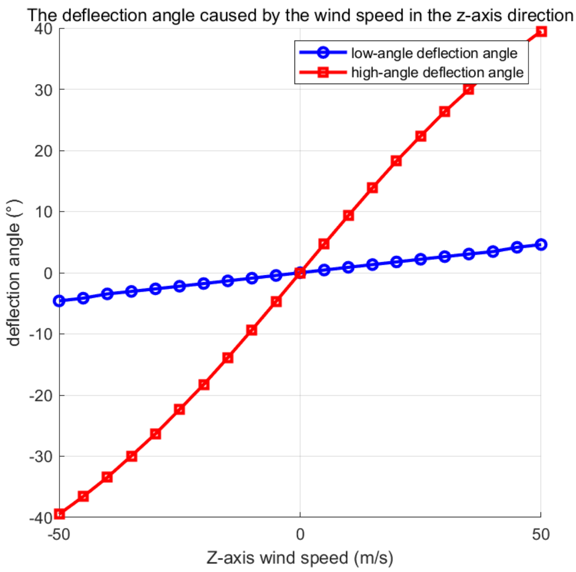

Figure 12: The deflection angle caused by the wind speed in the z-axis direction

As the Distance increases or decreases, in order to hit the designated target, the elevation angle of the gun barrel (i.e., the angle at which the shell is fired) needs to be adjusted as shown in the figure: For every 1m increase in target altitude, the gun barrel is raised by 0.0521° at high elevation angles and lowered by 0.0023° at low elevation angles

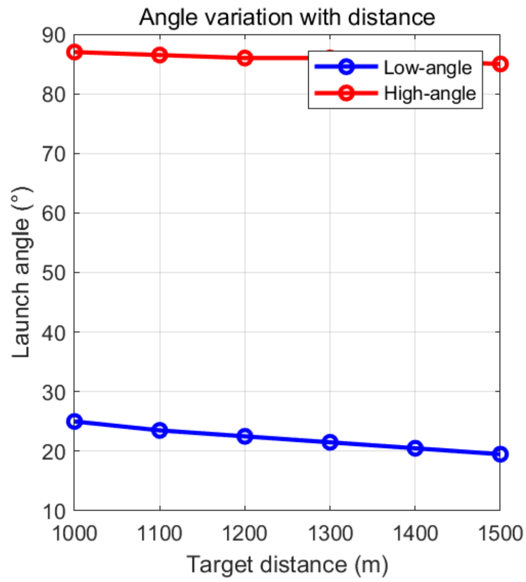

Figure 13: Angle variation with distance

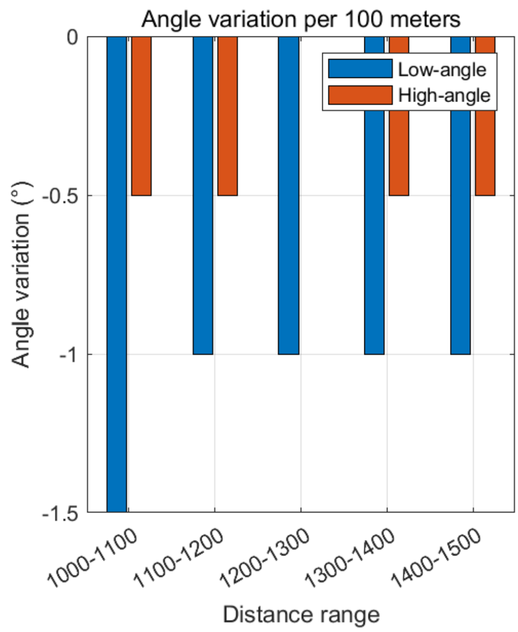

Figure 14: Angle variation per 100 meters

As the Altitude increases or decreases, in order to hit the designated target, the elevation angle of the gun barrel (i.e., the angle at which the shell is fired) needs to be adjusted as shown in the figure: For every 1m increase in target altitude, the gun barrel is raised by 0.0521° at high elevation angles and lowered by 0.0023° at low elevation angles

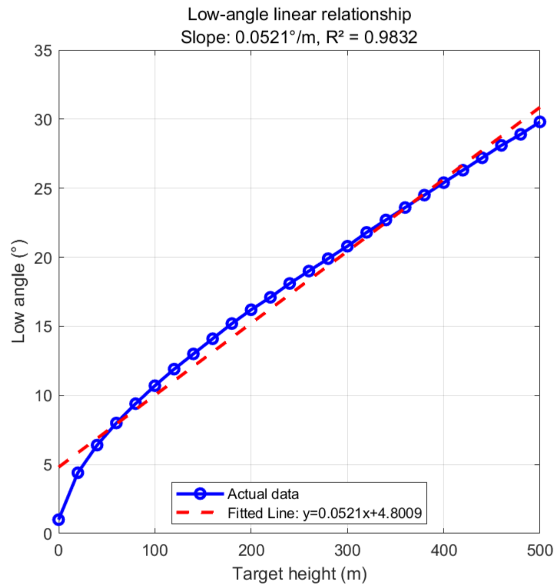

Figure 15: Low-angle linear relationship

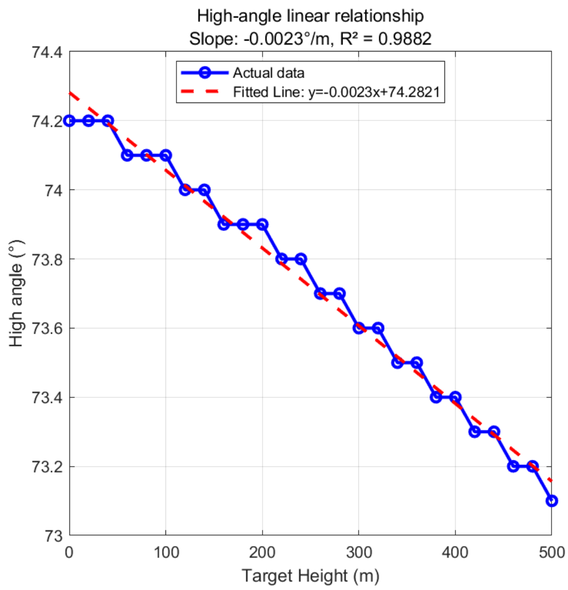

Figure 16: High-angle linear relationship

In Summmary:

1 For every 1 m/s increase in the x-axis component of the wind, the gun barrel is raised by 0.319° at high elevation angles and lowered by 0.0121° at low elevation angles

2 For every 1 m/s increase in the y-axis component of the wind, the gun barrel is lowered by 0.095° at high elevation angles and raised by 0.0854° at low elevation angles

3 For every 1 m/s increase in the z-axis component of the wind, the gun barrel is raised by 0.101° at high elevation angles and lowered by 0.7928° at low elevation angles

4 For every 1m increase in target altitude, the gun barrel is raised by 0.0521° at high elevation angles and lowered by 0.0023° at low elevation angles

5 For every 1m increase in target altitude, the gun barrel is raised by 0.0521° at high elevation angles and lowered by 0.0023° at low elevation angles

4 Model Validation

Model Validation Under the premise of an initial velocity of 300m/s, a certain angle was randomly selected for simulation, and it was calculated that the impact point of a shell with a firing angle of 73.5 ° was approximately (995.54, 1.97), at the same altitude as the gun. It is now known that the target to be hit is 100m above it and 1095.54m away from the gun, with wind speeds of (-10, -5, 10). Based on the linear relationship proved in the previous text, taking into account the height difference, distance, and wind speed factors, the calculated firing angle should be 71.5 °. As shown in the figure below, the actual strike coordinate is (1110.98100.28), with an error of about 5m. Taking into account factors such as rough estimation on the battlefield, the error is within an acceptable range. The main reason for the error is the linear analysis of the influence of various factors on the angle, and the correlation coefficient of the fitted curve cannot be greater than 0.98, especially for the distance factor

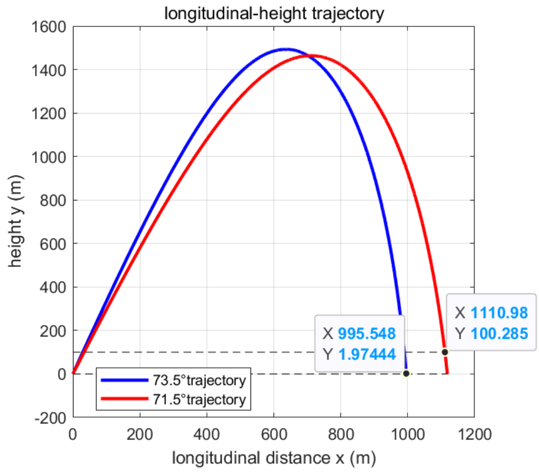

Figure 17: Low-angle linear relationship

5 Model Improvement

5.1 Refine Aerodynamic Model

Optimize the drag coefficient C_d calculation by incorporating more precise transonic and supersonic fitting data, replacing the segmented empirical formula with a continuous function to reduce numerical errors at segment boundaries. Introduce the Magnus effect model to account for projectile rotation (even with smooth surfaces, minor rotation may occur), which helps improve trajectory accuracy for long-range shots (1000–1500 meters).

5.2 Refine Aerodynamic Model

Add a time-varying wind field model to simulate dynamic wind speed and direction changes, instead of the static wind assumption, making the simulation closer to actual battlefield conditions. Improve the air density formula by integrating temperature and humidity corrections, replacing the single exponential decay model with a comprehensive atmospheric parameter model that considers multi-environmental variables.

5.3 Upgrade Numerical Calculation Method

Replace the Euler method with a higher-precision numerical integration method (e.g., Runge-Kutta method of order 4) to reduce cumulative errors from discrete time steps, especially for high-velocity projectiles with rapid acceleration changes. Optimize the trajectory search algorithm by adopting adaptive step sizes for launch angle and initial velocity (instead of fixed 0.1° and 1m/s), improving solution efficiency while maintaining accuracy.

5.4 Supplement Secondary Physical Effects

Incorporate the Coriolis force model for long-range engagements (exceeding 1500 meters) to address the Earth's rotation impact on trajectory deviation. Add a rolling resistance model for the projectile's initial flight phase, considering slight surface roughness and air viscosity effects to refine the initial acceleration calculation.

6 Model Evaluation

6.1 Strengths

The segmented drag coefficient model aligns with engineering practice for blunt objects (spherical projectiles), ensuring reliable aerodynamic force calculation in different Mach number ranges. Comprehensive parameter setting and clear formula derivation cover key physical factors (air resistance, gravity, wind effect), forming a complete 2D-to-3D trajectory calculation system. Rigorous numerical simulation with trajectory search and wind/altitude/distance sensitivity analysis provides practical guidance for actual artillery operations. The Euler method-based discrete motion equations balance computational efficiency and accuracy, suitable for rapid trajectory estimation scenarios.

6.2 Weakness

Simplified assumptions (static wind, no rotation, neglecting Coriolis force) limit the model's applicability in complex environmental and long-range engagement scenarios. The fixed time step Euler method leads to inevitable cumulative errors, which may affect trajectory accuracy for high-speed projectiles with large acceleration variations. The trajectory search algorithm uses fixed step sizes, resulting in redundant calculations and lower efficiency when searching for optimal launch parameters. Lack of experimental validation data; the model's accuracy relies solely on theoretical derivation and simulation, without comparison with actual firing test results.

6.3 Applicability Scope

The model is highly applicable for medium-range (1000–1500 meters) spherical projectile trajectory estimation under standard atmospheric conditions (stable wind, normal temperature and humidity). It provides effective decision support for rapid artillery deployment and target engagement in scenarios where high-precision trajectory correction is not required. It is not suitable for extreme environments (strong dynamic winds, extreme temperatures) or long-range (exceeding 1500 meters) engagements that require consideration of secondary physical effects.

7 Conclusion

This study focuses on the rapid trajectory estimation of spherical projectiles, aiming to solve the practical problem of precise target engagement under different environmental conditions. Through in-depth theoretical derivation, systematic numerical simulation, and comprehensive sensitivity analysis, the research achieves the expected objectives and forms a complete technical method system. In terms of model construction, the study establishes a set of trajectory calculation models covering both two-dimensional and three-dimensional scenarios. By adopting a segmented drag coefficient formula suitable for spherical blunt objects, combining Newton's second law and fluid dynamics principles, the model accurately characterizes the key physical effects such as air resistance, gravity, and wind interference during projectile flight. The parameter setting is comprehensive and reasonable, and the formula derivation logic is rigorous, laying a solid theoretical foundation for subsequent trajectory simulation. In terms of simulation results, the research successfully identifies multiple groups of optimal launch parameter combinations (launch angle and initial velocity) that can accurately hit the 1200-meter target under no-wind conditions. For the influence of environmental variables (wind speed in x/y/z axes) and target parameter changes (horizontal distance, altitude), the study quantifies the required muzzle angle adjustment rules and reveals the different response characteristics of high and low launch angles to external changes. These results provide direct and practical operational guidance for actual artillery deployment and target engagement. In terms of technical performance, the model balances computational efficiency and result reliability through the Euler method-based discrete numerical calculation. It can quickly complete trajectory estimation and parameter search, meeting the demand for rapid decision-making in battlefield scenarios. Meanwhile, the research clearly defines the model's strengths, weaknesses, and applicability scope, providing a clear reference for users to select appropriate technical tools according to actual needs. Although the current model achieves good results in medium-range (1000–1500 meters) trajectory estimation under standard atmospheric conditions, there is still room for improvement in adapting to complex environments and enhancing calculation accuracy. Future research can further optimize the model by refining the aerodynamic model, upgrading numerical calculation methods, and supplementing secondary physical effects as proposed in the model improvement section. Additionally, combining actual firing test data to conduct experimental validation will help improve the model's engineering application value. Overall, the trajectory estimation method proposed in this study effectively integrates theoretical ballistics and practical constraints, providing a reliable technical solution for precise artillery shelling. The research results have important reference significance for related fields such as military operations, engineering simulation, and projectile design.

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